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MODELING OF THE COOPERATIVE NEURAL DYNAMICS THROUGH MAP BASED APPROACH

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Abstract

In last decades map-based neural models have attracted much attention. These models replicate the most important features of neurons dynamics and allow to simplify the analysis of the cooperative dynamics appearing in the complicate neural system. In this work we have studied the cooperative dynamics appearing in the ensembles of map-based neurons. Synchronization regimes of two coupled map-based neurons are analyzed. The possible mechanism for control of the synchronous properties of the distant elements in the neural ensemble is proposed.

Key words

Neuron, chaos, synchronization, control.

1 Introduction

The nervous system is an extremely complex system comprising nerve cells (or neurones) and glial cells. By electrical and chemical synapses of different polarity neurones form a great variety of large-scale networks. Dynamical modeling approach is effective tool for the analysis of this kind of networks. The observed types of neural activity are extremely various. Understanding dynamical mechanism of such activity in biological neurones has stimulated the development of models on several levels of the complexity. To explain biophysical membrane processes in a single cell, it is generally used ionic channel-based models. The prototype of those models is the Hodgkin-Huxley (HH) system which was originally introduced in the description of the membrane potential dynamics based on voltage-gated ion channels in the giant squid axon. In 1952 in a series of five articles Alan Hodgkin, Andrew Huxley and Bernard Katz made detection of the several types of ionic conductances that generate the nerve action potential (electric impulse). Besides they have developed the dynamical model of the propagation of action potentials along the axon. Although HH model was obtained for a specific problem, its base ideas have universal nature and it induced new approach in neurophysiological modeling. Experimental background to create the HH model was experiments with respect to division of ionic currents through the membrane of the axon.

The HH model is extremely complex dynamical system in four-dimensional phase space, and its detailed analysis is a very difficult problem, which hasn't been completely solved even today. However the existence of some properties of the ionic channels permits using of the simplified models which have less complicate mathematical structure but still replicates the key features of neural dynamics. They are based on separation of the fast and slow processes on the membrane and their modeling with different fullness of description. The HH model lay down the foundation of the series of the models (so-called conductance based models). They describe basic property of neuron dynamics, but these models do not take into account the large number of voltage-gated ion channels of neurones. As a rule it involves generalized variables which mimic the dynamics of the some number of ionic currents at the same time. The example of this type models are FitzHugh-Nagumo, Hindmarsh-Rose, Morris-Lecar etc. They have the form of the differential equation systems. However, there is another class of models in the form of point maps. Such type of models are called "map-based" models. In the last decade this kind of neural models has attracted much attention. For example, using a map-based approach Rulkov et al. [Rulkov, 2002; Rulkov, 2004] have studied dynamics of one- and two-dimensional large-scale cortical networks. It has been found that such map-based models produce spatio-temporal regimes similar to those exhibited by Hodgkin-Huxley type models.

In this paper we report about modeling of the

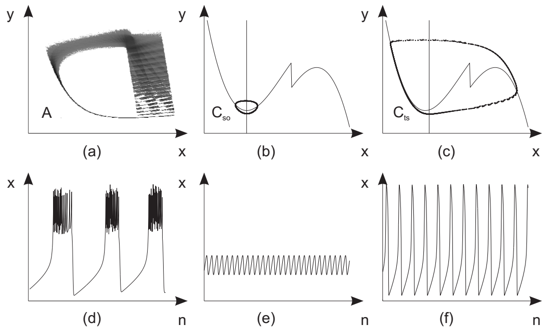

Figure 1. Dynamics of the single map-based neuron.

cooperative regimes of the neural activity in the ensembles of map-based neurons. In doing so, we use the map-based neurons model which has been suggested recently in [Courbage, 2007; Nekorkin, Vdovin, 2007; Nekorkin, Vdovin, 2007].

2 Map based model of the neural activity

In order to model basic regimes of neural activity we design a new family of maps that are two-dimensional and based on the discrete FitzHugh-Nagumo system in which we introduce the Heaviside step function. Let $f : R^2 \to R^2$ be the map $(x, y) \to (\overline{x}, \overline{y})$ given by the following equations

$$\begin{cases} \overline{x} = x + F(x) - y - \beta H(x - d), \\ \overline{y} = y + \varepsilon(x - J), \end{cases} \tag{1}$$

where the x-variable describes the evolution of the membrane potential of the neuron, y - variable describe the dynamics of the outward ionic currents (recovery variable), the functions $F(x)$ and $H(x - d)$ are of the form

$$F(x) = x(x - a)(1 - x), 0 < a < 1, \tag{2}$$

$$H(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0. \end{cases} \tag{3}$$

The parameter ε ($\varepsilon > 0$) defines the time scale of recovery variable, the parameter J is a constant stimulus, the parameter d, ($d > 0$) controls threshold property of the model and parameter β controls amplitude of the fast oscillations.

Due to the smallness of the parameter ($\varepsilon > 0$) the evolution of variable y is much slower then x and therefore the dynamics of the map (1) is a relaxation. The distinctive characteristic of these systems is two time and velocity scales, so-called "fast" and "slow" motions. Basically fast motions are provided by a "frozen" system in which slow variables are regarded as parameters, and it is assumed that a small parameter of the system equals zero. Slow motions with the size of order of the small parameter are given by the evolution of the "frozen" variable.

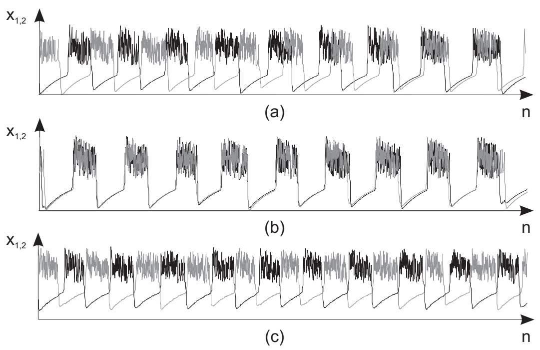

Figure 2. Regimes of synchronization of two coupled map-based neurons.

The dynamics of the system was studied in [Courbage, 2007; Nekorkin, Vdovin, 2007; Nekorkin, Vdovin, 2007]. The conditions on the parameters under which different oscillations modes are produced by map f were found. It was shown that the chaotic attractor A exists inside invariant domain (Fig.1 (a)) and its fractal dimension is a rational value. Chaotic attractor provides simulating of spike-bursting oscillations (Fig.1(d)) which are commonly observed in a wide variety of neurons. It was shown that two close invariant curves C_{so} (Fig.1(b)) and C_{ts} (Fig.1(c)) exist under different values of the parameter J. The invariant curve C_{so} corresponds to quasiperiodic subthreshold oscillations (Fig.2(e)) and the invariant curve C_{ts} corresponds to periodic single spikes generation (Fig.2(f)). The system (1) is able to demonstrate excitable dynamics and generate the single or burst of spikes as the response on the external stimulus as well.

By using this simple and flexible model we have studied different cases of the cooperative dynamics of the neural systems.

3 Regimes of synchronization of two coupled map-based neurons

Let us consider a simple network of two electrically coupled map-based neurons. The model of such network can be written as follows

$$\begin{cases} \overline{x_1} = x_1 + F(x_1) - y_1 - \beta H(x_1 - d) + c(x_2 - x_1), \\ \overline{y_1} = y_1 + \varepsilon(x_1 - J_1), \\ \overline{x_2} = x_2 + F(x_2) - y_2 - \beta H(x_2 - d) + c(x_1 - x_2), \\ \overline{y_2} = y_2 + \varepsilon(x_2 - J_2). \end{cases} \tag{4}$$

The study of synchronization regimes in coupled chaotically spiking-bursting neurons is of most interest for us. First, consider the main regimes of synchronization between the elements generating chaotic bursts of spikes. Let $J_1 = J_2 = J$. When the neurons are uncoupled ($c = 0$) they produce chaotic spiking-bursting oscillations (Fig. 2(a)). In the case of strong positive coupling bursts of spikes becomes synchronized, but single spikes in the burst are generated asynchronously (Fig. 2 (b)). If the neurons are connected via inhibitory coupling with negative value of the parameter c, the elements demonstrate synchronous antiphase bursts of spikes (Fig. 2(c)). These regimes were experimentally observed in neurophysiological studies of coupled neurons.

To study in-phase synchronization in relation to the strength of coupling and the external stimulus we calculate the coefficient of synchronization defined as

$$D^2(c, J) = \frac{1}{N}\sum_{k=1}^{N}[x_1(k) - x_2(k)]^2, \qquad (5)$$

where N is the number of iterations in the numerical experiment (Fig.3). According to our analysis, there is a minimal value of the parameter c under which in-phase synchronization takes place. Critical value has the increasing dependency versus parameter J.

4 Control of the synchronous properties in the ensemble with long-range distribution of the neurons

We have studied possible dynamical mechanism for long-range synchronicity driving of the neural oscillations by using external stimulus. It is expected that the neurons are widely spaced from each other. As the result the interaction of the neurons takes place with large delay. Because of this,l synaptic coupling makes no sense in first approximation.

Let us consider a neural network consisting of M map-based neurons under external stimulus. This network could be designed in the following form:

$$\begin{cases} \overline{x_i} = x_i + F(x_i) - y_i - \beta H(x_i - d), \\ \overline{y_i} = y_i + \varepsilon[x_i - J_i - J^*(n)], \end{cases} \qquad (6)$$

where external pulse stimulus $J^*(n)$ is defined as follows

$$J^*(n) = \begin{cases} A, & n \in [n_1, n_2], \\ 0, & n \in (-\infty, n_1) \cup (n_2, \infty). \end{cases} \qquad (7)$$

We have studied response of the ensemble generating chaotic bursting oscillations on the common pulse stimulus. Two situations are discussed: synchronization of the "slow" and "fast" oscillations under inhibitory stimulus and resetting of the synchronous oscillations under activator stimulus. We have observed dynamical mechanisms for these processes and studied dependencies of the synchronous oscillations properties on the parameters of the neurons and external stimulus.

It was shown that there is the threshold value for inhibitory stimulus under which synchronous "slow" oscillations appears due to stimulation. We have found the conditions for synchronization of "fast" oscillations in the ensemble. Synchronous modes are not stable and disappear on the time scale kT, where T is a time scale for "slow" oscillations and $k > 1$ because attractor is chaotic. We have found condition on the parameters under which activator stimulus magnifies asynchronous properties of the oscillations as well. Due to this effect time scale for disappearing of the synchronous mode could be reduced ($k \sim 1$).

Let us consider neural ensemble consisting of the set of groups of the elements. For each of them parameters J_i are equal. Different groups have the different value for parameter J_i, but all elements in ensemble produces the chaotic spiking-bursting oscillations. Such system demonstrates selective response to the external stimulus depending on stimulation amplitude. This process is de-

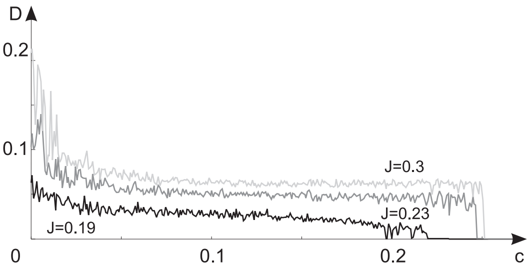

Figure 3. Coefficient of synchronization versus coupling strength.

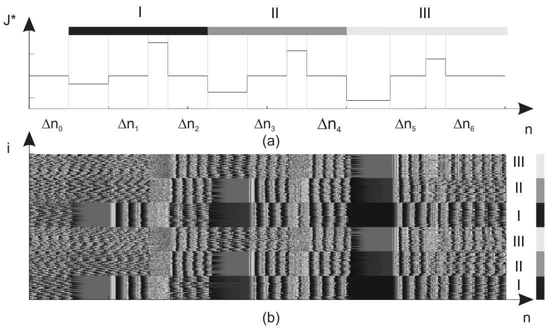

Figure 4. Driving synchronicity properties in the ensemble.

scribed on Fig.4. On the time interval Δn_0 ensemble produce asynchronous oscillations. For time interval Δn_1 after applying the inhibitory stimulus with amplitude optimal for synchronization of the group I, this group becomes synchronized, but for the groups II and III this stimulus makes no sense. For time interval Δn_2 synchronous properties for the group I are minimized due to activator stimulus applying. For time interval Δn_3 "slow" oscillations are synchronized for both the groups I and II, but the first burst produced by the group II is synchronized for "fast" oscillations also. This feature is obtained due to applying of the inhibitory stimulus with amplitude optimal for synchronization of the group II. This mechanism allows driving the phase for chaotic oscillation in the ensemble by using external stimulus. As the result of this research we have found the dependencies of the synchronous properties of the oscillations in the ensemble on the parameters of the elements and external stimulus.

5 Conclusion

In this paper a phenomenological model of neural activity is proposed. It can reproduce basic activity modes of real biological neurons such as spiking, chaotic spiking-bursting, subthreshold oscillations etc. The model is a discontinuous two-dimensional map based on the discrete version of the FitzHugh-Nagumo system and dynamical properties of the Lorenz-type map. We have investigated the cooperative dynamics of electrically coupled map-based neurons generating spiking-bursting oscillations. We have analyzed how synchronization regimes of two coupled map-based neurons depends on coupling strength and other parameters. We have studied the possible underlying mechanism for controling the synchronous properties of neural ensemble. In spite of model's sufficiently simple structure, the dynamical modes which are demonstrated in our model are in agreement with neural activity regimes experimentally found in real biological systems. We hope that our model will be useful to understand the mechanisms of neural pattern formation in large ensembles.

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