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CONTROL AND HIGH FIDELITY QUANTUM STATE TRANSFER BETWEEN ATOMS COUPLED TO A DISTINCT MICRO-TOROIDAL CAVITIES

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Abstract
Hybrid quantum systems usually composed by electromagnetic systems have providing an efficient route for build scalable integrated photonic circuit techniques. Here, we have proposed, to control and transfer the entangled quantum states between two atoms, a hybrid system comprising of micro-toroidal cavities and atoms connected via electromagnetic fields. Each cavity supports two counter-propagating whispering-gallery modes coupled simultaneously to an atom through their evanescent fields. The superposition state of atom 1 coupled the micro-toroidal cavity 1 can be transferred to the atom 2 coupled to micro-toroidal cavity 2 with high fidelity. Another interesting fact is that, presence of structural deformation in cavity can induces interaction between the gallery modes and generating entanglement between them and allowing the transferring these entangled states between the cavities. The results have shown that the quantum state transfer under dissipation is still trustworthy and providing an effective path to communication and quantum information.

Key words
Micro-toroidal cavity, two-atoms, quantum state transfer, entanglement

1 Introduction

Transferring quantum information between two or more sites without disturbing the stored information is one of the great challenges for development of the quantum technologies. Basically, the quantum information processing can be understood as a quantum network, these which in turn consist of distant nodes connected by quantum communication channels. Each nodes can process, store and distribute the information under the network via reversible and irreversible processes channels [Boozer et al., 2007]. Numerous proposals have been realized - both experimental and theoretical - for implementation of the atoms coupled to optical cavities to become the nodes that make up a quantum network [Raimond, Brune, and Haroche, 2001]. In this case, coupled optical cavities can be implemented as quantum channels [Cirac et al., 2009]. Different architectures of cavities have been developed (microfabricated) due to need for ultrahigh factors (Q) and scalability to large number of devices [Kimble, 2008]. For this propose, micro-toroidal or micro-spherical resonators have present a technical feature to achieve efficient optical communications, such as, small-mode-volume, ultrahigh quality factors and coherent strong interactions between matter and light [Spillane et al., 2003]. Therefore it would be interesting to extend their research to consider the dynamics of quantum state transfer in a system formed by two-level atoms coupled to a distinct micro-toroidal cavities.

In this paper, we investigate the dynamics of two coupled micro-toroidal cavities via the evanescent field of two intracavity modes and where each one of them is coupled to a single two-level atom. Our main result consist in high-fidelity transfer a superposition state of one atom which is coupled to a cavity for the other atom which is coupled to another cavity, taking into account mechanism of the system losses. The possibility of the coupled between the intracavity modes (generated for cavity imperfection) for the dynamical entanglement between the atoms is also studied.

2 The Model

Our system consist of two coupled micro-toroids interacting with two-level atoms as shown in Fig. 1. The micro-toroids and atoms are depicted by label $i = 1, 2$. The two degenerate counter-propagating whispering-gallery modes (WGM's) of frequency ω_{Ci} , with annihilation (creation) operators $\hat{a}_i$ ($\hat{a}_i^\dagger$) and $\hat{b}_i$ ($\hat{b}_i^\dagger$) of each one of the cavities, are coupled simultaneously to a single two-level atom with coupling constant g_i and transition frequency ω_{eg}^i . We assume that the interaction of the atoms and toroid with the surrounding environment is described by spontaneous emission rate of the atoms (γ_A) and cavity decay rate (κ). We also consider the intermode backscattering between the two WGM's (strength constant J_i) induced by small deformation in the toroid [Spillane et al., 2003].

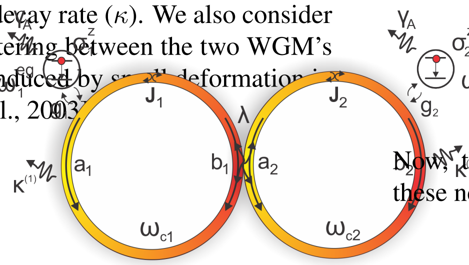

Figure 1. Scheme experimental of the two atoms- two micro-toroidal cavity system. Each cavity consists of two modes coupled to a two-level atom.

According to the above scheme, the Hamiltonian of the atom-microtoroid system is given by

$$H = H_{A1C1} + H_{A1C2} + H_{C1C2} \qquad (1)$$

where

$$\begin{aligned} H_{A1C1} = {} & \hbar\omega_1^{eg}\sigma_1^+\sigma_1^- + \hbar\omega_{C1}(\hat{a}_1^\dagger\hat{a}_1 + \hat{b}_1^\dagger\hat{b}_1) + \\ & \hbar J_1(\hat{a}_1^\dagger\hat{b}_1 + \hat{b}_1^\dagger\hat{a}_1) + \hbar(g_1^*\hat{a}_1^\dagger\sigma_1^- + g_1\hat{a}_1\sigma_1^\dagger) + \\ & \hbar(g_1\hat{b}_1^\dagger\sigma_1^- + g_1^*\hat{b}_1\sigma_1^\dagger), \qquad (2a) \\ H_{A2C2} = {} & \hbar\omega_2^{eg}\sigma_2^+\sigma_2^- + \hbar\omega_{C2}(\hat{a}_2^\dagger\hat{a}_2 + \hat{b}_2^\dagger\hat{b}_2) + \\ & \hbar J_2(\hat{a}_2^\dagger\hat{b}_2 + \hat{b}_2^\dagger\hat{a}_2) + \hbar(g_2^*\hat{a}_2^\dagger\sigma_2^- + g_2\hat{a}_2\sigma_2^\dagger) + \\ & \hbar(g_2\hat{b}_2^\dagger\sigma_2^- + g_2^*\hat{b}_2\sigma_2^\dagger), \qquad (2b) \\ H_{C1C2} = {} & \hbar\lambda(e^{-i\phi}\hat{a}_1^\dagger\hat{b}_2 + e^{i\phi}\hat{b}_2^\dagger\hat{a}_1 + e^{-i\phi}\hat{b}_1^\dagger\hat{a}_2 + \\ & e^{i\phi}\hat{a}_2^\dagger\hat{b}_1) \qquad (2c) \end{aligned}$$

with $\hbar\omega_i^{eg}$ denotes the energy required of separation between of the atom i ($i = 1, 2$) for excited and ground states by $|e\rangle_i$ and $|g\rangle_i$, $\sigma_i^+ = |e\rangle_i\langle g|$ and $\sigma_i^- = |g\rangle_i\langle e|$ are the raising and lowering operators of the atom i, λ is the coupling constant between the two micro-toroid and determine the speed of the energy transfer between them [Zhou et al., 2014]. The phase ϕ take into account the propagation distance between the micro-toroids. For neglect effects of the retardation at time of flight of the light, a short distance limit between the toroids should be imposed.

In the Eqs.(2) H_{A1C1} and H_{A2C2} describe the first and second atom-toroid interacting systems, respectively, and H_{C1C2} describes the coupling between the toroids. In the resonate regime ($\omega_{C1} = \omega_{C2} = \omega$) and $\phi = 0$, it is possible to diagonalize the Hamiltonian that represents the interaction between the cavities using the basis of new bosonic operators giving by the unitary transformation:

$$\begin{aligned} \hat{B}_1 &= \frac{1}{\sqrt{2}}(-\hat{a}_1 + \hat{b}_2); & \hat{B}_2 &= \frac{1}{\sqrt{2}}(-\hat{b}_1 + \hat{a}_2); \\ \hat{B}_3 &= \frac{1}{\sqrt{2}}(\hat{a}_1 + \hat{b}_2); & \hat{B}_4 &= \frac{1}{\sqrt{2}}(\hat{b}_1 + \hat{a}_2); \end{aligned} \qquad (3)$$

Now, the Hamiltonian of the system, in function of these new operators, can be rewritten in the form:

$$\begin{aligned} H = {} & \sum_{i=1}^{4}\hbar\Omega_i\hat{B}_i^\dagger\hat{B}_i + \sum_{i=1}^{2}\hbar\omega_i^{eg}\sigma_i^+\sigma_i^- + \\ & \hbar(g_1^*\hat{a}_1^\dagger\sigma_1^- + g_1\hat{a}_1\sigma_1^\dagger) + \hbar(g_1\hat{b}_1^\dagger\sigma_1^- + g_1^*\hat{b}_1\sigma_1^\dagger) + \\ & \hbar(g_2^*\hat{a}_2^\dagger\sigma_2^- + g_2\hat{a}_2\sigma_2^\dagger) + \hbar(g_2\hat{b}_2^\dagger\sigma_2^- + g_2^*\hat{b}_2\sigma_2^\dagger) \qquad (4) \end{aligned}$$

where $\Omega_1 = \omega - \lambda - J$, $\Omega_2 = \omega - \lambda + J$, $\Omega_3 = \omega + \lambda - J$ and $\Omega_4 = \omega + \lambda + J$ are the frequencies of the normal modes $\hat{B}_1$, $\hat{B}_2$, $\hat{B}_3$ and $\hat{B}_4$, respectively. In order to obtain a Hamiltonian in interaction picture, we imposed two conditions on the system: $\Omega_3 = \omega_i^{eg}$ and $\lambda \gg g, J$. Under these two conditions, we obtained the following Hamiltonian

$$\begin{aligned} \tilde{H}_I = {} & \frac{\hbar}{2}(g_1^*\hat{B}_3^\dagger\sigma_1^- + g_1\hat{B}_3\sigma_1^\dagger) + \\ & \frac{\hbar}{2}(g_1\hat{B}_3^\dagger\sigma_1^- + g_1^*\hat{B}_3\sigma_1^\dagger) + \\ & \frac{\hbar}{2}(g_2^*\hat{B}_3^\dagger\sigma_2^- + g_2\hat{B}_3\sigma_2^\dagger) + \\ & \frac{\hbar}{2}(g_2\hat{B}_3^\dagger\sigma_2^- + g_2^*\hat{B}_3\sigma_2^\dagger). \qquad (5) \end{aligned}$$

The temporal evolution of the system is obtained using the Schrödinger equation in the interaction picture, i. e.:

$$\frac{d}{dt}|\tilde{\psi}(t)\rangle = -\frac{i}{\hbar}\tilde{H}_I|\tilde{\psi}(t)\rangle \qquad (6)$$

where $|\tilde{\psi}(t)\rangle$ is the state vector of the system at time t (in the interaction picture). The pure state of the system at time t, in the base $\{|k_1\rangle_{B1}|k_2\rangle_{B2}|k_3\rangle_{B3}|k_4\rangle_{B4}\}$, of states associated to operators $\hat{B}_1$, $\hat{B}_2$, $\hat{B}_3$ and $\hat{B}_4$, is given by

$$|\tilde{\psi}(t)\rangle = \sum_{k1=0}^{\infty}\sum_{k2=0}^{\infty}\sum_{k3=0}^{\infty}\sum_{k4=0}^{\infty}[C^{gg}_{k1,k2,k3,k4}(t)|g\rangle_1|g\rangle_2 + C^{ge}_{k1,k2,k3,k4}(t)|g\rangle_1|e\rangle_2 + C^{eg}_{k1,k2,k3,k4}(t)|e\rangle_1|g\rangle_2 + C^{ee}_{k1,k2,k3,k4}(t)|e\rangle_1|e\rangle_2]|k_1\rangle|k_2\rangle|k_3\rangle|k_4\rangle, \quad (7)$$

where the coefficients obey the following set of differential equations,

$$\begin{aligned}
\frac{d}{dt}C^{gg}_{k_1,k_2,k_3,k_4}(t) &= -ig[\sqrt{k_3}C^{eg}_{k_1,k_2,k_3-1,k_4}(t) + \sqrt{k_3}C^{ge}_{k_1,k_2,k_3-1,k_4}] \\
\frac{d}{dt}C^{ge}_{k_1,k_2,k_3,k_4}(t) &= -ig[\sqrt{k_3}C^{ee}_{k_1,k_2,k_3-1,k_4}(t) + \sqrt{k_3+1}C^{gg}_{k_1,k_2,k_3+1,k_4}] \\
\frac{d}{dt}C^{eg}_{k_1,k_2,k_3,k_4}(t) &= -ig[\sqrt{k_3+1}C^{gg}_{k_1,k_2,k_3+1,k_4}(t) + \sqrt{k_3}C^{ee}_{k_1,k_2,k_3-1,k_4}] \\
\frac{d}{dt}C^{ee}_{k_1,k_2,k_3,k_4}(t) &= -ig[\sqrt{k_3+1}C^{ge}_{k_1,k_2,k_3+1,k_4}(t) + \sqrt{k_3+1}C^{eg}_{k_1,k_2,k_3+1,k_4}]
\end{aligned} \quad (8)$$

and $|g\rangle_i$ and $|e\rangle_i$ correspond, respectively, to the fundamental and excited states of atoms i and $|k\rangle_l$ is the Fock state of the mode related to the operator $\hat{l}$ ($\hat{a}_1$,$\hat{a}_2$, $\hat{b}_1$,$\hat{b}_2$, $\hat{B}_3$). For sake of simplify, we assume that $g_1 = g_2 = g$. Having in mind the quantum state processing our main objective here is the quantum state transfering between the two atoms, following the quantum transmission between two qubit as defined by [Cirac et al., 2009], i.e.,

$$(c_a|1\rangle_1 + c_b|0\rangle_1) \otimes |0\rangle_2 \Rightarrow |0\rangle_1 \otimes (c_a|1\rangle_2 + c_b|0\rangle_2)$$

where c_a and c_b are complex numbers. We can understanding the quantum state transmission process matching tomographically the evolved state of the system or subsystem on the initial state, such as, a subsystem having the first qubit in a superposition state (in Fock bases) and the second in vacuum state, and at certain elapsed time t the first qubit of subsystem is projected on the vacuum state and second in a superposition state, performing a perfect and complete transmission. In our case, the transmission of the quantum state is obtained if the system makes the transition between the following states:

$$(\cos\theta|g\rangle_1 + e^{i\alpha}\sin\theta|e\rangle_1)|g\rangle_2|00\rangle_{c1}|00\rangle_{c2} \Rightarrow |g\rangle_1(\cos\theta|g\rangle_2 + e^{i\alpha}\sin\theta|e\rangle_2)|00\rangle_{c1}|00\rangle_{c2}$$

where $|00\rangle_{c1} = |0\rangle_{a1} \otimes |0\rangle_{b1}$ and $|00\rangle_{c2} = |0\rangle_{a2} \otimes |0\rangle_{b2}$. Note that, according to the quantum state transfer above, preparing the initial state of our system as $|\psi\rangle_i = (\cos\theta|g\rangle_1 + e^{i\alpha}\sin\theta|e\rangle_1)|g\rangle_2|00\rangle_{c1}|00\rangle_{c2}$ the goal is to obtain the final state $|\psi\rangle_f = |g\rangle_1(\cos\theta|g\rangle_2 + e^{i\alpha}\sin\theta|e\rangle_2)|00\rangle_{c1}|00\rangle_{c2}$. Following the time evolution of the quantum state it is possible find the exact time when the state of the atom 1 is completely transferred to the atom 2. We have exploited this dynamics using the *Fidelity* as defined by [Nielsen, Chuang, and Grover, 2000]

$$\mathcal{F} = \langle\psi_f|\hat{\rho}_{A1A2}(t)|\psi_f\rangle \quad (9)$$

where $\hat{\rho}_{A1A2}(t)$ is the reduced density matrix of the two atoms defined by $\hat{\rho}_{A1A2}(t) = Tr_{C1}\{Tr_{C2}\{|\tilde{\psi}(t)\rangle\langle\tilde{\psi}(t)|\}\}$. In order to observe the dynamics of state transfering and degree of entanglement between the atoms, we also have used the *Negativity*, as proposed by [Vidal and Werner ,2002]

$$\mathcal{N} = \sum_i |\mu_i^-| \quad (10)$$

where μ^- are the negative eigenvalues of $\hat{\rho}_{A1A2}(t)$. When $\mathcal{N} = 0$ indicates that the atomic states of the system are separables and for $\mathcal{N} = 1$ the two atoms are in a maximally entanglement state. These two measures cited (eq.(9) and eq. (10)) are important to certify that, when the atomic system is initially prepared in a product state, at instant of time that occur the complete quantum state transfer the *Fidelity* will be equal to one and in the same instant the *Negativity* must be zero.

3 Quantum State Transfer

In this section we present results about the dynamics of the quantum state transfer in a system of two coupled resonators. Firstly, we examine the implementation of a swap gate between the two atoms (analogous the swap gate two-qubit) observing time evolution of the *Fidelity*. Then, we extend our investigation for the case of a transference of superposition state from atom 1 to atom 2, when the modes of the resonators are in vacuum state. The influence of the dissipation effects and the dynamics entanglement between the two atoms are also considered. Besides, we also consider in our scheme the transference of entangled state.

3.1 Swap Gate Two-atoms

Quantum computation require the successful implementation of the quantum gates. In this way, we describe as a swap gate can be applied in our scheme, making use of the fidelity. For this purpose, we will use in the eq.(9) the initial state of the system ($|\psi_i\rangle$), e. g., the fidelity of the system related to its initial state, not the final state like there. The new equation is now represented by

$$\mathcal{F}_i = \langle\psi_i|\hat{\rho}_{A1A2}(t)|\psi_i\rangle = \cos^2\theta\times \Big[\cos^2\theta + \frac{1}{2}\sin^2(\sqrt{2}\tau)\sin^2\theta + \sin^2\theta\cos^4(\frac{\tau}{\sqrt{2}})\Big] + \frac{1}{2}\sin^2\theta\sin^2(\sqrt{2}\tau) + \sin^2\theta\cos\phi\cos^2\theta\sin^2(\sqrt{2}\tau) \quad (11)$$

We can observe in Fig. 2 that the function $\mathcal{F}_i$ at instant of time $\tau_n = 2n\frac{\pi}{\sqrt{2}}$ (n
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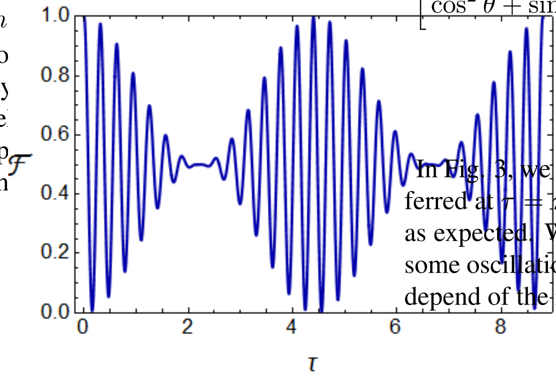

Figure 2. Time evolution of the Fidelity related to state $|\psi_i\rangle$ as a function of normalize time ($\tau = gt$) and $\theta = \pi/4$. The results were obtained for $\omega/g = 20$ and $\alpha = 0$.

3.2 Transmission of the Quantum State

Now, we study the possibility of the transferring of quantum state for two coupled micro-toroidal cavities via evanescent field, where each of them is coupled to a single two-level atom. For sake of simplify, we that the frequency of the two WGM's is equal the frequency of the atomic transition ($\omega_{Ci} = \omega^i_{eg} = \omega$). For this case, considering the initial state ψ_i and we obtain the following solution for state of the system in the Schödinger picture

$$|\psi(t)\rangle = [\cos\theta e^{\frac{i\omega}{g}\tau}|g\rangle_1|g\rangle_2 - e^{i\alpha}\sin^2(\frac{\tau}{\sqrt{2}})\sin\theta|g\rangle_1|g\rangle_2 + e^{i\alpha}[\cos^2(\frac{\tau}{\sqrt{2}})\sin\theta|e\rangle_1|g\rangle_2]|00\rangle_{c1}|00\rangle_{c2}] + \frac{i}{\sqrt{2}}[e^{i\alpha}\sin(\sqrt{2}\tau)\sin\theta|g\rangle_1|g\rangle_2|00\rangle_{c1}|01\rangle_{c2}] \quad (12)$$

In the instant of time $\tau_n = \frac{(2n+1)\pi}{\sqrt{2}}$ $(n = 0, 1, 2...)$ the state of the system is given by:

$$|\psi_f\rangle = |g\rangle_1[\cos\theta|g\rangle_2 + e^{-i\alpha'}\sin\theta|e\rangle_2]|00\rangle_{c1}|00\rangle_{c2} \quad (13)$$

where $\alpha' = \alpha - \frac{(2n+1)\pi}{\sqrt{2}}\frac{\omega}{g}$.

Under the condition $\omega/g = 2l$ (l integer), the state of the atom 2 at the instant of time τ_n is exactly same atom 1 superposition state, i.e., the atomic state was completely transferred from the atom 1 to the atom 2. To observe the temporal evolution of system in the process of transferring this state, we observed the fidelity of the system related to its final state ($|\psi_f\rangle$) as a function of the parameters θ and τ defined by eq.(9):

$$\mathcal{F} = \langle\psi_f|\hat{\rho}_{A1A2}(t)|\psi_f\rangle = \cos^2\theta\times \Big[\cos^2\theta + \sin^2\theta\cos^4(\frac{\tau}{\sqrt{2}})\Big] + \frac{1}{2}\sin^2\theta\sin^2(\sqrt{2}\tau) +$$

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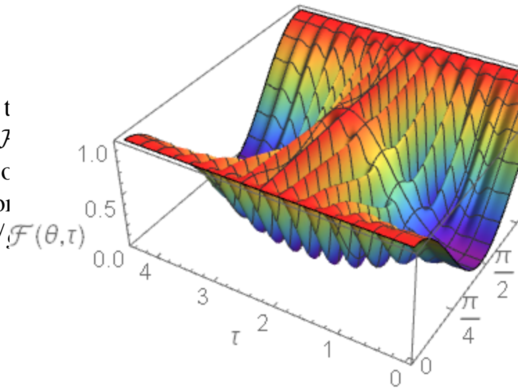

Figure 3. Time evolution of the Fidelity related to state $|\psi_f\rangle$ as a function of normalize time ($\tau = gt$) and θ. The results were obtained for $\omega/g = 20$ and $\alpha = 0$.

3.3 Transmission Under Dissipation

From an experimental point of view to transfer quantum state efficiently under realistic conditions, one must take into account the interaction of system with the surrounding environment, e.g., spontaneous emission of the atoms (γ_A) and decays of the two resonators (κ_1 and κ_2). In this form, we described the system in terms of the master equation (considering that the reservoir is in the temperature $T = 0$ and weak coupling) which can be written as [Carmichael, 1993]

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[\tilde{H}_I, \rho(t)] + \sum_{i=1}^{2}\frac{\kappa_i}{2}(2\hat{a}_i\rho(t)\hat{a}_i^\dagger - \hat{a}_i^\dagger\hat{a}_i\rho(t) - \rho(t)\hat{a}_i^\dagger\hat{a}_i) + \sum_{i=1}^{2}\frac{\kappa_i}{2}(2\hat{b}_i\rho(t)\hat{b}_i^\dagger - \hat{b}_i^\dagger\hat{b}_i\rho(t) - \rho(t)\hat{b}_i^\dagger\hat{b}_i) + \sum_{i=1}^{2}\frac{\gamma_A}{2}(2\sigma_-^{(i)}\rho(t)\sigma_+^{(i)} - \sigma_+^{(i)}\sigma_-^{(i)}\rho(t) - \rho(t)\sigma_+^{(i)}\sigma_-^{(i)}) \quad (15)$$

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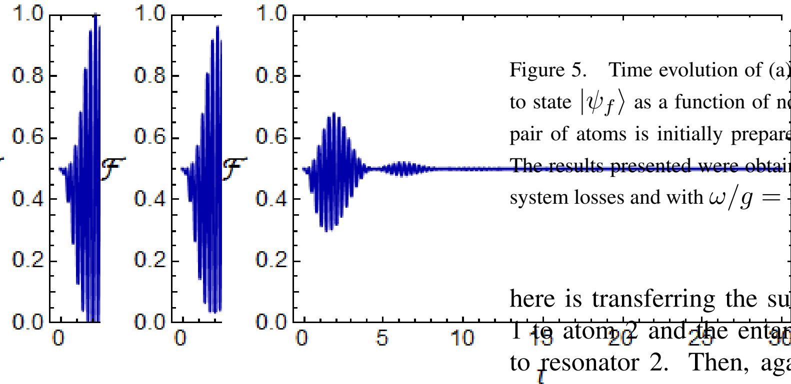

Figure 4. Time evolution of Fidelity related to state $|\psi_f\rangle$ as a function of normalize time ($\tau = gt$) for $\theta = \pi/4$ and with following rates (a) $\kappa = \gamma = 2 \times 10^{-3}g$, (b) $\kappa = 2 \times 10^{-1}g$,$\gamma = 1 \times 10^{-2}g$ and (c) $\kappa = \gamma = g$. The results present were obtained with $\omega/g = 20$ and $\alpha = 0$.

In Fig. 4 (a) represents the case when the system is coupled weakly with the surround environment, $\kappa = \gamma = 2 \times 10^{-3}g$, indicating that the transmission is reliable, in this regime. The Fig. 4 (b) represent the case of intermediate coupling with environment, when $\kappa = 2 \times 10^{-1}g$ and $\gamma = 1 \times 10^{-2}g$, reaching a maximum transmission of $F = 0.957$ (first maximum) indicating still, an efficient transmission. The Fig. 4 (c) illustrate the case of dissipation more intense when the system is coupled strongly with environment, as we can seen with $\kappa = \gamma = g$, the Fidelity reach at $F = 0.652$ (first maximum), indicating that the quantum state transfer is inefficient.

In order to confirm the complete transference of the superposition state, we observe the dynamics of entanglement between the atoms using, as witness of perfect transference, the *Negativity* (eq. 10) for initial state $|\psi\rangle_i$. As shown in the Fig. 5 (a), when $\mathcal{N} = 0$ at time $\tau = \pi/\sqrt{2}$ correspond to the situation where the atoms are in a separate state and occurring (in same moment) the maximum quantum state transfer (see the arrow connecting Fig. 5(a) with Fig. 5(b)), meaning that we have complete transference of the atomic state of the atom 1 to the atom 2. At this instant of time the state $|\psi_f\rangle$ is a separate state in accordance with a $\mathcal{N} = 0$, as expected. This allows us use projective measurement over one atomic state without disturb the state of the other atom, indicating that this system could be used for processing quantum information.

Besides the possibility of transfer an atomic superposition state, we also note that our system support the transference of a entangled state. In such a case, the atom 1 is initially prepared in a superposition state and the modes of resonator 1 is in maximality entangled state and other parts of the system are in their fundamental states, e.g., $|\psi_i^{(2)}\rangle = (\cos\theta|g\rangle_1 + e^{i\alpha'}\sin\theta|e\rangle_1)|g\rangle_2\frac{1}{\sqrt{2}}(|10\rangle_{c1} + |01\rangle_{c1})|00\rangle_{c2}$. The goal

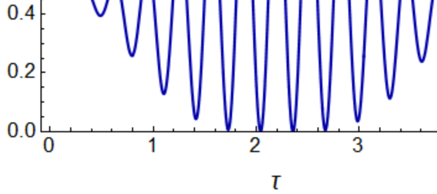

Figure 5. Time evolution of (a) Negativity and (b) Fidelity related to state $|\psi_f\rangle$ as a function of normalize time ($\tau = gt$) when the pair of atoms is initially prepared in state $|\psi_i\rangle$ and $\theta = \pi/4$. The results presented were obtained without taking into account the system losses and with $\omega/g = 20$ and $\alpha = 0$.

here is transferring the superposition state from atom 1 to atom 2 and the entangled state from resonator 1 to resonator 2. Then, again, we observed the evolution of the *Fidelity* as function of normalize time (τ) for $\theta = \pi/4$. This result is shown in the Fig. 6. In this figure we can see that, at time $\tau_n = \frac{(2n+1)\pi}{\sqrt{6}}$ $(n = 0, 1, 2...)$ the entangled state is completely transferred, showing more efficiency (decrease in transfer time in compared with separated state $|\psi_i\rangle$) for quantum information proce to the transfer of the [Nohama and Roversi, case instead of the vib performing the transm tween the modes of the

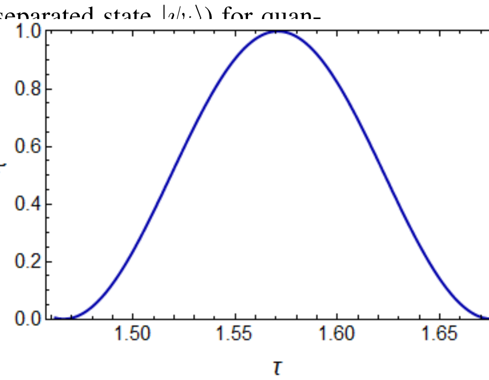

Figure 6. Time evolution of Fidelity as a function of normalize time ($\tau = gt$) for $\theta = \pi/4$, related to state $|\psi_f\rangle = |g\rangle_1(\cos\theta|g\rangle_2 + e^{i\alpha'}\sin\theta|e\rangle_2)|00\rangle_{c1}\frac{1}{\sqrt{2}}(|10\rangle_{c2} + |01\rangle_{c2})$. The results present were obtained with $\omega/g = 20$ and $\alpha = 0$.

4 Conclusion

We explored a system composed of two micro-toroidal cavities coupled by evanescent field, where each cavity interacts with a single two-level atom. It was observed that the set of quantum state (separate and entangled) is completely transferred from atom 1 coupled with the resonator 1 to atom 2 coupled with resonator 2. Even under the influence of interaction between the system and environment (reservoir at temperature $T = 0$) the transference could be done with high efficiency ($F = 0.957$). The system of two coupled micro-toroidal cavities shown that the period of separable state ($\mathcal{N} = 0$) during the transference is relatively large allowing quantum information processing without disturb the states of others subsystems involved in the process. It is also important to emphasize that the transference of the quantum state had shown more efficient when the initial state is an entangled state.

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