SELF-OPTIMAL STABILIZATION OF A CLASS OF POSITIVE COMPARTMENTAL SYSTEMS VIA VARIABLE-STRUCTURE CONTROL: BIOREACTORS

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Abstract

In this paper a class of positive compartmental systems is stabilized via variable-structure feedback in sense that the control signal is self-adjusted to reach the optimal set-point (unknown). The goal is to design a self-optimizing control scheme for the substrate stabilization in bioreactors with inhibition kinetics. Open-loop analysis for the bioreactor suggests that the optimal behavior, respect to maximal biomass production, occurs in an unstable region (structurally unstable). The controller is designed based on sliding modes techniques, such that the sliding regime on the optimal manifold is achieved. The self-optimizing comprises an uncertainty estimator which computes the unknown terms for increasing the robustness issues of the sliding-mode scheme. By exploiting the inhibitory effect of substrate concentration under the biomass growth, the controller proposed achieve the practical stabilization around to the optimal substrate concentration (unknown) and, consequently, the growth-rate achieves its maximum. Numerical experiments for a microbial growth process in continuous and fed-batch operating modes illustrate the performance and execution of the control scheme.

Key words

Self-optimizing Control; Positive compartmental systems; Bioreactors; Substrate inhibition.

1 Introduction

Positive compartmental systems are been used to different applicative fields of mathematical modeling, ranging from physiological systems, to the chemical processes. Since the state variables for these systems represent the amount of material contained in each compartment, they are bounded to be nonnegative over time. Then, compartmental systems belong to the class of positive systems [Benvenuti and Farina (2002)]. The biochemical reactors are a class of positive compartmental systems that have become widely used with different purposes, including: (i) the production of metabolite synthesized by a microorganism in fermentative processes, which has high value in marked [Luna-Flores et al., (2010)]; (ii) the biomass culture which is classical process, and more recently, (iii) for waste (water and solid) treatment of organic matter with production of renewable energy [Smith(1995); Bastin and Dochain(1990)]. The optimization of the operation of bioreactors is an interesting non-linear problem whose solution offers potential economic benefit [Bastin and Dochain(1990); González et al., (1998); Henson(2003); Hess and Bernard(2008)]. Moreover, the optimal operation for bioreactors can become undesired because of small changes or external perturbations can lead it towards the *operational instability* conditions as the washout [Henson(2003); Hess and Bernard(2008)]. In fact, it has been recently demonstrated that the inhibitory effect, induced by substrate or product concentration, under the biomass growth rate provokes structural instability for continuous bio-processes [Lara-Cisneros et al., (2012)]. Although the automatic control schemes for bioreactors are usually focused on regulation, servo set-points, or tracking reference trajectories [Bastin and Dochain(1990); Simeonov et al., (2007); Tan et al., (2009); Wang et al., (1999)], in some biotechnological applications, the control objective is to optimize an objective function that can be a function of unknown parameters keeping a performance variable at its extremum value [Wang et al., (1999); Dochain(2008)]. In this sense, the design of self-optimizing control strategies for stabilizing bioprocesses is a topic of academic and industrial relevance.

The problem of locating an optimal set-point on-line has motivated the design of sophisticated control strategies [Bastin and Dochain(1990); Simeonov et al., (2007); Tan et al., (2009); Wang et al., (1999)]. Roughly speaking, this problem consists in maximizing an objective function in face to uncertain internal dynamics. The self-optimizing controllers has been implemented in telecommunications and aerospace engineering; *e.g.*, in the radio telescope antenna adjustment to maximize the received signal, and blade adjustment in water turbines or wind mills to maximize the generated power [Krstić and Wang(2000); Tan et al., (2009)]. However, the real-time optimization by extremum-seeking control has been applied to bio-reacting systems is found in recent contributions [Bastin and Dochain(1990); Simeonov et al., (2007); Smets et al., (2002); Wang et al., (1999); Zhang et al., (2003)], which are mainly motivated by increasing demand of automatic control for biotechnological processes. The extremum-seeking control strategies include an excitation signal to extracts the gradient of the unknown output mapping, and include many tunable parameters [Krstić and Wang(2000); Wang et al., (1999)]. On the other hand, the self-optimizing controllers problem has also been studied in the sliding mode control framework for automotive control technologies [Utkin(1992); Perruquetti(2002); Tham et al., 2003); Hai and Ozguner(2006); Korovin and Utkin(1974)]. In such contributions, the idea is to ensure that a desired output follows a time function as closer as possible to its extremal value via discontinuous controls and sliding mode motions [Haskara(2000); Pan and Furuta(2007); Fu and Ozguner(2009)]. In this contribution, we present a self-optimizing control strategy based on variable-structure theory such that the optimal stabilization is achieved for bioreactors with inhibition kinetics. Unlike the previous schemes [Bastin and Dochain(1990); Haskara(2000); Fu and Ozguner(2009); Krstić and Wang(2000); Wang et al., (1999)], the proposed self-optimizing controller has three tuning parameters and dispenses with requirement of on-line measurement of the performance variable. As a first designing step, an ideal self-optimizing controller is found from the structure-variable theory, where we set an optimal manifold, determined by the maximum biomass growth rate, such that the states of the closed-loop system are driven to the optimum operation condition within the domain. In this first designing step we show that exact knowledge of the biomass growth gradient is needed to compute the optimal manifold. After that, we show this requirement can be solved using an observed-based uncertainty estimator to approach the unknown terms. Thus, the practical stabilization on the neighborhood of optimal unknown set-point is ensured by when estimator and controller are coupled. That is, the proposed scheme comprises the variable structure feedback and an uncertainty dynamical estimator. The closed-loop execution of the self-optimizing controller is shown by numerical experiments for different nominal growth rates. The paper is organized as follows. For completeness, the dynamical behavior in open-loop configuration of bioreactor is briefly discussed in the Section 2. Particulary, the structural properties to uncontrolled system on optimal conditions is discussed. The Section 3 contains the design of the self-optimizing controller and the stability analysis of the closed-loop system. The numerical experiments illustrate the performance and robustness to the self-optimizing control strategy considering different nominal values for biomass growth rate and small changes for the kinetics parameters (see the Section 4). Additionally, numerical simulation shown the self-optimizing control performance in the optimal regularization of a microbial growth process in fed-batch operating mode. Finally, some concluding remarks are discussed in the Section 5.

2 Open-loop analysis

In order to study the dynamical behavior at the optimal operating conditions, the following open-loop analysis is considered. For this purpose, consider us a biomass culture processes occurring within a continuously stirred tank bioreactor (CSTB), where the microorganisms of a single species (X) prosper by consuming a substrate (S). Such a bioreaction is a first approximation of a more complicated chemical reaction network and is written as

$$S \overset{\mu(\cdot)x}{\looparrowright} X$$

The reaction rate is denoted by $\mu(\cdot)x$ where $\mu : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is a smooth function standing for the specific growth rate and the symbol $\looparrowright$ indicates that the biomass (X) is an autocatalyst; i.e. the microorganism are produced from the substrate consumption by microorganism. The following two-dimensional dynamical system can be found from a mass balance into the CSTB,

$$\dot{s} = u(s_f - s) - \frac{1}{Y}\mu(\cdot)x \qquad (1)$$

$$\dot{x} = [\mu(\cdot) - u]x \qquad (2)$$

The state vector $z = [s, x]^T \in \Omega \subset \mathbb{R}^2_{\geq 0}$ has components representing the concentrations of substrate and biomass respectively and the superscript T stands for transpose. The control input u is called dilution rate and corresponds to the ratio between the feeding flow with respect to the reaction volume. The feeding substrate concentration is denoted by $s_f \in \mathbb{R}_+$ whereas $Y \in \mathbb{R}_+$ is the yield factor. Seeking completeness, we discuss the dynamical properties of the system (1-2) (see [Lara-Cisneros et al., (2012)] for further details).

Let $z = [s, x]^T$ be the state vector of the system (1-2). It is said that the set $\Omega \subset \mathbb{R}^2$ the *physically realizable domain* of the bioreactor model (1) if

$$\Omega = \left\{ (s, x) \in \mathbb{R}^2 \middle| 0 \le s \le s_f, 0 \le x \le x_m \right\}$$

where $s_f, x_{max} < \infty$. The following properties of the specific growth rate μ hold on:

Property 1. $\mu \in C^\infty(S)$, where $S = \left\{ s \in \mathbb{R} \middle| 0 \le s \le s_m \right\}$ with $s_m < \infty$.

Property 2. $\exists\, s^* \in S$ such that $\mu \le \mu(s*) \triangleq \overline{\mu} \in \mathbb{R}\, \forall s \in S$. $\overline{\mu} < \infty$ is the upper bound of μ.

Property 3. The first derivative of μ with respect to s, denoted by μ', satisfies the follows: (a) $\mu' > 0\ \forall \tilde{s} < s^*$; $\mu' = 0$ at $\tilde{s} = s^*$ and (b) $\mu' < 0\ \forall \tilde{s} > s^*$, where $\tilde{s} \in S$.

Property 4. The domain Ω is a positively invariant set.

It is important to remark that the Properties 1-4 hold independently of the kinetics governing the bioreaction. Actually, kinetics with substrate saturation, substrate inhibition or product inhibition are continuous which means Property 1 holds. Property 2 regards the existence of the parameter named maximum growth rate, which has a very important biotechnological meaning. Properties 3 and 4 are more related to mathematical concepts about continuous functions and function domain, respectively.

As previously demonstrated by [Lara-Cisneros et al., (2012)], the structure and equilibrium bifurcation for bioreactor (1-2) in open-loop configuration have important issues respect to the optimal operation under a closed-loop control. The following Lemma from [Lara-Cisneros et al., (2012)] states the equilibrium bifurcation for the system (1-2) in open-loop configuration.

Lemma 2.1. *[Lara-Cisneros et al., (2012)] Consider the dynamical system (1-2) on the physically realizable domain $\Omega \subset \mathbb{R}^2$, where $\mu : S \to \mathbb{R}$ is a locally smooth function satisfying the properties* $1, 2, 3$ *and $u \in \mathbb{R}$ is a constant parameter. Then, the system (1-2) satisfies the following:*

1. *If $u > \overline{\mu}$, then, there is only one equilibria on $\overline{z}_w = (s_f, 0)$ which is asymptotically stable.*
2. *If $\mu(s_f) < u < \overline{\mu}$ there are three equilibrium points: the trivial washout equilibria $\overline{z}_w$ and two non-trivial equilibria $\overline{z}_1$ y $\overline{z}_2$, with $\overline{z}_1 < \overline{z}_2$, given by $\overline{z}_{1,2} = (\overline{s}, Y(S_f - \overline{s}))$ in where $\overline{s}$ satisfies $\mu(\overline{s}) - u = 0$. Moreover, the called equilibrium of inhibition $\overline{z}_2$ is unstable (saddle point), and the called conventional operation equilibrium $\overline{z}_2$ is locally stable.*
3. *If $0 < u < \mu(s_f)$, then the trivial washout equilibria $\overline{z}_w = (s_f, 0)$ becomes unstable (saddle point), and $\overline{z}_1$, $\overline{z}_2$ are locally stable.* $\square$

The above result is depicted as a bifurcation map for substrate concentration in equilibrium, with the dilution rate as bifurcation parameter (Fig. 2). The solid line represents the branch of stable equilibrium, while dashed line represent the unstable equilibrium (saddle type). A transcritical and saddle node bifurcations are shown at $u = \mu(s_f)$ and $u = \overline{\mu}$ respectively (see Fig 2).

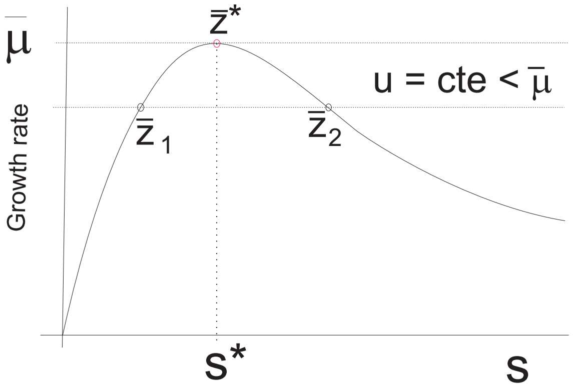

Figure 1. Nontrivial equilibria for $u \le \overline{\mu}$.

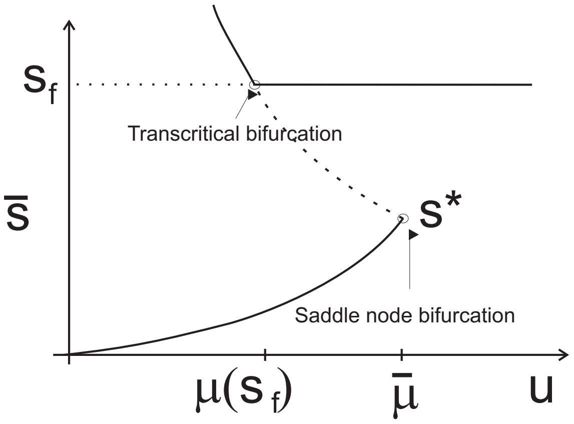

Figure 2. Bifurcation map for substrate equilibria to open-loop configuration ($u \in \mathbb{R}$), at $s = s^* \Leftrightarrow u = \overline{\mu}$ the system is structurally unstable.

Now, in regard the optimal operating conditions, a goal is to hold substrate at s^*, because of the (active) biomass production reach their maximal valued [Bastin and Dochain(1990)]. In this way, the following corollary establishes the structural properties of the system (1-2) at $u = \overline{\mu}$ corresponds to the *optimal equilibria* $\overline{z}^* = (\overline{s}^*, Y(s_f - \overline{s}^*))$.

Corollary 2.2. *The dynamical system (1-2) on the physically realizable domain $\Omega \subset \mathbb{R}^2$, where $\mu : S \to \mathbb{R}$ satisfies the properties* $1, 2, 3$, *is structurally unstable for open-loop configuration at $u = \overline{\mu}$.*

Proof : If $u = \overline{\mu}$ the system (1-2) has only one non-trivial equilibria $\overline{z}^* = (\overline{s}^*, Y(s_f - \overline{s}^*))$ (Lemma 2.1) whose eigenvalues are $\lambda_1 < 0$ and $\lambda_2 = 0$, since $\mu' = 0$ at $s = s^*$ (property 3). In this way, $\overline{z}^*$ is a non-hyperbolic equilibrium point. According to the topological structure of the trajectories on the state-space for smooth dynamical systems [Wiggins(2003)], the system (1-2) is structurally unstable. $\square$

The Corollary 2.2 provides conditions that the optimal behavior occurs on structural unstability for the system in open-loop configuration. In other words, small changes or external perturbations on the bioreactor operating in this conditions can lead to undesirable washout states. This motivates the design of an optimal stabilizer control strategy such that the system (1-2) is stabilized on the unknown optimal set-point s^*.

3 Control Design

Next the design of the self-optimizing control scheme is shown for the stabilization of the bioreactor (1-2) at the optimal set-point $\overline{z}^*$. The control goal is to design a controller with dilution rate u as the control command such that the growth rate achieves its maximum at $\overline{\mu}$. The main idea is to develop a variable structure feedback such that the gradient to growth rate (unknown output) is stabilized at zero. In order to design the self-optimizing controller, the following assumptions are considered:

Assumption 3.1. *The state $z = [s, x]^T$ and the feeding substrate concentration s_f are available for feedback.*

Assumption 3.2. *A nominal value of the yield coefficient $Y = \overline{Y}$ is known.*

Some comments in regard Assumptions 3.1 and 3.2 are in order. In some sense, an observer design is convenient to relax the Assumption 3.1. Actually, the soft-sensors or observers design for the estimation of biomass concentration is an interesting topic towards the bioprocesses control implementation. But, biomass estimation is beyond the goal of this manuscript. However, we assert the Assumption 3.1 is not restrictive in sense that there is available technology to measure online the total biomass via, for example, modern electrical impedance spectroscopy [Sarro et al., (2012)] or classical optical methods [Konstantinov(1994)]. Concerning Assumption 3.2, the nominal value $\overline{Y}$ can be derived from kinetic studies for a specific bioreactor.

The proposed variable structure feedback has a sliding mode control is in which an appropriate input is provided so that the system states are confined to a desired manifold on the state space [Hai and Ozguner(2006); Perruquetti(2002); Utkin(1992)]. The called sliding regime on the sliding manifold could be achieved by changing the controller structure, where the control input is switched between two control signals. Now, by exploiting the non-monotonic characteristic on the biomass growth rate (properties 2-3), we define the so called *optimal manifold* as:

$$\Sigma = \{(s, x) \in \Omega : \mu'(s) \triangleq \sigma(s) = 0\} \qquad (3)$$

where the switching function is chosen as the gradient to growth rate $\mu'(s)$ denoted by σ, note that $s^* \in \Sigma$. Based in sliding mode control techniques, the variable-structure feedback is proposed such that the system (1-2) achieved the sliding regime on the optimal manifold Σ. In order to design an ideal self-optimizing controller, the growth rate and their gradient are considered well known; later this assumption is relaxed to design an observed-based estimator for these terms.

Proposition 3.3. *The variable-structure controller:*

$$u(z) = \frac{\mu(s)x}{Y(s_f - s)} + K sign(\sigma); \qquad K > 0 \quad (4)$$

globally stabilizes the systems (1-2) towards the optimal manifold $\Sigma \subset \mathbb{R}^2$.

Proof : The biomass equilibria is given by $\overline{x} = Y(s_f - \overline{s})$ (Section 2). Then, for any $\overline{s} \in S$ the trajectory $x(t) \longrightarrow \overline{x}$ as $t \longrightarrow \infty$, the closed-loop stability analysis of the substrate concentration is only required.

Let $\tilde{s} \triangleq s - s^*$. Then, the error dynamics is given by

$$\dot{\tilde{s}} = \dot{s} = K(s_f - s)sign(\sigma) \qquad (5)$$

To analyze the reaching mode, i.e. the convergence towards the switching surface Σ, the following Lyapunov-like function of the surface coordinate $\tilde{s}$ is considered:

$$V(\tilde{s}) = \tfrac{1}{2}\tilde{s}^2$$

Differentiating with respect to time and replacing $u(x)$ in (4) with (1) yields

$$\dot{V}(\tilde{s}) = \tilde{s}\dot{\tilde{s}} = \tilde{s}K(s_f - s)sign(\sigma) \leq 0$$

Since $(s_f - s) \leq 0$ for all $s \in \Omega$, from property 3 is verified that $\sigma > 0\ \forall \tilde{s} < 0$ ($\Leftrightarrow s < s^*$) and $\sigma < 0\ \forall \tilde{s} > 0$ ($\Leftrightarrow s > s^*$). Since K is chosen to be positive, it follows $\dot{V}(\tilde{s}) < 0$ for $\tilde{s} \neq 0$ ($\Leftrightarrow \sigma \neq 0$) and $\dot{V}(\tilde{s}) = 0$ for $\tilde{s} = 0$ ($\Leftrightarrow \sigma = 0$). According to the invariance sliding mode properties $\tilde{s} = 0 \wedge \dot{\tilde{s}} = 0$ and to the equivalent control method [Korovin and Utkin(1974); Perruquetti(2002)], it follows from (5) that $u_{eq}(x) = \frac{\mu(s)x}{Y(s_f - s)}$. Therefore, a sliding regime is established on the manifold Σ for any initial condition $z(0) \in \Omega$. $\square$

Now, the variable-structure control (4) requires perfect knowledge of specific growth rate $\mu(\cdot)$ as well as its analytical expression for the gradient σ. However, the biomass growth rate is highly uncertain term for the most of real biotechnological processes. To solve this drawback, an uncertainty estimator scheme is proposed, from the extension of the previously reported ideas by Bastin, Dochain and Alvarez-Ramírez [Alvarez-Ramírez et al., (1999); Bastin and Dochain(1990)]. First, consider the following observer-based estimator of growth rate $\mu(\cdot)$ as proposed by [Bastin and Dochain(1990)].

$$\dot{\hat{x}} = (\hat{\mu} - u)x + \omega x(x - \hat{x}) \quad (6)$$
$$\dot{\hat{\mu}} = \gamma x(x - \hat{x}) \quad (7)$$

where $\hat{x}$, $\hat{\mu}$ denotes the estimated for biomass and growth rate respectively. The observer's gain $\omega, \gamma > 0$ are chosen as $\omega = 2\lambda$ and $\gamma = \lambda^2/x^2$, such that the dynamics of the corresponding estimation error system are governed by the double eigenvalue $-\lambda$, see [Bastin and Dochain(1990)]. Now, it is necessary the estimation for the gradient to specific growth rate σ. In this way, from the chain rule for scalar functions, the unknown gradient σ is computed from the expressions to $\dot{\hat{\mu}}$ in (12) and the estimated valued for the dynamics to substrate concentration as $\dot{\hat{s}} = u(s_f - s) - \hat{\mu}x/Y$. Then the unknown gradient σ is computed as $\hat{\sigma} = \frac{\dot{\hat{\mu}}}{\dot{\hat{s}}}$ by the following expression

$$\hat{\sigma} = \frac{\gamma x(x - \hat{x})}{u(s_f - s) - \hat{\mu}x/Y + \delta(t)} \quad (8)$$

where $\hat{\sigma}$ is the gradient to growth rate computed from the estimates values for $\dot{\hat{\mu}}$ and $\dot{\hat{s}}$. The signal $\delta(t)$ is incorporated to induce persistent excitation to the feedback controller avoiding the divergence on the estimated term $\hat{\sigma}$. Finally, the robust sliding self-optimizing control is given by

$$u(x) = \frac{\hat{\mu}x}{Y(s_f - s)} + K sign(\hat{\sigma}) \quad (9)$$

In this way, the resulting self-optimizing controller is composed by variable-structure feedback (9) where the gradient and growth rate are replaced by its estimate given by (7-8).

Proposition 3.4. *Consider the CSTBR (1-2) and suppose that the properties 1-4 and assumptions 3.1-3.2 are satisfied. Then, the variable-structure controller described in (6-9) achieves the practical asymptotic stabilization around of the optimal concentration $s^* \in \Omega$.*

Proof : From Proposition 3.3 it is direct to see that the control law (4) achieves the sliding regime on the optimal manifold Σ and, consequently, achieves the asymptotic substrate stabilization to the optimal concentration s^*. However, it cannot be directly implement due to unknown terms in growth rate and their gradient. Then, an observer-based estimator is proposed (6-8). Let $e(t) = [\mu - \hat{\mu}, \sigma - \hat{\sigma}]^T$ the estimation error vector, since the observer's gain $\omega, \gamma > 0$ are chosen as $\omega = 2\lambda$ and $\gamma = \lambda^2/x^2$, such that the dynamics of the corresponding estimation error system are governed by the double eigenvalue $-\lambda$; then is follows that the estimator scheme (9-11), with $\delta(t) \in C^1$ as an persistent excitation signal, guarantees the practical convergence of $e(t)$ to an arbitrary small neighborhood of the origin (i.e., $e(t) \to \epsilon$ as $t \to \infty$, with $\epsilon > 0$) [Teel & Praly, (1995)]. This implies that the self-optimizing controller (9) performance will tend asymptotically at (4) as the time tends to infinity. As a consequence, under the properties 1-4 and assumption 3.1-3.2, the self-optimizing controller (6-9) will be able to achieve the practical asymptotic substrate stabilization around an optimal concentration value s^*. □

4 Numerical Implementation

In order to illustrate the self-optimizing controller performance respect to changes in the optimal set-point value s^* due to modeling uncertainty, numerical experiments for CSTB (1-2) with different non-monotonic growth rates are presented. Nominal values for the CSTB model and the gains for the controller (6-9) are shown in Table 1, the gain observer are selected such that the dynamics for the estimation error has eigenvalues at $\lambda = -10$. For these numerical simulations were used three different growth kinetics models with corresponding values to optimal substrate concentration are 1, 3 and 5 g/L, respectively (see Table 2).

Table 1. Nominal values for CSTB and control parameters

s_f $[g/L]$	Y	K	ω	γ
10	0.5	0.005	6.0	0.5625

Table 2. Parameter values of the growth rates used

Model	K_S $[g/L]$	K_I $[L/g]$	μ_m $[1/h]$	s^*
a	0.5	0.5	1.6	1.0
b	0.9	0.1	1.3	3.0
c	1.0	0.04	1.13	5.0

The stabilization to neighborhood at the optimal manifold Σ for different initial conditions is shown in Fig. 3. It is observed that the trajectories to the closed-loop system are confined to the sliding manifold Σ (for growth rate with $s^* = 3.0$). The performance to the control strategy (6-9) is shown in the Figure 4. In Figure 4, robust stabilization on the optimal conditions to the CSTB system (1) is shown respect to changes in the optimal set-point value s^*. The substrate concentration to closed-loop system converge towards the unknown set-points s^* corresponding to 1, 3 and 5 respectively (Fig. 4a). Note that at optimal set-point s^* for the controlled systems, leads to maximum corresponding growth rate $\overline{\mu}$, as was established in the control objective. The Figure 4b shows the smooth control signal for adequate values at

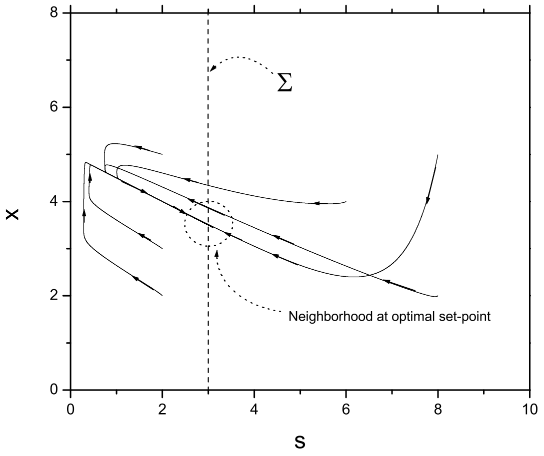

Figure 3. Stabilization on the optimal manifold Σ to different initial conditions for the growth model with $s^* = 3$.

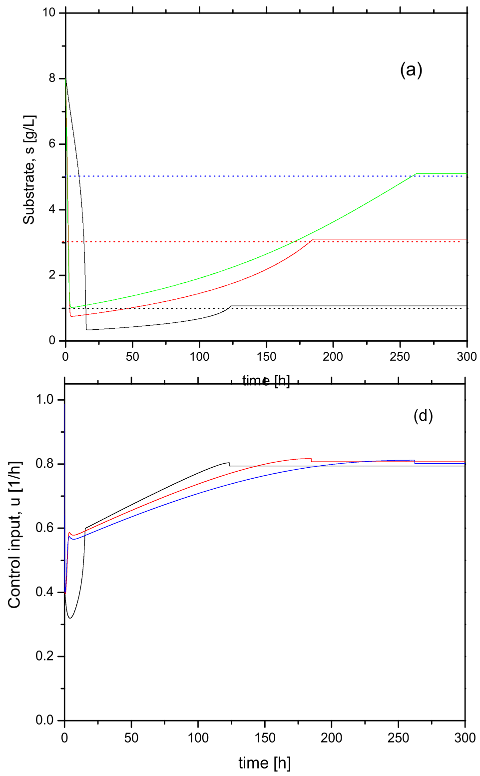

Figure 4. Robust stabilization for changes in the optimal set-point value s^*, the controlled system is driven to the neighborhood of the optimal manifold Σ.

$t = 0$, this is an interesting issue in contrast to others control structures reported in the literature. To improve the performance of self-optimizing strategy proposed in this work, the design of more sophisticated estimation algorithms to gradient to growth rate σ, that leads the asymptotic convergence features as incorporating integral action in the observer structure as proposed in [Aguilar(2002)]; or observers based in sliding modes to both substrate concentration and growth rate [Fu and Ozguner(2009); Pico et al., (2009)].

4.1 Comparison with linear controllers

It is well known that the linear structure controllers are very used in industrial applications involving growth of microorganism. In order to provide a brief comparison of the self-optimizing controller performance, the following linear-structure controllers have been considered.

4.1.1 Robust asymptotic feedback with linear structure (RLC)

Using ideas from modeling-error compensation techniques [Alvarez-Ramírez et al., (1999); Méndez-Acosta et al., (2005)], the robust asymptotic feedback for stabilization of the bioreactor (1-2) at the optimal substrate concentration value s^* is designed.

$$u = \frac{1}{s_f}[\hat{\eta} - K_c(s - s^*)], Kc > 0 \tag{10}$$

where $\hat{\eta}$ is the estimated value to the lumping uncertainties term defined as $\eta = Y^{-1}\mu(s) + su$. The estimated values for (s, η) are given by the following observer-based estimator

$$\dot{\hat{s}} = s_f u - \hat{\eta} + \kappa_1(s - \hat{s}) \tag{11}$$

$$\dot{\hat{\eta}} = \kappa_2 L^2(s - \hat{s}) \tag{12}$$

the observers parameters $\kappa_{1,2}$ are chosen such that the polynomial $P(\lambda) = \lambda^2 + \kappa_1\lambda + \kappa_2$ is Hurwitz, and $L > 0$ is a positive parameter (high-gain observer).

4.1.2 Linear Quadratic Regulator (LQR)

LQR is one of the most commonly used and power ways of controlling linear systems [Kwakernaak and Sivan (1972)]. Now, the LQR-controller will bee used as a reference controller for comparison the self-optimizing control (6-9). In order to design the LQR, the linear state space representation is obtained by linearization the bioreactor model (1-2) at the optimal equilibrium $(s^*, Y(s_f - s^*)) \in \mathbb{R}^2$; analyzing in [Lara-Cisneros et al., (2012)] $\dot{x} = Ax + bu$ where $A = \begin{pmatrix} -0.82 & -1.64 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 7 \\ 3.5 \end{pmatrix}$, with a quadratic cost function defined as $J(x,u) = \int_0^\infty [x^T Q x + u^T R u] dt$. The feedback control law that minimizes the value of the cost has the form $u = \overline{u} - Kx$. For the weighting matrices $Q = 10I_2$ and $R = 1$ the resulting state feedback gain is $K = [0\ 7.0711]$.

The comparison between the linear-structure controllers with the self-optimizing control law (6-9) are shown by numerical simulation in the Figure 5. The simulation was carried out with small parametric uncertainties in the kinetics parameters around of 10 percent of the nominal values, and changes on nominal value for s^* each 10 hours (Fig. 5). The self-optimizing controller (6-9) shows acceptable performance for the tracking of the optimal substrate concentration s^* whereas the LQR and RLC cannot track the optimal s^*.

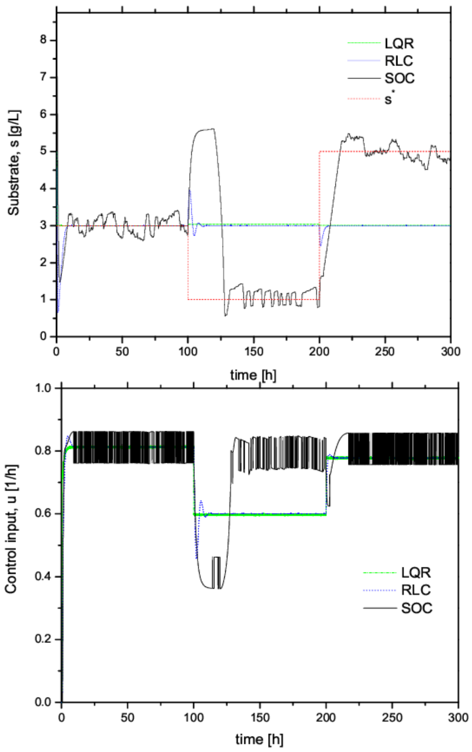

Figure 5. Closed-loop performance of the Self-optimizing controller (SOC) compared with the Robust asymptotic feedback with linear structure (RLC) and Linear Quadratic Regulator (LQR). The simulation was carried out with parametric uncertainties in the kinetics parameters, and changes on nominal value for s^* at interval on 100 hours.

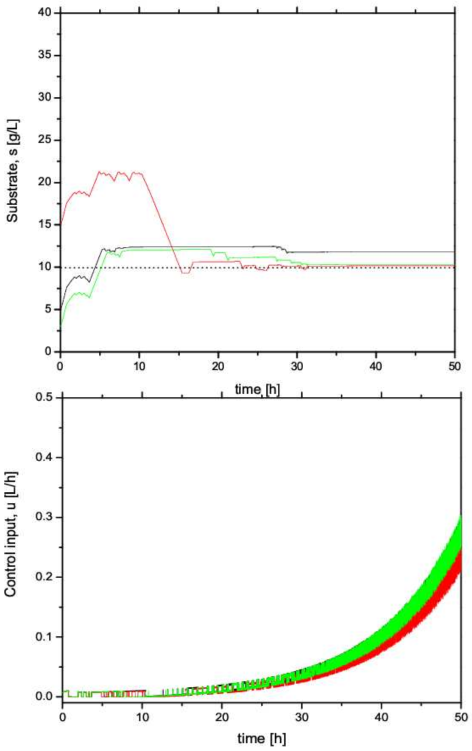

Figure 6. Optimal stabilization of fed-batch bioreactor (13-15) around of $S^* = 10g/L$ with perturbations on the kinetics parameters for different initial conditions.

4.2 Microbial growth process in feed-batch bioreactor

In order to show the applicability of the controller proposed, in this section the optimal stabilization problem for a class of microbial growth process in fed-batch operating mode is considered. The model has been proposed in [Pict(1965)] from experimental data for glycerol-limited growth of Aerobacter aerogenes.

$$\frac{dX}{dt} = \mu(S) - \frac{F}{V}X \tag{13}$$

$$\frac{dS}{dt} = -[k_1\mu(S) + m] + \frac{F}{V}(S_f - S) \tag{14}$$

$$\frac{dV}{dt} = F \tag{15}$$

where states X and S hold for bacterial and glycerol concentrations, respectively, μ is the specific growth rate, k_1, m are the yield coefficient and the maintenance constant, respectively, and F, the control input, is the inflow. The control objective is to design a self-optimizing controller of the form (6-9) such that the optimal stabilization around of S^* is achieved, and the growth rate into the fed-batch bioreactor is maximized. The self-optimized controller for the inflow is given by

$$F = \frac{V(k_1\hat{\mu} + m)}{S_f - S}X + Ksign(\hat{\sigma}) \tag{16}$$

$$\hat{\sigma} = \frac{\gamma x(x - \hat{x})}{-[\bar{k}_1\mu(S) + \bar{m}] + \frac{F}{V}(S_f - S) + \delta(t)} \tag{17}$$

$$\dot{\hat{x}} = (\hat{\mu} - F/V)x + \omega x(x - \hat{x}) \tag{18}$$

$$\dot{\hat{\mu}} = \gamma x(x - \hat{x}) \tag{19}$$

The performance of the controller (16-19) is evaluated for numerical simulations with aleatory perturbations of the kinetics parameters k_1 and m around of 10 percent on the nominal values reported in [Smets et al., (2002)]. The controller (16-19) shown well performance for the optimal stabilization of fed-batch bioreactor (13-15) with perturbation on the kinetics parameters (Fig. 6).

5 Conclusion

From the approach of the biochemical reactors as positive compartmental systems, a simple self-optimizing controller has been developed for stabilization towards the unknown optimal set-point (respect to maximal biomass production) to a class of bioreactors with a non-monotonic biomass growth rate. From previous analysis for bioreactor in open-loop mode, showed that the optimal operation conditions are structurally unstable. In this way, the control strategy developed, which is based in sliding modes techniques, reaches the optimal stabilization on the neighborhood at the optimal manifold, defined in the sense of maximizing the growth rate to microorganism into the bioreactor. The self-optimizing control is coupled with uncertainty estimator scheme for computing the growth rate kinetics as well as its gradient. The proposed self-optimizing controller contain only three tuning parameters and does not require on-line measurement of the performance variable. It is important to note that the designed controller may be applied for the optimal stabilization of bio-processes when biomass growth rate is governed by a different growth law. Numerical experiments for different growth rates showed the performance of the controller.

The generalization of these classes of self-optimizing controller for bioprocesses with multi-substrate and product inhibition need to be looked into more deeply.

Acknowledgements

This work is supported by the project "SPectroscopies for the Evaluation and ConTrol in Real time of biological cells physiological statE *(SPECTRE)* S-3933"

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