AN ADAPTIVE OBSERVER FOR CHAOTIC DUFFING SYSTEM

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Abstract
Problem of unknown encoded parameter reconstruction is solved by means of procedure of design of adaptive observer for chaotic Duffing system. Unlike known analogues, the problem in question is only solved using measurements of output of chaotic system and in conditions of full parametrical uncertainty.

Key words
Chaotic system, adaptive observer, data transmitting.

1 Introduction

Problem of an adaptive observer design for nonlinear dynamic systems has been in the centre of attention for the last years. One of reasons of this interest is that there is a possibility of use of adaptive observers for information encoding and transmission. One of new directions of data transmission is encoding information with parameters of a dynamic system ("transmitter"). Output signal of that system is transmitted to "receiver", which is intended to reconstruct unmeasured signals and model parameters of "transmitter". Structural scheme of such system is shown in Fig.1, where θ is parameter vector of "transmitter" system model, encoding transmitted information; y is "transmitter" output transmitted via communication channel; $\hat{\theta}$ is estimation of vector θ, produced by receiver.

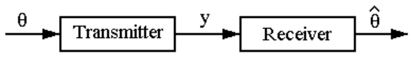

Fig 1. Structural scheme of data transmitting system

It is especially prospective to use chaotic dynamic systems as models of "transmitters" because output signal of a chaotic system has, on the one hand, wide frequency range and, on the other hand, solutions of such systems show weak dependence on initial conditions that increases protection of the system from unauthorized reconstruction of signal information component. In this case "receiver" must construct an adaptive observer of chaotic system.

A few groups of methods (Fradkov *et al.*, 1997, Markov *et al.*, 1996) are usually used for design of adaptive observers. Most of the methods are based on possibility of passification of transmitter system model via feedback in assumption that this model has relative degree equal to zero or one. Other solutions imply accessibility for measurement full state vector of the "transmitter" system (Fradkov *et al.*, 1998, Huijberts *et al.*, 2000). In paper (Nikiforov *et al.*, 2002) the solution which allows to design adaptive observers for "transmitter" models of high (more than one) relative degree and not passificated via output feedback was proposed. These results are based on use of new canonical form of nonlinear adaptive observers proposed in (Nikiforov *et al.*, 2002).

Result of (Nikiforov *et al.*, 2002) was strengthened in (Efimov, 2004, Efimov *et al.*, 2005) where problem of design of adaptive partial observers for non-autonomous nonlinear dynamic systems was considered. Use of external exiting signal is one of approaches to create chaotic modes of operation in nonlinear systems. Examples of such systems are Duffing model and model of brusselator (Nikolis *et al.*, 1977) demonstrating chaotic behaviour only in presence of proper harmonic disturbance. Propagation of classical results on adaptive observers design problem for non-autonomous systems allows essential extending of class of possible models for “transmitter” system and increases protection of the system from unauthorized access.

In this paper we consider problem of an adaptive observer design for chaotic signals generated by Duffing chaotic system. The heart of the problem involves separation of useful information transmitted via communication channel from chaotic signal. Unlike known results, in this paper problem of observer design using only measurements of output variable of chaotic signal in conditions of full parametrical uncertainty of its model is considered.

2 Problem statement

Consider chaotic Duffing system described by equation of the following form

$$\ddot{y}(t) + c_1\dot{y}(t) + c_2 y(t) - \bar{\theta}\, f(y) - w(t) = 0 \qquad (1)$$

where c_1, c_2 and $\bar{\theta}$ are unknown numbers, nonlinear function $f(y) = y^3$ and $w(t) = A\sin(\omega t + \varphi)$ is unmeasured harmonic signal.

It is required to design an observer ensuring reconstruction of unknown parameter $\bar{\theta}$ of model (1). Let us assume that only output variable $y(t)$ of model (1) is measured. We also assume that parameters of chaotic system c_1, c_2, $\bar{\theta}$, A, ω and φ are unknown numbers.

3 Design of an adaptive observer

Let us rewrite model (1) the following way to derive the main result

$$y(t) = \frac{1}{a(p)}\left[\bar{\theta} y^3 + w(t)\right], \qquad (2)$$

where polynomial $a(p) = p^2 + c_1 p + c_2$ and $p = d/dt$.

Passing to Laplace images in equation (1) we obtain

$$Y(s) = \frac{\bar{\theta} F(s)}{s^2 + c_1 s + c_2} + \frac{W(s)}{s^2 + c_1 s + c_2} + \frac{D(s)}{s^2 + c_1 s + c_2}, \qquad (3)$$

where s is complex variable, $Y(s) = L\{y(t)\}$, $F(s) = L\{f(y(t))\}$, $W(s) = L\{w(t)\}$ are Laplace images of functions $y(t)$, $f(y(t))$ and $w(t)$ respectively, polynomial $D(s)$ denotes sum of all terms containing nonzero initial conditions.

Let us transform model (3) the following way

$$Y(s) = \frac{\bar{\theta} F(s)}{s^2 + c_1 s + c_2} + \frac{W(s)}{s^2 + c_1 s + c_2} + \frac{D(s)}{s^2 + c_1 s + c_2},$$

$$(s + \lambda)^2 Y(s) = a_1(s) Y(s) + \bar{\theta} F(s) + W(s) + D(s),$$

whence

$$Y(s) = \frac{a_1(s)}{(s+\lambda)^2} Y(s) + \frac{\bar{\theta} F(s)}{(s+\lambda)^2} + \frac{W(s)}{(s+\lambda)^2} + \frac{D(s)}{(s+\lambda)^2}, \qquad (4)$$

where λ is any positive number, polynomials $a_1(s) = (s+\lambda)^2 - a(s)$ and $a(s) = s^2 + c_1 s + c_2$.

From equation (4) we obtain

$$y(t) = \frac{a_1(p)}{(p+\lambda)^2} y(t) + \frac{\bar{\theta}}{(p+\lambda)^2} y^3(t) + \frac{1}{(p+\lambda)^2} w(t) + \varepsilon_y, \qquad (5)$$

where $f(y) = y^3$ and $\varepsilon_y(t) = L^{-1}\left\{\frac{D(s)}{(s+\lambda)^2}\right\}$ is exponentially decaying function of time caused by nonzero initial conditions. Neglecting exponentially decaying item $\varepsilon_y(t) = L^{-1}\left\{\frac{D(s)}{(s+\lambda)^2}\right\}$, let us parameterize model (5).

Remark 1. As exponentially decaying function $\varepsilon_y(t) = L^{-1}\left\{\frac{D(s)}{(s+\lambda)^2}\right\}$ depends on parameter λ, it is possible to accelerate convergence of $\varepsilon_y(t)$ to zero by increasing λ.

Consider auxiliary filters of the following form

$$\xi_1(t) = \frac{1}{(p+\lambda)^2} y(t), \qquad (6)$$

$$\xi_2(t) = \frac{1}{(p+\lambda)^2} y^3(t). \qquad (7)$$

Substituting (6) and (7) into equation (5), we obtain

$$y(t) = a_1(p)\xi_1(t) + \bar{\theta}\xi_2(t) + \bar{w}(t), \qquad (8)$$

where function $\bar{w}(t) = \frac{1}{(p+\lambda)^2} w(t)$.

From equation (8) we have

$$y(t) = \delta(t) + \bar{w}(t), \qquad (9)$$

where function $\delta(t) = a_1(p)\xi_1(t) + \bar{\theta}\xi_2(t)$.

Consider filter (6)

$$\xi_1(t) = \frac{1}{(p+\lambda)^2} y(t) = \frac{1}{(p+\lambda)^2}\delta(t) + \frac{1}{(p+\lambda)^2}\bar{w}(t),$$

whence

$$\frac{1}{(p+\lambda)^2}\bar{w}(t) = \xi_1(t) - \frac{1}{(p+\lambda)^2}\delta(t). \qquad (10)$$

As signal $w(t) = A\sin(\omega t + \varphi)$, and polynomial $(p+\lambda)^2$ is Hurwitz then function $\bar{w}(t)$ can be represented the following way

$$\bar{w}(t) = \sigma \cdot \sin(\omega t + \phi),$$
$$p^2\bar{w}(t) = -\sigma\omega^2 \sin(\omega t + \phi) = \theta\bar{w}(t),$$

where $\theta = -\omega^2$.

Let us rewrite the last equation

$$(p+\lambda)^2\bar{w}(t) = p^2\bar{w}(t) + 2\lambda p\bar{w}(t) + \lambda^2\bar{w}(t) =$$
$$= \theta\bar{w}(t) + 2\lambda p\bar{w}(t) + \lambda^2\bar{w}(t) =$$
$$= (2\lambda p + \lambda^2)\bar{w}(t) + \theta\bar{w}(t),$$
$$(p+\lambda)^2\bar{w}(t) = (2\lambda p + \lambda^2)\bar{w}(t) + \theta\bar{w}(t).$$

As $\bar{w}(t) = y(t) - \delta(t)$, then

$$(p+\lambda)^2(y(t)-\delta(t)) =$$
$$= (2\lambda p + \lambda^2)(y(t)-\delta(t)) + \theta(y(t)-\delta(t)) =$$
$$= (p+\lambda)^2 \cdot \left[\frac{(2\lambda p + \lambda^2)}{(p+\lambda)^2}(y(t)-\delta(t)) + \right.$$
$$\left. + \frac{\theta}{(p+\lambda)^2}(y(t)-\delta(t))\right].$$

From the last equation we obtain

$$\bar{w}(t) = (y(t)-\delta(t)) =$$
$$= (2\lambda p + \lambda^2)\xi_1 + \theta\xi_1 + \frac{(2\lambda p + \lambda^2)}{(p+\lambda)^2}(-\delta(t)) +$$
$$+ \frac{\theta}{(p+\lambda)^2}(-\delta(t)) =$$
$$= (2\lambda p + \lambda^2 + \theta)\xi_1 + \frac{(2\lambda p + \lambda^2)}{(p+\lambda)^2}(-\delta(t)) +$$
$$+ \frac{\theta}{(p+\lambda)^2}(-\delta(t)) =$$
$$= (2\lambda p + \lambda^2 + \theta)\xi_1 + \frac{(-p^2 - 2\lambda p - \lambda^2)}{(p+\lambda)^2}\delta(t) +$$
$$+ \frac{p^2 - \theta}{(p+\lambda)^2}\delta(t) =$$
$$= (2\lambda p + \lambda^2 + \theta)\xi_1 - \delta(t) + \bar{\delta}(t), \qquad (11)$$

where function

$$\bar{\delta}(t) = \frac{p^2 - \theta}{(p+\lambda)^2}\delta(t) = \frac{p^2}{(p+\lambda)^2}\delta(t) - \frac{\theta}{(p+\lambda)^2}\delta(t) =$$
$$= \frac{p^2 a_1(p)}{(p+\lambda)^2}\xi_1(t) - \frac{\theta a_1(p)}{(p+\lambda)^2}\xi_1(t) +$$
$$+ \frac{p^2\bar{\theta}}{(p+\lambda)^2}\xi_2(t) - \frac{\bar{\theta}\theta}{(p+\lambda)^2}\xi_2(t). \qquad (12)$$

Let us transform model (11)

$$\theta\xi_1(t) + \bar{\delta}(t) = \bar{w}(t) + \delta(t) - 2\lambda\dot{\xi}_1(t) - \lambda^2\xi_1(t) =$$
$$= y(t) - 2\lambda\dot{\xi}_1(t) - \lambda^2\xi_1(t), \qquad (13)$$

denoting

$$z(t) = \theta\xi_1(t) + \bar{\delta}(t) = y(t) - 2\lambda\dot{\xi}_1(t) - \lambda^2\xi_1(t), \qquad (14)$$

where function $z(t)$ is measured by virtue of measurability of signals $y(t)$, $\xi_1(t)$ and $\dot{\xi}_1(t)$.

Substituting equation (12) into (14), we obtain

$$z(t) = \theta\xi_1(t) + \frac{p^2 a_1(p)}{(p+\lambda)^2}\xi_1(t) - \frac{\theta a_1(p)}{(p+\lambda)^2}\xi_1(t) +$$
$$+ \frac{\bar{\theta}p^2}{(p+\lambda)^2}\xi_2(t) - \frac{\bar{\theta}\theta}{(p+\lambda)^2}\xi_2(t) =$$
$$= \theta\left(1 - \frac{a_1(p)}{(p+\lambda)^2}\right)\xi_1(t) + \bar{\theta}\frac{p^2}{(p+\lambda)^2}\xi_2(t) -$$
$$-\bar{\theta}\theta\frac{1}{(p+\lambda)^2}\xi_2(t) + \frac{p^2 a_1(p)}{(p+\lambda)^2}\xi_1(t). \qquad (15)$$

Taking into consideration that $a_1(p) = a_1 p + a_0$, we have

$$\theta\left(\frac{p^2 + 2\lambda p + \lambda^2 - a_1 p - a_0}{(p+\lambda)^2}\right)\xi_1(t) =$$
$$= \theta\frac{p^2}{(p+\lambda)^2}\xi_1(t) + (2\lambda\theta - a_1\theta)\frac{p}{(p+\lambda)^2}\xi_1(t) +$$
$$+(\lambda^2\theta - a_0\theta)\frac{1}{(p+\lambda)^2}\xi_1(t), \qquad (16)$$

$$\frac{p^2(a_1 p + a_0)}{(p+\lambda)^2}\xi_1(t) =$$

$$= a_1 \frac{p^2}{(p+\lambda)^2}\dot{\xi}_1(t) + a_0 \frac{p^2}{(p+\lambda)^2}\xi_1(t) . \qquad (17)$$

Substituting (16), (17) into equation (15), we have

$$z(t) = (\theta + a_0)\frac{p^2}{(p+\lambda)^2}\xi_1(t) + \bar{\theta}\frac{p^2}{(p+\lambda)^2}\xi_2(t) -$$
$$-\bar{\theta}\theta\frac{1}{(p+\lambda)^2}\xi_2(t) + a_1\frac{p^2}{(p+\lambda)^2}\dot{\xi}_1(t) +$$
$$+(2\lambda\theta - a_1\theta)\frac{p}{(p+\lambda)^2}\xi_1(t) +$$
$$+(\lambda^2\theta - a_0\theta)\frac{1}{(p+\lambda)^2}\xi_1(t) . \qquad (18)$$

From equation (18) we obtain

$$z(t) = \Psi^T(t)\Theta = \psi_1(t)\theta_1 + \psi_2(t)\theta_2 + \psi_3(t)\theta_3 +$$
$$+\psi_4(t)\theta_4 + \psi_5(t)\theta_5 + \psi_6(t)\theta_6 , \qquad (19)$$

where vector $\Psi(t) = col\{\psi_i(t), i = \overline{1,6}\}$ of known functions

$$\psi_1(t) = \frac{p^2}{(p+\lambda)^2}\xi_1(t) , \ \psi_2(t) = \frac{p^2}{(p+\lambda)^2}\xi_2(t) ,$$
$$\psi_3(t) = \frac{1}{(p+\lambda)^2}\xi_2(t) , \ \psi_4(t) = \frac{p^2}{(p+\lambda)^2}\dot{\xi}_1(t) ,$$
$$\psi_5(t) = \frac{p}{(p+\lambda)^2}\xi_1(t) , \ \psi_6(t) = \frac{1}{(p+\lambda)^2}\xi_1(t)$$

and vector $\Theta = col\{\theta_i(t), i = \overline{1,6}\}$ of unknown parameters

$$\theta_1 = \theta + a_0 , \ \theta_2 = \bar{\theta} , \ \theta_3 = -\bar{\theta}\theta , \ \theta_4 = a_1 ,$$
$$\theta_5 = 2\lambda\theta - a_1\theta , \ \theta_6 = (\lambda^2\theta - a_0\theta) .$$

Let us use an adaptive observer of the view (20), (21) for estimation of unknown parameters of model (19)

$$\hat{z}(t) = \Psi^T(t)\hat{\Theta} = \psi_1(t)\hat{\theta}_1 + \psi_2(t)\hat{\theta}_2 + \psi_3(t)\hat{\theta}_3 +$$
$$+\psi_4(t)\hat{\theta}_4 + \psi_5(t)\hat{\theta}_5 + \psi_6(t)\hat{\theta}_6 , \qquad (20)$$
$$\dot{\hat{\theta}}_i = k_i\psi_i(t)e(t) = k_i\psi_i(t)(\ddot{\xi}_1 - \hat{z}(t)) , \qquad (21)$$

where constant coefficient $k_i > 0$, $i = \overline{1,6}$.

It is easy to show that an adaptive observer (20), (21) is equal to ensures

$$\lim_{t\to\infty}\left|z(t) - \hat{z}(t)\right| = 0 , \qquad (22)$$
$$\lim_{t\to\infty}\left|\theta_i - \hat{\theta}_i(t)\right| = 0 . \qquad (23)$$

Execution of (23) guarantees convergence of estimation of parameter $\bar{\theta}$ to true value of “transmitter” system model.

Remark 2. It is easy to show that an adaptive observer (20), (21) is robust.

4 Simulation results

Let us simulate scheme of adaptive estimation of unknown parameter $\bar{\theta}$ for the following parameters of chaotic Duffing system (1): $c_1 = 0$, $c_2 = 0,5$, $w(t) = 8\sin(0,5t)$. Algorithm of parameters tuning takes the form

$$\dot{\hat{\theta}}_1 = 18\psi_1(t)(\ddot{\xi}_1 - \hat{z}(t)) , \ \dot{\hat{\theta}}_2 = 14\psi_2(t)(\ddot{\xi}_1 - \hat{z}(t)) ,$$
$$\dot{\hat{\theta}}_3 = 14\psi_3(t)(\ddot{\xi}_1 - \hat{z}(t)) , \ \dot{\hat{\theta}}_4 = 14\psi_4(t)(\ddot{\xi}_1 - \hat{z}(t)) ,$$
$$\dot{\hat{\theta}}_5 = 14\psi_5(t)(\ddot{\xi}_1 - \hat{z}(t)) , \ \dot{\hat{\theta}}_6 = 14\psi_6(t)(\ddot{\xi}_1 - \hat{z}(t)) .$$

Fig. 2-8 show simulation results for $\bar{\theta} = -1$ and $\lambda = 1$ (i.e. $\theta_1 = 0,25$, $\theta_2 = -1$, $\theta_3 = -0,25$, $\theta_4 = 2$, $\theta_5 = 0$, $\theta_6 = -0,125$).

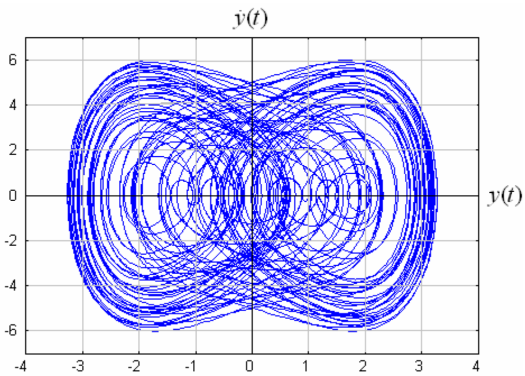

Fig 2. Phase path of chaotic Duffing system (1)

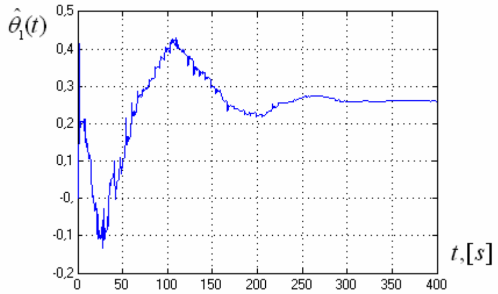

Fig 3. Transients for variable $\hat{\theta}_1$

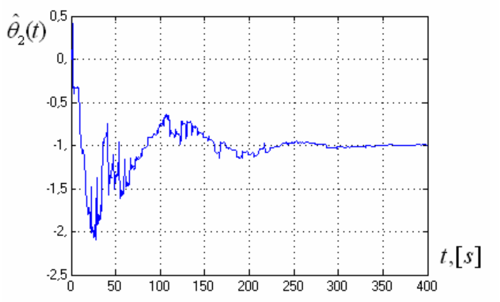

Fig 4. Transients for variable $\hat{\theta}_2$

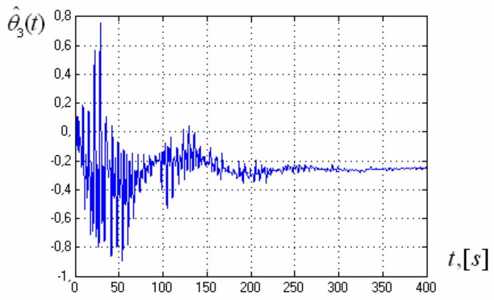

Fig 5. Transients for variable $\hat{\theta}_3$

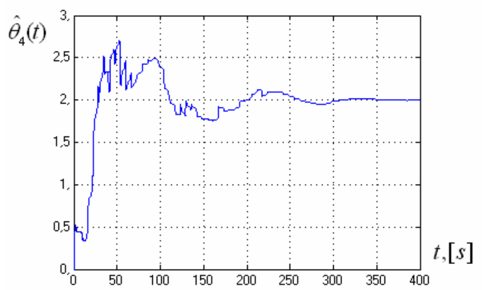

Fig 6. Transients for variable $\hat{\theta}_4$

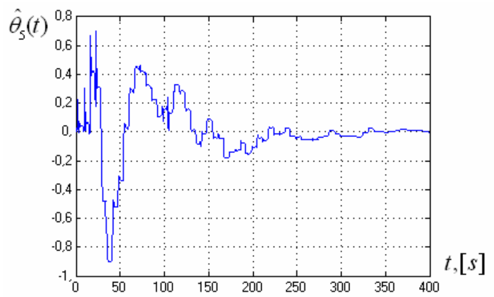

Fig 7. Transients for variable $\hat{\theta}_5$

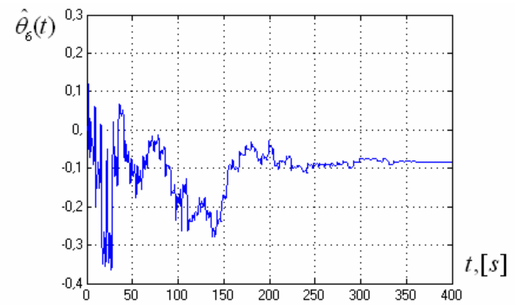

Fig 8. Transients for variable $\hat{\theta}_6$

Simulation results show that $\hat{\theta}_1 \to 0,25$, $\hat{\theta}_2 \to -1$, $\hat{\theta}_3 \to -0,25$, $\hat{\theta}_4 \to 2$, $\hat{\theta}_5 \to 0$, $\hat{\theta}_6 \to -0,125$.

Let us simulate scheme of adaptive estimation of unknown parameter $\overline{\theta}$ for the following parameters of chaotic Duffing system (1): $c_1 = 0$, $c_2 = 1$, $w(t) = 12\sin(t)$. Algorithm of parameters tuning takes the form

$$\dot{\hat{\theta}}_1 = 18\psi_1(t)(\ddot{\xi}_1 - \hat{z}(t)),\ \dot{\hat{\theta}}_2 = 14\psi_2(t)(\ddot{\xi}_1 - \hat{z}(t)),$$
$$\dot{\hat{\theta}}_3 = 10\psi_3(t)(\ddot{\xi}_1 - \hat{z}(t)),\ \dot{\hat{\theta}}_4 = 24\psi_4(t)(\ddot{\xi}_1 - \hat{z}(t)),$$
$$\dot{\hat{\theta}}_5 = 16\psi_5(t)(\ddot{\xi}_1 - \hat{z}(t)),\ \dot{\hat{\theta}}_6 = 12\psi_6(t)(\ddot{\xi}_1 - \hat{z}(t)).$$

Fig. 9-15 show simulation results for $\overline{\theta} = -2$ and $\lambda = 0,5$ (i.e. $\theta_1 = -1,75$, $\theta_2 = -2$, $\theta_3 = -2$, $\theta_4 = 1$, $\theta_5 = 0$, $\theta_6 = -1$).

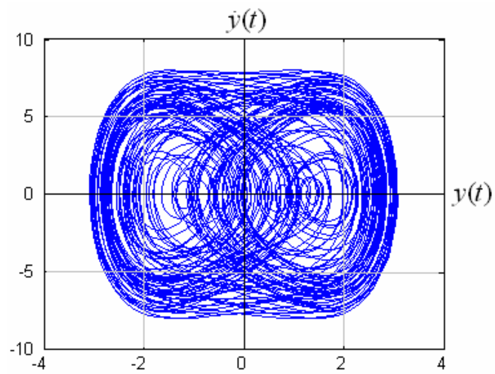

Fig 9. Phase path of chaotic Duffing system (1)

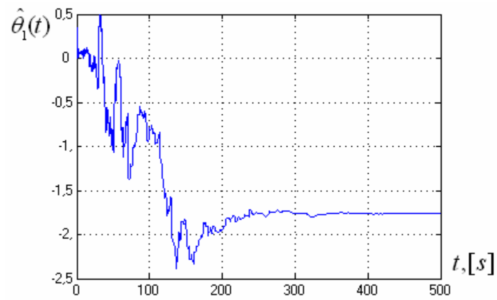

Fig 10. Transients for variable $\hat{\theta}_1$

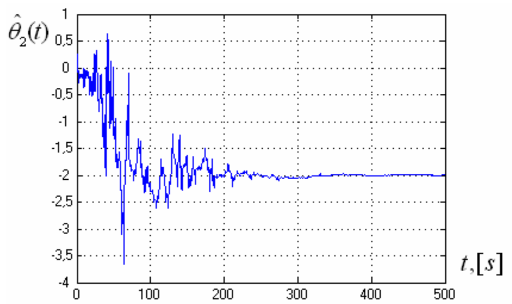

Fig 11. Transients for variable $\hat{\theta}_2$

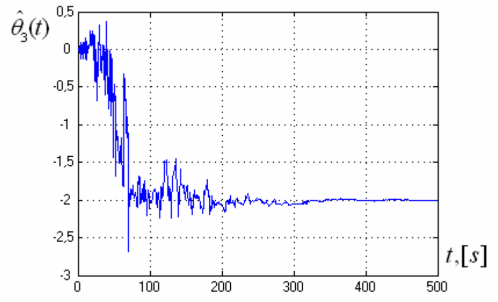

Fig 12. Transients for variable $\hat{\theta}_3$

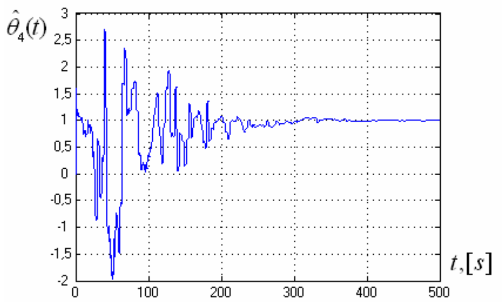

Fig 13. Transients for variable $\hat{\theta}_4$

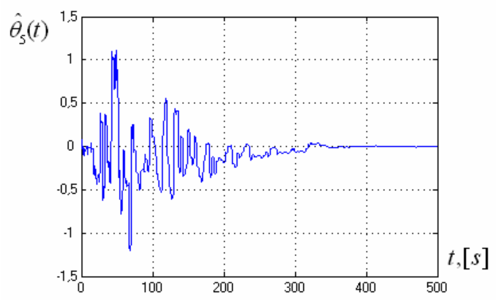

Fig 14. Transients for variable $\hat{\theta}_5$

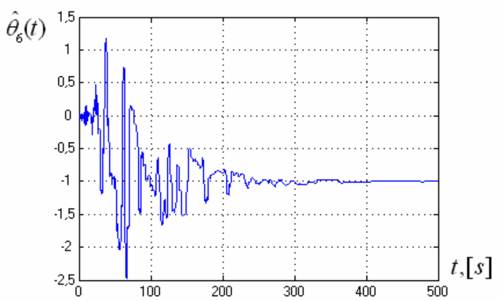

Fig 15. Transients for variable $\hat{\theta}_6$

Simulation results show that $\hat{\theta}_1 \to -1{,}75$, $\hat{\theta}_2 \to -2$, $\hat{\theta}_3 \to -2$, $\hat{\theta}_4 \to 1$, $\hat{\theta}_5 \to 0$, $\hat{\theta}_6 \to -1$.

Simulation results illustrate efficiency of proposed scheme of adaptive estimation of unknown parameter $\bar{\theta}$ of the Duffing model (1).

5 Conclusion

Problem of estimation of unknown encoded parameter is solved using an adaptive observer (20), (21) for chaotic Duffing system. Unlike known analogues, this result uses only measurements of output signal of the chaotic system and also allows to find unknown encoded parameter $\bar{\theta}$ in conditions of full parametric uncertainty of the model (1).

References

Efimov, D.V. and A.L. Fradkov (2005). Adaptive partial observers with applications to time-varying chaotic systems. *Proc. IUTAM symp. On chaotic dynamics and control of systems and processes in mechanics, Springer*, pp. 27-35.

Efimov, D.V. (2004). Robust adaptive nonlinear partial observers for time-varying chaotic systems. *Proc. IEEE CDC, Atlantis, Paradise Island, Bahamas*, Dec, pp. 13-15.

Fradkov, A.L. and D. Hill (1998). Exponential feedback passivity and stabilizability of nonlinear systems, *Automatica*, **6**, pp. 697-703.

Fradkov, A.L., H. Nijmeijer and A. Markov (1997). Adaptive observer-based synchronization for communications. *Int. J. Bifurcations and Chaos*, **12**, pp. 2807-2814.

Huijberts, H., H.Nijmeijer and R. Willems (2000). System identification in communication with chaotic system, *IEEE trans. Circ. Syst. – I*, **47**, pp. 800-808.

Markov, A.Yu. and A.L. Fradkov (1996). Adaptive synchronization of coupled chaotic system, *Proc. Int. Conf. "Fractals and Chaos in Chemical Engineering", Rome*, pp. 153-154.

Nikiforov, V.O. and B.R. Andrievsky (2002). Adaptive observers for nonlinear nonpassifiable systems with applications to signal transmittion. *Proc. 41th IEEE Conf. Decision and Control, Las Vegas*, Dec, pp. 10-13.

Nikolis, G. and I.R. Prigogine (1977). *Self-organization in non-equilibrium systems*. Wiley, New York.