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INVESTIGATION OF NONLINEAR DYNAMICS OF ELECTRON BEAM INSTABILITY IN THREE-DIMENSIONAL PERIODICAL STRUCTURES

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Abstract
Nonlinear phenomena originating in interaction of relativistic electron beams with three-dimensional periodical structures in Volume Free Electron Lasers (VFEL) are investigated by methods of mathematical modelling. In particular, it is considered chaotic dynamics of such interaction in the space of the following control parameters: length of interacting zone and electron beam current density. Bifurcation diagrams and two-parametric maps show onset of oscillations and destruction of different chaotic regimes under bifurcational parameters changing. Such regimes are periodic, quasiperiodic and chaotic self-oscillations with standard routes to chaos such as period doubling, Hopf bifurcations and intermittency as well as transitions between large-scale and small-scale amplitude regimes. Study of chaotic nature of electron beam instability in three-dimensional periodical structures will be useful for providing experimental investigations.

Key words
Chaos, nonlinear dynamics, electron beam instability

1 Introduction

Nonlinear oscillations meet not only in mechanics but also in chemistry, biology, society just as in electrical and radio engineering. It is well-known nonlinear oscillations in vacuum and plasma electronics, microwave electronics, conventional lasers, free electron lasers (FEL) etc.

The main principle of microwave tubes [Vainshtein and Solncev, 1973] and FEL [Roberson and Sprangle, 1989] is based on radiation of bunches of charged particles which are not bounded in atoms. Such devices should be considered consisting of coupled nonlinear oscillators. Under conditions of synchronism they begin to transform the kinetic energy of charged particles to energy of electromagnetic radiation. An elementary oscillator here is a single electron or an electron bunch, moving uniformly in the slow-wave structure.

In considered types of devices an interaction of moving forward electrons in the bunch with a stream of energy of electromagnetic waves occurs under conditions of distributed feedback. In conventional microwave tubes and lasers such interaction is as a rule one-dimensional. I. e. electrons in a bunch and electromagnetic waves spread along one straight line: in one direction or in opposite directions.

In unmodulated system individual radiations of separate electrons destroy one another and electromagnetic fields are not exited. In the modulated system at presence of an external electromagnetic wave radiation of electron bunches combines in a phase as a result of long interaction of moving electrons with a field and raises a strong electromagnetic wave. Such type of devices is an amplifier. Other type of electronic devices is a generator where distributed feedback provides generation of electromagnetic radiation by electrons without an external wave.

It is well-known [Rabinovich and Trubetskov, 1992] that exceeding by an electron beam current of some threshold value leads to self-oscillating regime. Then at excess in 3–4 times of the threshold current value the regime of self-modulation with periodic sequence of pulses is realized. Further increasing of beam current can lead to chaotic generation with non-recurring in form and duration pulses. All this is caused by non-uniform distribution of intensity of electromagnetic field and field of spatial charge of electron beam that leads to significant perturbations in movement of electrons through resonator. This results to reduction of average velocity of electrons and to deformation of bunches during overtaking some electrons by other ones, to stratification of electron beam etc. The higher harmonics of these fields and their combinations are excited and raised. This leads to chaos in the system.

Investigation of origin of chaos in electronic generators and amplifiers is very topical in modern physics

[Bruni et al., 2004]–[Pae and Hahn, 2003]. In [Bruni et al., 2004] theoretical and experimental investigations of the super-ACO FEL have been made to observe clear bifurcation and chaos sequences in the responce to a detuning modulation that is a changing of the synchronism between electron beam and optical pulse. In [Kuznetsov and Trubetskov, 2004] different regimes of so-called ”weak” chaos and ”hyperchaos” or self-oscillations were investigated for backward-wave tube (BWT). In [Kuznetsov, 2006] it was investigated its nonlinear dynamics in the presence of energy dissipation at wave transmission, field of space charge, wave reflection at system edges. It was depicted main principles of chaos control in BWT via supression of self-modulation. Paper [Ginzburg et al., 2002] is devoted to experimental investigation of chaotic nature of powerful BWT. In [Antoniazzi et al., 2005] the chaotic sea model for FEL dynamics is considered for investigation of phase space portraits of evolution of radiation intensity at different times. Experimental and theoretical investigation of generation of chaotic radiation in a driven travelling wave tube amplifier with time-delayed feedback is a topic of the paper [Marchewka et al., 2006]. Here the feasibility of chaotic communications using two tubes is demonstrated. Three main routes to chaos for nonlinear systems such as FEL and driven plasma diodes under control parameter changing were investigated [Lee et al., 1998]. There are period doubling, quasiperiodicity and intermittency. In [Pae and Hahn, 2003] besides these traditional routes to chaos it was observed competing multistability in plasma diodes. This phenomenon consists in coexistence of several dynamical states with the same initial conditions. It is observed in the very narrow region of parameter space.

So, investigation of chaos in electronic devices is of great interest in modern physics.

2 Physical principles of interaction of relativistic electron beam with periodical structures

We consider interaction of relativistic electron beam with three-dimensional periodical structures in Volume Free Electron Lasers (VFEL). Its basic difference from the above described electronic devices is the availability of volume (non-one-dimensional) distributed feedback (VDFB) when an electron beam and some strong coupled electromagnetic waves spread angularly one to other in the system.

The principles and theoretical foundations of VFEL operation based on mechanism of multi-wave VDFB were proposed in [Baryshevsky and Feranchuk, 1984] and [Baryshevsky, 1988]. There it was shown that the increment of instability for an electron beam passing through a spatially-periodic target in degeneration points essentially increased in comparison with the single-wave system. This means the noticeable reduction of electron beam current density necessary for achievement the generation threshold. This law is universal and valid for all wavelength ranges regardless the spontaneous radiation mechanism. First lasing of VFEL in mm wavelength range obtained recently [Baryshevsky et al., 2002] put the beginning of experimental developing of new type of electronic generators.

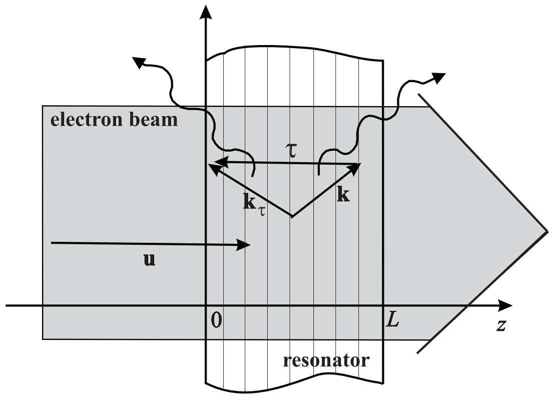

Figure 1. Scheme of two-wave VFEL

Common scheme of VFEL in two-wave geometry is depicted in Fig.1. Here an electron beam with electron velocity $\mathbf{u}$ and current density j_0 passes through a spatially periodic resonator of the length L. Under diffraction conditions strong electromagnetic waves are excited in the resonator. If simultaneously electrons of the beam are under synchronism condition, they emit electromagnetic radiation in directions depending on diffraction conditions.

We met with oscillations and chaos during our VFEL investigation. Here reasons of initiation of chaotic dynamics remain the same as in other electronic devices: significant perturbation in movement of electrons and deformation of bunches leading to generation of higher harmonics in the system and vice versa.

There exists a whole spectrum of external operating (bifurcational) parameters in the system. Their changing leads to qualitative changing of its behavior. Such parameters are electron beam current, length of the resonator, geometry parameters, factors of asymmetry etc. Investigation of chaos is important in the light of experimental development of VFEL. In [Batrakov and Sytova, MMA, 2005]–[Sytova, 2007] a gallery of different chaotic regimes for VFEL laser intensity with corresponding phase space portraits, attractors and Poincaré maps was proposed. There are periodic, quasiperiodic regimes and chaotic self-oscillations. Bifurcation points corresponding to transitions between different regimes of generation were investigated. There were investigated the largest Lyapunov exponents and sensibility of the system behavior to initial conditions. It was analyzed dependence of chaotic lasing on the following couples of parameters: electron beam current and factor of asymmetry, beam current and detuning from exact synchronism condition.

The main goal of the paper presented is the further investigation of different aspects of chaotic nature of interaction of relativistic electron beams with periodical structures.

3 Mathematical model and computer simulation

In interaction of electron beams with periodical structures the linear stage investigated analytically [Baryshevsky, Batrakov and Dubovskaya 1991] quickly changes into the nonlinear one where most of the electron beam energy is transformed into electromagnetic radiation. System of equations describing such interaction is obtained from Maxwell equations in the slowly-varying envelope approximation. Electromagnetic field is represented in the common case in the form:

$$\mathbf{E}=\sum \mathbf{e}_j E_j \exp\left\{i(\mathbf{k}_j\mathbf{r}-\omega t)\right\}, \quad j=1,...,n,$$

where n is the number of waves generated in the system with wave vectors $\mathbf{k}_j$ and frequency ω. $\mathbf{e}_j$ are vectors of polarization. i is the imaginary unit.

Let us consider two-wave geometry that is depicted in Fig.1. Electric field strength $\mathbf{E}$ and electron beam current density $\mathbf{j}$ are considered in the form:

$$\begin{aligned}&\mathbf{E}=\mathbf{e}_\sigma(E\exp\left\{i(\mathbf{kr}-\omega t)\right\}+\\&E_\tau\exp\left\{i(\mathbf{k}_\tau\mathbf{r}-\omega t)\right\}),\\&\mathbf{j}=\mathbf{e}_\sigma j\exp\left\{i(\mathbf{kr}-\omega t)\right\}.\end{aligned} \tag{1}$$

Here $\mathbf{e}_\sigma$ is a vector of sigma polarization [Chang, 1984]. τ is the reciprocal lattice vector. $\mathbf{k}$, $\mathbf{k}_\tau=\mathbf{k}+\tau$ are wave vectors of two strong coupled electromagnetic waves, that are under diffraction conditions $2k_z\tau_z\approx-2\mathbf{k}_\perp\tau_\perp+\tau^2$. Electromagnetic wave with vector $\mathbf{k}$ is transmitted and one with vector $\mathbf{k}_\tau$ is diffracted. E and E_τ are complex-valued amplitudes of these waves.

It was proposed here that the electron beam is synchronous with the transmitted wave E only. This means the following synchronism condition is fulfilled: $|\omega-\mathbf{ku}|\le\delta\omega$.

From two first Maxwell equations: we obtain one differential equation

$$\triangle\mathbf{E}-\nabla(\nabla\mathbf{E})-\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial\mathbf{j}}{\partial t}. \tag{2}$$

Let us take into account that the electric displacement vector $\mathbf{D}$ can be presented in the following way [Chang, 1984]:

$$\mathbf{D}(\mathbf{r},t)\approx\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},t),$$

where $\varepsilon(\mathbf{r},\omega)=\sum_\tau\varepsilon(\tau,\omega)\exp(-i\tau\mathbf{r})$ is a the resonator dielectric susceptibility. $\varepsilon(0,\omega)=1+\chi_0$, $\varepsilon(\tau,\omega)=\chi_{\pm\tau}$, $\varepsilon(-\tau,\omega)=\chi_{-\pm\tau}$. χ_0, $\chi_{\pm\tau}$ are the zero and $\pm\tau$ Fourier components of the resonator dielectric susceptibility.

Then using an expansion in series of the reciprocal lattice vector τ we use the following expression for the vector $\mathbf{D}$

$$\begin{aligned}&\mathbf{D}=(1+\chi_0)\mathbf{E}+\chi_\tau\mathbf{e}_\sigma E\exp\{i(\mathbf{kr}-\omega t)\}+\\&\chi_{-\tau}\mathbf{e}_\sigma E_\tau\exp\{i(\mathbf{k}_\tau\mathbf{r}-\omega t)\}\\&.\end{aligned} \tag{3}$$

Slowly-varying envelope approximation means that

$$|\frac{1}{k}\frac{\partial E}{\partial\mathbf{r}}|<<|E|, \quad |\frac{1}{k}\frac{\partial E}{\partial t}|<<|E|,$$

where $k=\omega/c$. So, second derivatives with respect to time and space can be neglected.

Substituting (1), (3) into (2) and taking into account diffraction conditions and (4) we obtain the following system of equations:

$$\left.\begin{aligned}&\frac{\partial E}{\partial t}+\gamma_0 c\frac{\partial E}{\partial z}+0.5i\omega lE-0.5i\omega\chi_\tau E_\tau=\\&2\pi j_0\Phi\int_0^{2\pi}\frac{2\pi-p}{8\pi^2}\left(\exp(-i\Theta(t,z,p)+\right.\\&\left.\exp(-i\Theta(t,z,-p))\right)dp,\\&\frac{\partial E_\tau}{\partial t}+\gamma_1 c\frac{\partial E_\tau}{\partial z}+0.5i\omega\chi_{-\tau}E-\\&0.5i\omega l_1 E=0.\end{aligned}\right\} \tag{4}$$

Here $l_{0,1}=\left(k_{\cdot,\tau}^2c^2-\omega^2\varepsilon_0\right)/\omega^2$, $l=l_0+\delta$. δ is detuning from exact synchronism condition.

γ_0, γ_1 are distributed feedback cosines having form $\gamma_0=\frac{k_z}{k}$, $\gamma_1=\frac{k_{\tau z}}{k}$. $\Phi=\sqrt{l_0+\chi_0-1/(\beta\gamma)^2}$. γ is the Lorentz factor of electron beam.

Initial and boundary conditions for the system (4) depends on coupled waves directions and can have different form including external reflectors. Their simplest form is the following

$$\left.\begin{aligned}&E(t,z=0)=E_0, \quad E_\tau(t,z=L)=E_1,\\&E(t=0,z)=0, \quad E_\tau(t=0,z)=0.\end{aligned}\right\} \tag{5}$$

System (4)–(5) must be supplemented by equations of phase dynamics of electron beam.

$$\left.\begin{aligned}&\frac{d^2\Theta}{dz^2}=\frac{e\Phi}{m\gamma^3\omega^2}\left(k_z-\frac{d\Theta}{dz}\right)^3\cdot\\&\cdot\mathrm{Re}\left(E\exp(i\Theta)\right),\\&\frac{d\Theta(t,0,p)}{dz}=k_z-\omega/u, \quad \Theta(t,0,p)=p.\end{aligned}\right\} \tag{6}$$

In (4)–(6) $t > 0$, $z \in [0, L]$, $p \in [-2\pi, 2\pi]$.

The function $\Theta(t, z, p)$ describes the phase of electron beam relative to electromagnetic field. Let us explain derivation of (6) and right-hand side of (4).

We use the method of averaging over initial phases of electrons that is well-known [Vainshtein and Solncev, 1973] and widely used in simulation of electronic vacuum devices. We consider magnetized electron beam which propagation can be considered as one-dimensional. The motion equation of each relativistic electron in the wave has the next form:

$$\ddot{z} = \frac{e}{m\gamma^3}(\mathbf{e}_\sigma \mathbf{n}) Re\{E \exp(i\mathbf{k}_\perp \mathbf{r}_\perp + ik_z z - i\omega t)\},$$

where e and m are electron charge and mass respectively. $\mathbf{n}$ is a normal vector to the back plane of the resonator. Initial phase is an individual mark of the electron in beam. At moment t_0 it has the form

$$\Theta(t, t_0, \mathbf{r}_\perp) = k_z z + \mathbf{k}_\perp \mathbf{r}_\perp - \omega t(z, t_0).$$

$t(z, t_0)$ is a trajectory of electron emerged in the resonator. Initial phase of electron in interaction region at $z = 0$ has the form:

$$\Theta(t = t_0, t_0, \mathbf{r}_\perp) = \mathbf{k}_\perp \mathbf{r}_\perp - \omega t_0 = \Theta_1 - \Theta_0.$$

$\Theta_0, \Theta_1 \in [0, 2\pi]$. Equation (6) depends on these two initial phases only in combination $\mathbf{k}_\perp \mathbf{r}_\perp - \omega t_0$ (that appears in initial condition for phase at $z = 0$).

Averaging over this phase allows to pass from microscopical description to macroscopical one. Applying Liuville's Theorem leads to the following expression:

$$j \sim j_0 \int d\Theta_0 d\Theta_1 \exp\{-i\Theta(t, \Theta_1 - \Theta_0)\} \sim$$
$$j_0 \int_0^{2\pi} (2\pi - p)(\exp(-i\Theta(t, z, p)) +$$
$$\exp(-i\Theta(t, z, -p)))dp.$$

Numerical methods for solving (4)–(6) and its versions nonlinear stage simulation were proposed [Batrakov and Sytova, MMA, 2005], [Batrakov and Sytova, CMMP, 2005]. They are implemented in computer code VOLC [Batrakov and Sytova, 2006]. VOLC means "VOLume Code". Its dimensionality is 2D (one spatial coordinate and one phase space coordinate) plus time. VOLC (see Fig.2) allows for two-wave geometry to obtain different distributions of electromagnetic field intensities as well as dynamical regimes recognition and intensity Fourier transforms. Different VFEL geometries were investigated [Batrakov and Sytova, MMA, 2005]–[Sytova, 2007] numerically. All numerical results are in good agreement with analytical predictions and experimental results [Baryshevsky et al., 2006].

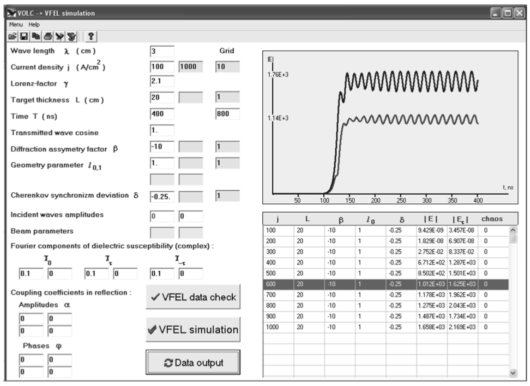

Figure 2. Interface of computer code VOLC

Using VOLC interface one should set values of different system parameters such as wave length, electron beam current density, Lorenz factor, resonator length, different geometry parameters, dielectric susceptibility of periodical structure, reflection coupling coefficients etc. (see (4)–(6)). User can set ranges for the following parameters: current density, resonator length and some geometry parameters. After computation he obtains a table with output results that can be visualized in plots.

4 Results of numerical study

Analytical investigation of chaos in the system considered seems to be impossible because of its nonlinearity. Electron beam moving through periodical structure leads to a diversity of features of generation dynamics that is due to non-local nature of interaction between electron beam and electromagnetic field under VDFB. In the paper presented the chaotic behavior in two-wave geometry was investigated for the following set of parameters: wavelength λ = 3 cm, $L = 10 \div 40$ cm, $j = 400 \div 3000$ A/cm^2.

Let us consider parametric maps of chaotic lasing presented in Fig.3 and Fig.4 for transmitted and diffracted waves respectively. On edges the most typical dependencies of electromagnetic field intensities $|E(t, L)|$ in Fig.3 and $|E_\tau(t, 0)|$ in Fig.4 on time (in ns) are presented. 0 depicts a domain under generation threshold where generation of electromagnetic radiation is not realized. After overcoming this threshold by parameters j or/and L the radiation gain of generating mode becomes equal to absorption and generation begins and develops actively. All main dependencies of different threshold points were investigated analytically [Baryshevsky, Batrakov and Dubovskaya 1991] and numerically in [Batrakov and Sytova, NPCS, 2005], [Batrakov and Sytova, 2006].

After small overcoming this threshold periodical self-oscillations are developed (see Fig.3-4, plots 1–3). Here regimes with different scale of amplitudes (compare these plots) are realized. At small overcoming the threshold values the initiation of quasiperiodic regimes is possible in VFEL. This is demonstrated in Fig.3-4 when transitions from periodic to quasiperiodic regimes than through non-periodical chaotic self-oscillations with some lines in spectrum to periodic regime again are founded.

Then plots 4, 8 and 9 depict examples of quasiperiodic regimes. First one appears by traditional Hopf bifurcation introducing new incommensurate frequency in the spectrum. The last one is a result of intermittency (plot 10). Plots 12 and 13 is a good illustration of period doubling for quasiperiodic regime (Fig.3) and periodic one (Fig.4).

In [Kuznetsov and Trubetskov, 2004] for the backward-wave tube with strong reflections parametric maps with large-scale and small-scale amplitude regimes were adduced. In our simulation besides domains with simply large-scale and small-scale amplitudes we obtained domains with transitions between them (see examples in Fig.3-4, plots 5, 7). After such transition regimes with periodicity, quasiperiodicity and chaos can be established. Last type of transition is realized as a result of tangent bifurcation from chaotic attractor to another one. In the case of transition from large-scale to small-scale amplitude regime when quasiperiodic regime is obtained, we deal with the tangent bifurcation from chaotic attractor to quasiperiodic movement on the torus. All this can be explained by the nonlinear mode competition mechanism in the system.

Bifurcation diagrams presented in Fig.5-6 were obtained for $j = 1000$ A/cm^2 under section of attractor $(E(t), E(t+d\tau), E(t+2d\tau))$ reconstructed in 2D space on (x, y) plane by line $y = x$. $d\tau$ is the delay remaining constant for all considered values L. In these figures one can see transitions from large-scale to small-scale amplitude regimes and vice versa as well as principle bifurcational points.

Domains with transition from large-scale to small-scale amplitudes are situated at relatively large values of j and L. Reverse transition is carried out gradually without breaking up of one regime by another.

Domains of quasiperiodicity for transmitted wave are situated mainly not far from generation threshold. Here transition from periodicity to chaos is carried out through appearance of additional spectrum lines incommensurate in frequency. For diffracted wave wide band of periodicity is observed. In general, spectrum of transmitted wave has much more lines. This confirms theoretical and experimental investigations of VFEL capability to generate at several frequencies [Baryshevsky et al., 2006].

5 Conclusion

Since VFEL physical principles differ from ones using in other electronic vacuum devices we deal with a new subject of inquiry. So, each step in investigation of its nonlinear dynamics will profit valued results.

Investigation of nonlinear dynamics of electron beam instability in three-dimensional periodical structures showed the complicated nature of such interaction leading to intricate changing of regimes of operation

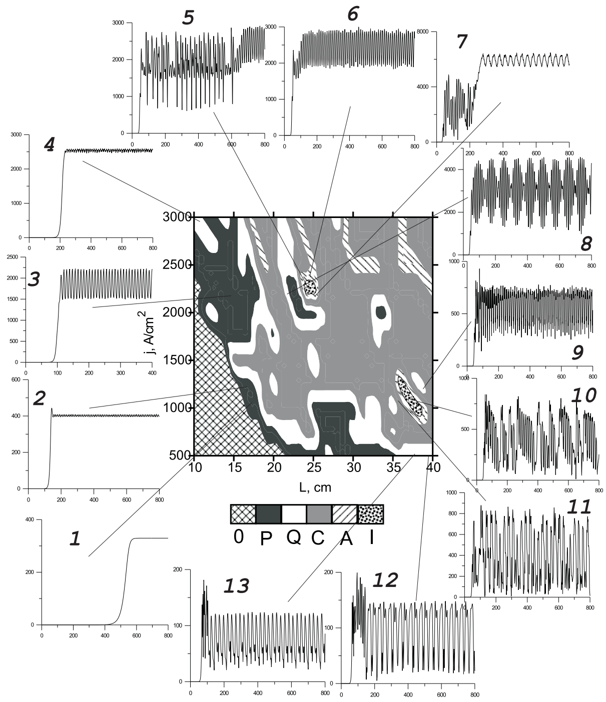

Figure 3. Two-parametric map of chaotic lasing for transmitted wave. 0 depicts a domain under generation threshold. P, Q, C correspond to periodic regimes, quasiperiodicity and chaos, respectively. A describes domains with transitions between large-scale and small-scale amplitudes. I stands for intermittency. On edges the most typical dependencies of $|E(t, L)|$ on time (in ns) are presented.

under changing of control parameters.

Numerical experiments show the possibility of the following transitions between different regimes: "period doubling – chaos", "intermittency – chaos", "quasiperiodicity – chaos", "chaos – chaos", large-scale and small-scale amplitude transitions and vice versa.

It is obvious that due to big amount of computational work some regimes are hard to find because their bands in parametric space are very narrow. And in reality the full picture of root to chaos in electron beam interaction with three-dimensional periodical structures is much more complicated than presented here. But such investigations are in progress and will be useful for providing experiments on the installations created at the Research Institute for Nuclear Problems of Belarusian State University.

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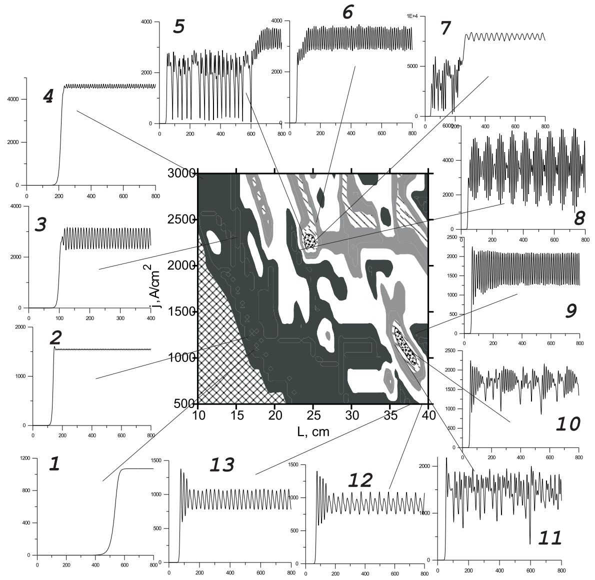

Figure 4. Two-parametric map of chaotic lasing for diffracted wave. See designations in Fig.3. On edges the most typical dependencies of $|E_\tau(t,0)|$ on time (in ns) are presented.

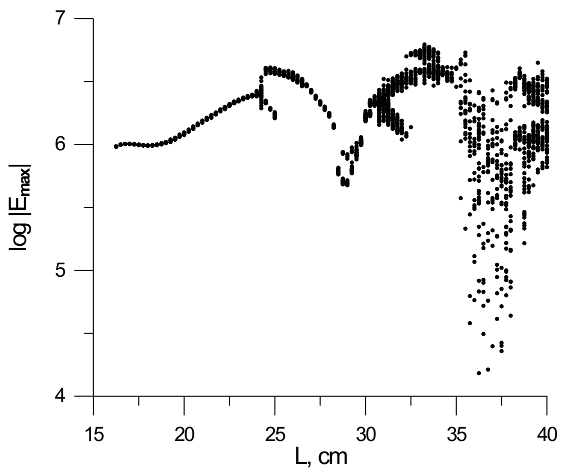

Figure 5. Bifurcation diagram for transmitted wave

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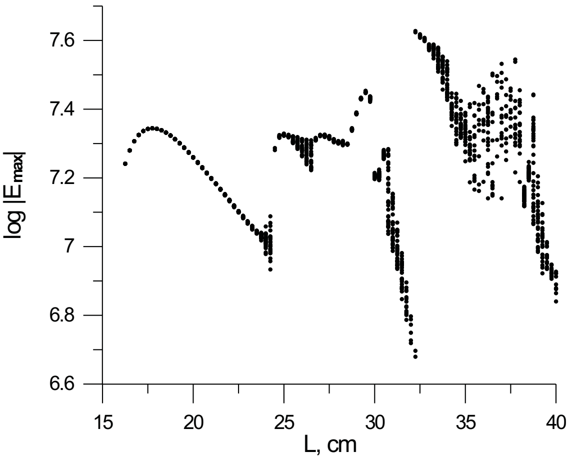

Figure 6. Bifurcation diagram for diffracted wave

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