

PARAMETRIC IDENTIFICATION OF REACTION WHEEL PENDULUMS WITH ADAPTIVE CONTROL

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Abstract

The paper proposes a new adaptive method for identification of the axial moment of inertia for a two degrees of freedom reaction wheel pendulum with viscous friction in suspension bearings. The pendulum contains a rod, a controlled DC motor and a flywheel attached to the rotor shaft. One end of the rod rotates in hinge bearing. The controlled DC motor with a flywheel is attached to the other end of the rod. The angles of rotation of the rod and of the controlled flywheel are measured by encoders. The proposed adaptive method is based on an energy algorithm with reversible symmetric motions of the pendulum system. The symmetric motions are used to eliminate the influence of dissipative factors on the results of identification. The results of computer modelling and experimental results show high accuracy of the proposed identification method.

Key words

Moment of inertia identification, reaction wheel pendulum, consecutive compensator, adaptive control.

1 Introduction

The problem of precise identification of *moments of inertia* (MOI) of solid systems operating under conditions of unknown friction and environmental resistance is an actual problem of mechanics and control theory. Identification of inertial parameters of the various constructions or of their complicated components is especially important in robotics. In some cases it is possible to estimate inertia parameters theoretically by using CAD programs, but in practice the thorough model of the system is often unknown, or requires a time-consuming process to obtain it.

The inertial parameters can be estimated experimentally [Tikhonov and Tkhai, 2016, Ovchinnikov et al., 2017, Andrievskiy and Boikov, 2017] using various

time and frequency domain methods [Almeida et al., 2007, Ivanov and Melnikov, 2015]. In the time domain methods including the pendulum method or its various modifications [Pandit et al., 1992], the inertial properties of a weighted body are determined relative to the period of its oscillations. Despite the fact that these methods are widely used, they are not accurate in the case of significant friction in the system or significant resistance of the environment. Frequency domain methods (IRM and DPPIM) and modal methods [Almeida et al., 2008] are based on measuring the motion of a solid body in bi-filar or multi-filar suspensions, Stewart platforms, etc. [Malekjarfarian et al., 2016, Barreto and Muñoz, 2010, Hou et al., 2009]. These methods also have drawbacks. For example, multi-filar suspensions create a significant measurement error due to additional unwanted movements in different directions, require accurate calibration and additional time for preparation of the test. In control theory, inertial parameters are estimated using various identification methods based on the analysis of the motion [Gobbi et al., 2011, García-Alarcón et al., 2012].

In this article, we present a new adaptive method for identifying inertial parameters of solids on special symmetrical program motions. The method uses an energy approach [Fradkov and Andrievsky, 2006, Fradkov and Andrievsky, 2004] and is based on an energy algorithm [Alyshev et al., 2015, Melnikov, 2012, Dudarenko et al., 2014]. This algorithm uses a two-step reversible symmetric motion, containing two motions with approximately equal energy dissipation. We use robust adaptive control with the high-gain feedback principle to provide the desired program motions. Using this method, we do not need a friction evaluation to accurately identify the MOI.

The paper is organized as follows. In Section 2 we describe the construction of the pendulum. In Section 3 we obtain a general equation for calculating the MOI of

the rod. In Section 4 we present an algorithm for controlling the program symmetrical motions of the pendulum and give the results of mathematical modelling and identification of MOI of the rod in MATLAB. After that we compare the obtained results with the results of existing methods. In Section 5 we present the results obtained at our experimental device. In Section 6 we conclude our paper with concluding remarks.

2 Problem Formulation

We will present and demonstrate our method on a system with two *degrees of freedom* (DOF) called *the reaction-wheel pendulum* [Block et al., 2007, Spong et al., 2001].

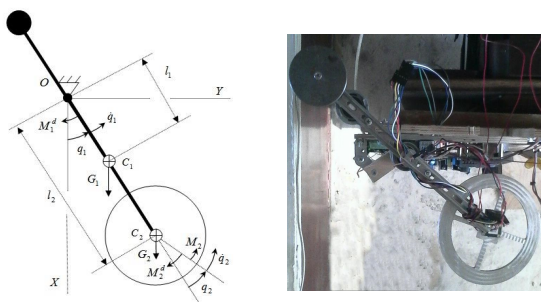


Figure 1. Reaction Wheel Pendulum

The system consists of two kinematic groups of elements. The first group consists of a rod with the stator of the motor and the housing of the motor gearbox with the central MOIs that are denoted by J_{rod} , J_s and J_{gh} . The first group performs a rotation motion around the fixed horizontal axis O in the plane OXY . The second group consists of a rotor of the motor and a flywheel (a homogeneous disk) with the central MOIs that are denoted by J_r and J_2 . The second group performs a plane motion. Note that we will use the second group as a measuring device to identify the element of the first group, so the parameters of the second group should be known.

The device is shown in Fig. 1, where q_1 is the absolute angle of rotation of the rod; q_2 is the relative angle of rotation of the flywheel; \dot{q}_1 is the angular velocity of the rod; \dot{q}_2 is the angular velocity of the flywheel. We assume that the position l_1 of the *center of gravity* (COG) of the rod C_1 and the position l_2 of the COG of the second element C_2 and of the motor C_2' are known. We denote the torque of viscous friction in the suspension bearing by M_1^d ; the electromagnetic torque of the motor by M_2 ; the gravity forces of the elements by G_1 , G_2 and of the motor G_2' .

We will identify the MOI $J_1 = J_{rod} + J_{gh} + J_s + m_1 l_1^2 + (m_2' + m_2) l_2^2$ and dissipative parameter using symmetric program motion and then we will proof the

MOI identification accuracy by adding the additional body with known moment of inertia $J_e = J_{body} + m_{body} l_2^2$. The program motion consists of a slowed part and a symmetrical inverse accelerated part. Both parts are performed in the same angular interval $q_{11} \leq q_1 \leq q_{12}$ after the preliminary accelerated motion. We use the high-resolution sensors to measure the angle of rotation of the flywheel and the pendulum. The program motion is performed under conditions of substantial viscous friction in bearings.

3 Energy Algorithm in MOI Identification Problem

Here we introduce the MOI identification procedure for a system with a measuring drive with a flywheel. The work of inertial torques changes its sign when the pendulum passes into reverse motion; the work of dissipative torques is always negative. This leads to the possibility of an analytical purification of MOI identification procedure from dissipative torques. Using the principle of the energy theorem for two motions, we get

$$(T_{12} + \Pi_{12}) - (T_{11} + \Pi_{11}) = A_1 + B_1 + D_1 \quad (1)$$

$$(T_{22} + \Pi_{22}) - (T_{21} + \Pi_{21}) = A_2 + B_2 + D_2, \quad (2)$$

here T_{11} , T_{12} , T_{21} , T_{22} and Π_{11} , Π_{12} , Π_{21} , Π_{22} are the nodal values of the kinetic and potential energies of the pendulum system; A_1 and A_2 are the work of the motor torque in the considered angular interval on the forward $[q_{11}, q_{12}]$ and on the inverse $[q_{13}, q_{14}]$ motions; B_1 and B_2 are negative work of unknown torques of viscous friction in the bearing O on two program motions; D_1 and D_2 are negative work of known torques of viscous friction in the bearings of the measuring motor. Note that the potential energy of the system at the same angular positions has the same values, i.e. $\Pi_{11} = \Pi_{22} \equiv \Pi_1$ and $\Pi_{12} = \Pi_{21} \equiv \Pi_2$.

We assume that the work of dissipative torques in bearings on symmetrical pendulum motions are approximately the same, i.e. $B_1 \approx B_2 < 0$. Subtracting (2) from (1), we obtain an equation that does not contain dissipative torques:

$$\begin{aligned} T_{12} - T_{11} + T_{21} - T_{22} = \\ = 2 \Pi_1 - 2 \Pi_2 + A_1 - A_2 + D_1 - D_2. \end{aligned} \quad (3)$$

The equation (3) contains the nodal values of the kinetic and potential energies and consumption of elec-

trical energy at the angular interval $[q_{12}, q_{11}]$:

$$\begin{aligned} 2T_{11} &= a_{11}\dot{q}_{11}^2 + 2a_{12}\dot{q}_{11}\dot{q}_{21} + a_{22}\dot{q}_{21}^2, \\ 2T_{12} &= a_{11}\dot{q}_{12}^2 + 2a_{12}\dot{q}_{12}\dot{q}_{22} + a_{22}\dot{q}_{22}^2, \\ 2T_{21} &= a_{11}\dot{q}_{13}^2 + 2a_{12}\dot{q}_{13}\dot{q}_{23} + a_{22}\dot{q}_{23}^2, \\ 2T_{22} &= a_{11}\dot{q}_{14}^2 + 2a_{12}\dot{q}_{14}\dot{q}_{24} + a_{22}\dot{q}_{24}^2, \end{aligned}$$

$$\begin{aligned} a_{11} &= J_1 + J_2 + J_r; \quad a_{12} = J_r i^{-1} + J_2 \\ a_{22} &= J_r i^{-2} + J_2, \end{aligned}$$

$$\Pi_1 = b(1 - \cos q_{11}) \quad (4)$$

$$\Pi_2 = b(1 - \cos q_{12}), \quad (5)$$

where m_1 , m_2' and m_2 are the masses of the rod, the motor and second elements, g is the gravitational acceleration; \dot{q}_{21} , \dot{q}_{22} and \dot{q}_{23} , \dot{q}_{24} are the angular velocities of the flywheel at the beginning and at the end of two symmetrical movements.

Substituting (4) for (3) and collecting the terms, we obtain the equation for MOI \hat{J}_1 :

$$\begin{aligned} \hat{J}_1 &= (2(A_1 - A_2) + 2(D_1 - D_2) + 4b(\cos q_{12} - \cos q_{11}) - \\ &- a_{22}(\dot{q}_{22}^2 + \dot{q}_{23}^2 - \dot{q}_{21}^2 - \dot{q}_{24}^2) - 2a_{12}(\dot{q}_{12}\dot{q}_{22} + \dot{q}_{13}\dot{q}_{23} - \dot{q}_{11}\dot{q}_{21} - \\ &- \dot{q}_{14}\dot{q}_{24}))/(\dot{q}_{12}^2 + \dot{q}_{13}^2 - \dot{q}_{11}^2 - \dot{q}_{14}^2) - J_2 - J_r - m_2 l_2^2, \end{aligned} \quad (6)$$

where $b = g(m_1 l_1 + (m_2' + m_2) l_2)$; the works A_1, A_2, D_1, D_2 on the time interval $t \in [t_{10}, t_{11}]$ of the direct motion and on the time interval $t \in [t_{20}, t_{21}]$ of the inverse motion are

$$\begin{aligned} \text{defined by } A_j &= \int_{t_{j0}}^{t_{j1}} M(t) i^{-1} \dot{q}_2(t) dt, \quad D_j = \\ &- \int_{t_{j0}}^{t_{j1}} M_2^d(t) i^{-1} \dot{q}_2(t) dt, \quad j = 1, 2 \text{ or, we can} \\ &\text{calculate the work } A_1, A_2, D_1, D_2 \text{ from the very} \\ &\text{beginning of the symmetric motion using } A_j = \\ &i^{-1} \left(\int_0^{t_{j1}} M(t) \dot{q}_2(t) dt - \int_0^{t_{j0}} M(t) \dot{q}_2(t) dt \right). \end{aligned}$$

Suppose that the friction torque M_1^d acting in the bearing O has an unknown significant viscous friction coefficients f_1 : $M_1^d = f_1 i^{-1} \dot{q}_1$. So, the works B_1 and B_2 are defined by: $B_i = - \int_{t_{i0}}^{t_{i1}} M_1^d(t) \dot{q}_1(t) dt, \quad i = 1, 2.$

Summarize (2) and (1), supposing that the inertial parameter is eliminated from the calculation, we obtain

an equation for the coefficients f_1 :

$$\begin{aligned} \hat{f}_1 &= (2(A_1 + A_2 + D_1 + D_2) - 2a_{12}(\dot{q}_{12}\dot{q}_{22} - \\ &- \dot{q}_{11}\dot{q}_{21} + \dot{q}_{14}\dot{q}_{24} - \dot{q}_{13}\dot{q}_{23}) - a_{22}(\dot{q}_{22}^2 - \dot{q}_{21}^2 + \\ &+ \dot{q}_{24}^2 - \dot{q}_{23}^2))/ \left(2i^{-1} \left(\int_{t_{10}}^{t_{11}} \dot{q}_1^2(t) dt + \int_{t_{20}}^{t_{21}} \dot{q}_1^2(t) dt \right) \right). \end{aligned} \quad (7)$$

4 Example

The example demonstrates a proposed adaptive energy method for MOI identification.

4.1 Mathematical Model of the Pendulum

Using the Lagrange method, we obtain a mathematical model of a pendulum with two DOFs in the form of a system of two differential equations with constant parameters

$$\begin{aligned} a_{11}\ddot{q}_1 + a_{12}\ddot{q}_2 &= -b \sin q_1 - M_1^d \\ i a_{12}\ddot{q}_1 + i a_{22}\ddot{q}_2 &= M - M_2^d. \end{aligned}$$

Let the torque of the motor M_2 be an approximately linear function of the angular velocity of the rotor: $M_2 = c_1 u - c_2 \dot{q}_2$; M is the torque with deadzone of the motor; i^{-1} is the transmission gear ratio; $M_2^d = f_2 i^{-1} \dot{q}_2 + f_3 \text{sign}(i^{-1} \dot{q}_2)$ is the motor friction. The DC motor has the deadzone

$$M = \begin{cases} 0, & \text{if } d_2 \leq \eta M_2 \leq d_1 \\ \eta M_2 - d_1, & \text{if } \eta M_2 > d_1, \\ \eta M_2 + d_2, & \text{if } \eta M_2 < d_2, \end{cases} \quad (8)$$

where η is the electrical efficiency of the motor.

We assume that the coefficients f_2 and f_3 of viscous and Coulomb friction in the motor bearings with the gearbox; deadzone parameters d_1, d_2 and the coefficients η, c_1 and c_2 are known from the preliminary identification of the motor mathematic model.

We use the control signal $u_1 = (c_1 \eta)^{-1} f_3 \text{sign}(i^{-1} \dot{q}_2)$ to neutralize Coulomb friction in the bearing O and than we get

$$\begin{aligned} \ddot{q}_1 &= (a_{22} a_{11} - a_{12}^2)^{-1} (-a_{22}(b \sin q_1 - M_1^d) - \\ &- a_{12} i^{-1} (\eta M_2 - f_2 i^{-1} \dot{q}_2)), \\ \ddot{q}_2 &= (a_{22} a_{11} - a_{12}^2)^{-1} (a_{11} i^{-1} (\eta M_2 - f_2 i^{-1} \dot{q}_2) + \\ &+ a_{12}(b \sin q_1 + M_1^d)), \end{aligned} \quad (9)$$

or in the Cauchy form:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= -\alpha_3 x_2 - \alpha_2 \sin x_1 + \alpha_7 x_4 - \alpha_4 u_2, \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \alpha_8 x_2 + \alpha_6 \sin x_1 - \alpha_9 x_4 + \alpha_5 u_2, \end{aligned} \quad (10)$$

where

$$\begin{aligned}
x_1 &= q_1, & x_2 &= \dot{q}_1, & x_3 &= q_2, & x_4 &= \dot{q}_2, \\
u_0 &= u_1 + u_2 + u_3, & \alpha_1 &= (a_{22}a_{11} - a_{12}^2)^{-1}, \\
\alpha_2 &= \alpha_1 a_{22} b, & \alpha_3 &= \alpha_1 a_{22} f_1 i^{-1}, \\
\alpha_4 &= \alpha_1 a_{12} i^{-1} \eta c_1, \\
\alpha_5 &= \alpha_4 a_{11} a_{12}^{-1}, \\
\alpha_6 &= \alpha_2 a_{12} a_{22}^{-1}, \\
\alpha_7 &= \alpha_1 a_{12} i^{-1} (c_2 \eta + f_2 i^{-1}), \\
\alpha_8 &= \alpha_3 a_{12} a_{22}^{-1}, & \alpha_9 &= \alpha_7 a_{11} a_{12}^{-1},
\end{aligned} \tag{11}$$

here $\alpha_1 \dots \alpha_9$ are positive coefficients; x_1 is the output of the system; u_3 is the DC-motor deadzone compensation signal defined as:

$$u_3 = \begin{cases} d_1(c_1 \eta)^{-1}, & \text{if } u_1 + u_2 > 0 \\ 0, & \text{if } u_1 + u_2 = 0 \\ d_2(c_1 \eta)^{-1}, & \text{if } u_1 + u_2 < 0 \end{cases} \tag{12}$$

The initial deviation from the equilibrium position of the pendulum can be set manually, or by using an additional device or by using preliminary controlled motions [Andrievskiy, 2004, Andrievskiy, 2011, Bobtsov et al., 2011, Fradkov et al., 2012, Tikhonov, 2000]. The compact form of the equations of motion with respect to dimensionless time is presented in [Beznos et al., 2003a, Beznos et al., 2003b].

Here we develop a control for nonlinear system (10). The control should ensure the symmetry of the program motion $y(t) = x_1(t)$ of the pendulum for the limited reference signal $y_m(t)$ as $\lim_{t \rightarrow \infty} |y(t) - y_m(t)| < \delta$. We use the algorithm proposed in [Bobtsov et al., 2011, Bobtsov and Pyrkin, 2008], where only the output variable is measured. The non-linear system (10) in the input-output representation has the form

$$y = \frac{r(p)}{a(p)} u_2 + \frac{h(p)}{a(p)} \varphi(y), \tag{13}$$

where $r(p) = r_m p^m + r_{m-1} p^{m-1} + \dots + r_1 p + r_0$, $r_m > 0$, $a(p) = p^n + a_{n-1} p^{n-1} + \dots + a_1 p + a_0$, $h(p) = p^n + h_{n-1} p^{n-1} + \dots + h_1 p + h_0$ are polynomials with unknown parameters; $m \leq n - 1$; $\rho = n - m$ is the maximum relative degree of transfer function $\frac{r(p)}{a(p)}$. The polynomial $r(p)$ in (13) is Hurwitz and $\varphi(y)$ is the unknown function that satisfies the assumptions $\varphi(0) = 0$, $-C_0 \leq \frac{\varphi(y)}{y} \leq C$ for any $y \neq 0$. The equation for the tracking error $e(t)$ has the form

$$e(t) = y_m(t) - x_1(t). \tag{14}$$

We define the control $u(t)$ in the form

$$u(t) = -(\nu + \kappa) \alpha(p) \xi_1(t), \quad \nu > \nu_0 \tag{15}$$

$$\begin{cases} \dot{\xi}_1(t) = \sigma \xi_2(t), \\ \dot{\xi}_2(t) = \sigma \xi_3(t), \\ \dots \\ \dot{\xi}_{\rho-1}(t) = \sigma(k_1 e(t) - \\ -k_1 \xi_1(t) - \dots - k_{\rho-1} \xi_{\rho-1}(t)), \end{cases} \tag{16}$$

where the polynomial $\alpha(p)$ of order $\rho - 1$ is chosen so that the polynomial $a(p) + \nu r(p) \alpha(p)$ is Hurwitz. The positive parameter κ is determined from the conditions for neutralizing the nonlinearity $\varphi(y)$. The coefficients $\sigma > \nu + \kappa$, k_i are determined from the asymptotic stability conditions for the system (16).

Now we define the coefficients ν , κ , σ of the controller. The parameter $\tilde{k} = \nu + \kappa$ can be tuned as follows:

$$\tilde{k}(t) = \int_{t_0}^t \lambda(\tau) d\tau, \tag{17}$$

where the function $\lambda(t)$ is defined as follows

$$\lambda(t) = \begin{cases} \lambda_0, & |e(t)W_f(p)| > \delta_0 \\ 0, & |e(t)W_f(p)| \leq \delta_0 \end{cases}; \quad \lambda_0 > 0. \tag{18}$$

The parameter σ is calculated using the following algorithm

$$\sigma(t) = \sigma_0 \tilde{k}^2(t), \quad \sigma_0 > 0. \tag{19}$$

Since the system (13), where $h(p) = -\alpha_2 p + \alpha_6 \alpha_7 - \alpha_2 \alpha_9$, $a(p) = p^3 + (\alpha_9 + \alpha_3) p^2 + (\alpha_3 \alpha_9 - \alpha_7 \alpha_8) p$, $r(p) = -\alpha_4 p - \alpha_4 \alpha_9 + \alpha_5 \alpha_7$, $\varphi(y) = \sin y$ has the maximum relative degree $\rho = 2$, then the controller (15) - (16) with $\alpha = p + 1$, $k_1 = 1$ has the form

$$\begin{aligned} u(t) &= -(p+1) \tilde{k}(t) \xi_1(t) = \\ &= -(\dot{\tilde{k}}(t) \xi_1(t) + \tilde{k}(t) \dot{\xi}_1(t) + \tilde{k}(t) \xi_1(t)) \\ &\quad \dot{\xi}_1(t) = \sigma(-\xi_1(t) + e(t)) \end{aligned} \tag{20}$$

4.2 Alternative Method

Another approach is to estimate MOI using the inverse controller (adaptive backstop control) proposed in [Krstic et al., 1995, Benaskeur and Desbiens, 1998, Chiu et al., 2011, Ebrahim and Murphy, 2005] with stability estimation by Lyapunov function [Alyshev et al., 2018, Melnikov, 2010]. Let e_1 be the tracking error (14) and e_2 be the second error defined as

$$e_2 = \hat{x}_2 - \beta_1 - \dot{y}_m,$$

where t is omitted for brevity. Then the derivatives of the tracking errors e_1 and e_2 have the form

$$\dot{e}_1 = \dot{\hat{x}}_1 - \dot{y}_m, \quad \dot{e}_2 = \dot{\hat{x}}_2 + n_1 \dot{e}_1 - \ddot{y}_m,$$

where $\beta_1 = -n_1 e_1$ is the virtual control signal.

We choose the first Lyapunov function as follows

$$V_1 = \frac{1}{2} e_1^2.$$

Taking the derivative of the Lyapunov function V_1 , we obtain

$$\begin{aligned} \dot{V}_1 &= e_1 \dot{e}_1 = e_1 (\dot{\hat{x}}_1 - \dot{y}_m) = \\ &= e_1 (\hat{x}_2 - \dot{y}_m) = e_1 (e_2 + \beta_1) = e_1 e_2 - n_1 e_1^2. \end{aligned}$$

If $e_2 = 0$, then we get $\dot{V}_1 = -n_1 e_1^2 \leq 0$ where $n_1 > 0$ and $\hat{x}_1 = \hat{x}_2$.

We choose a new candidate for the Lyapunov function V_2 as

$$V_2 = V_1 + \frac{1}{2} e_2^2 + \frac{1}{2\gamma_1} e_{\alpha_3}^2 + \frac{1}{2\gamma_2} e_{\alpha_2}^2$$

Taking the derivative of the function, we get

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + e_2 \dot{e}_2 + \frac{1}{\gamma_1} e_{\alpha_3} \dot{e}_{\alpha_3} + \frac{1}{\gamma_2} e_{\alpha_2} \dot{e}_{\alpha_2} = \\ &= -n_1 e_1^2 + e_2 (e_1 + \hat{x}_2 + n_1 \dot{e}_1 - \ddot{y}_m) - \frac{1}{\gamma_1} e_{\alpha_3} \dot{\alpha}_3 - \\ &- \frac{1}{\gamma_2} e_{\alpha_2} \dot{\alpha}_2 = -n_1 e_1^2 + e_2 (e_1 - \alpha_3 \hat{x}_2 - \alpha_2 \sin x_1 + \\ &+ \alpha_7 x_4 - \alpha_4 u_2 + n_1 \dot{e}_1 - \ddot{y}_m) + e_{\alpha_3} \left(\frac{-\dot{\alpha}_3}{\gamma_1} - e_2 \hat{x}_2 \right) + \\ &+ e_{\alpha_2} \left(\frac{-\dot{\alpha}_2}{\gamma_2} - e_2 \sin x_1 \right), \end{aligned}$$

where $e_{\alpha_3} = \alpha_3 - \hat{\alpha}_3$, $e_{\alpha_2} = \alpha_2 - \hat{\alpha}_2$ are the parameter estimation errors; $\dot{e}_{\alpha_3} = -\dot{\hat{\alpha}}_3$, $\dot{e}_{\alpha_2} = -\dot{\hat{\alpha}}_2$ are the derivatives of parameter estimation errors.

We apply the control signal u in the following form

$$\begin{aligned} u &= \hat{\alpha}_4^{-1} (e_1 - \hat{\alpha}_3 \hat{x}_2 - \hat{\alpha}_2 \sin x_1 + \hat{\alpha}_7 x_4 + n_1 \dot{e}_1 - \ddot{y}_m + \\ &+ n_2 e_2) = \hat{\alpha}_4^{-1} (x_1 - y_m - \hat{\alpha}_3 \hat{x}_2 - \hat{\alpha}_2 \sin x_1 + \\ &+ \hat{\alpha}_7 x_4 + n_1 (\hat{x}_2 - \dot{y}_m) - \ddot{y}_m + n_2 e_2) \end{aligned} \quad (21)$$

where the adaptation laws have the form $\dot{\hat{\alpha}}_3 = -\gamma_1 e_2 \hat{x}_2$, $\dot{\hat{\alpha}}_2 = -\gamma_2 e_2 \sin x_1$ with parameters $\hat{\alpha}_4 = \hat{\alpha}_2 (a_{22} b i)^{-1} a_{12} \eta c_1$, $\hat{\alpha}_7 = \hat{\alpha}_4 c_1^{-1} (c_2 + f_2(\eta i)^{-1})$, $n_2 > 0$, $\gamma_1 > 0$, $\gamma_2 > 0$.

Substituting (21) for \dot{V}_2 we get a negative definite function $\dot{V}_2 = -n_1 e_1^2 - n_2 e_2^2$, so in the $e_1 = 0$ and $e_2 = 0$ system is globally asymptotically stable.

We can obtain the estimation \hat{J}_1 and then \hat{f}_1 from the following equations:

$$\begin{aligned} \hat{J}_1 &= \frac{b}{\hat{\alpha}_2} + \frac{a_{12}^2}{a_{22}} - J_2 - J_r, \\ \hat{f}_1 &= \frac{\hat{\alpha}_3 (a_{22} (\hat{J}_1 + J_2 + J_r) - a_{12}^2)}{a_{22} i^{-1}}. \end{aligned} \quad (22)$$

Since the velocity of the flywheel changes rapidly, we will use the following [Xian et al., 2004] adaptive velocity observers for the pendulum and for the flywheel:

$$\begin{cases} \dot{\hat{x}}_2(t) = \hat{x}_1(t) + p_{x_2}(t) + \sigma_{x_2} \tilde{x}_1(t) \\ \dot{p}_{x_2}(t) = \varepsilon_{x_2} \text{sign}(\hat{x}_1(t)) + \rho_{x_2} \tilde{x}_1(t) \end{cases}$$

$$\tilde{x}_1(t) = x_1(t) - \hat{x}_1(t)$$

$$\begin{cases} \dot{\hat{x}}_4(t) = \hat{x}_3(t) + p_{x_4}(t) + \sigma_{x_4} \tilde{x}_3(t) \\ \dot{p}_{x_4}(t) = \varepsilon_{x_4} \text{sign}(\hat{x}_3(t)) + \rho_{x_4} \tilde{x}_3(t) \end{cases}$$

$$\tilde{x}_3(t) = x_3(t) - \hat{x}_3(t) \quad (23)$$

where $p(t)_{x_2, x_4}$ is an auxiliary variable; $\sigma_{x_2, x_4}, \varepsilon_{x_2, x_4}, \rho_{x_2, x_4}$ are constants.

This systems can be used to calculate the angular velocities \dot{q}_1 and \dot{q}_2 in (6) instead of directly measuring it by the velocity sensor. For the observer delay compensation the special filters [?] can be used.

5 Experimental and Simulation Results with Analysis

In this section we check the algorithms of parameters identification problem.

An experimental device (stand) is shown in Fig.1. We use a 4.5 W Maxon DC motor; two 14-bit magnetic angular position encoders AS5048A (SPI); an analog amplifier; and a ICP DAS PCI-1202LU data acquisition board. The DC motor is connected to the computer via an isolation converter.

Some parameters of the measuring motor (10) were obtained by Nonlinear Least Squares method separately on the set of free pendulum oscillations and multiharmonic testing signals for the motor: $J_r = 0.21 \cdot 10^{-5} \text{kg} \cdot \text{m}^2$, $J_2 = 11.34 \cdot 10^{-5} \text{kg} \cdot \text{m}^2$, $c_1 = 52.04 \cdot 10^{-4} \text{N} \cdot \text{m} \cdot \text{V}^{-1}$, $c_2 = 3.697 \cdot 10^{-6} \text{N} \cdot \text{m} \cdot \text{rad}/\text{s} \cdot \text{V}^{-2}$, $f_2 = 3.687 \cdot 10^{-5} \text{N} \cdot \text{m} \cdot \text{s}$, $f_3 = 302.02 \cdot 10^{-5} \text{N} \cdot \text{m}$, $\eta = 0.9$, $i^{-1} = 4.4$, $d_1 = 5.485 \cdot 10^{-4}$, $d_2 = -2.996 \cdot 10^{-4}$.

Due to the small parameter f_1 next we experimentally check the accuracy only of the MOI identification.

To calibrate the device, we use an additional test body with a known moment of inertia $J_e = 96.1 \cdot 10^{-5} \text{kg} \cdot \text{m}^2$ and mass $m_{body} = 68.0 \cdot 10^{-3} \text{kg}$, $l_2 = 117.0 \cdot 10^{-3} \text{m}$. This test body can balance the pendulum ($b = 0$) if it is connected to the rod at the opposite distance from point O away from the motor.

In the first case of $J_e = 0$ which leads to $b = m_{body}gl_2 = 78 \cdot 10^{-3} \text{kg} \cdot \text{m}^3 \cdot \text{s}^{-2}$, using the NLS, the estimated parameter $\hat{J}_1 = 272.6 \cdot 10^{-5} \text{kg} \cdot \text{m}^2$ and using the proposed method $\hat{J}_1 = 302.2 \cdot 10^{-5} \text{kg} \cdot \text{m}^2$. Here we use reference trajectory $y_{m1} = 0.8 \sin(8.5t)$.

Next we add the test body at the point C_2 , so the parameter $b = 2m_{body}gl_2$ can be easily recalculated, $a'_{11} = J_1 + J_2 + J_r + J_e$. Parameter obtained using NLS $\hat{a}'_{11} = 381.0 \cdot 10^{-5} \text{kg} \cdot \text{m}^2$ and using the proposed method $\hat{a}'_{11} = 405.1 \cdot 10^{-5} \text{kg} \cdot \text{m}^2$. For this case we use reference trajectory $y_{m2} = 0.8 \sin(6t)$.

So, we get the estimation of parameter $\hat{J}_e = \hat{a}'_{11} - \hat{J}_1 - J_2 - J_r$ using the NLS $\hat{J}_e = 118.4 \text{kg} \cdot \text{m}^2$ and by the proposed method $\hat{J}_e = 102.9 \text{kg} \cdot \text{m}^2$. One can see that the experimental identification result is close to the known value with the errors 12% and 7%, respectively. This results are proven with the balanced pendulum and slow oscillations with the reference trajectory $y_{m3} = 0.8 \sin(2t)$.

The position tracking errors of the system with the control algorithm (20), system state vector and parameter \tilde{k} are shown in Fig. 2-6. Suppose that the control signal is bounded $|u_0| \leq 10V$, parameters $\delta_0 = 0.08$, $\sigma_0 = 0.1$, $\lambda_0 = 2$, $\tilde{k}(0) = 0$. For the postprocessing the measured angles in the proposed method passed through a observer in the direct and reverse time for the delay compensation; the control signal passed through a filter with transfer function $W_f(p) = (0.0125p + 1)^{-1}$ which is also used in (18) for the smoothing error e in the adaptation.

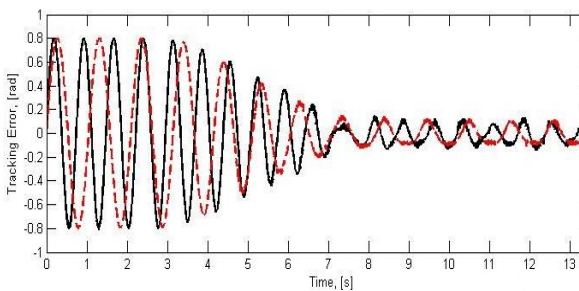


Figure 2. Tracking errors e with (dashed) and without (solid) body (no filtering)

In the proposed method we use series of MOI calculations on a symmetric angular intervals at the time interval from 15 to 22 seconds, where the tuning parameter is no longer changing.

Lets compare the proposed and alternative methods

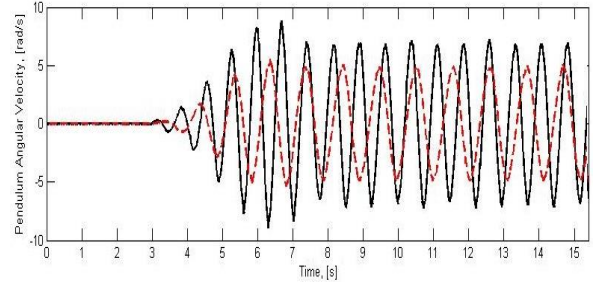


Figure 3. Pendulum angular velocity \dot{q}_1 with (dashed) and without (solid) body

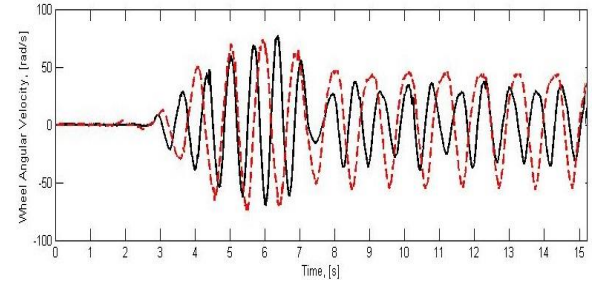


Figure 4. Wheel angular velocity \dot{q}_2 with (dashed) and without (solid) body

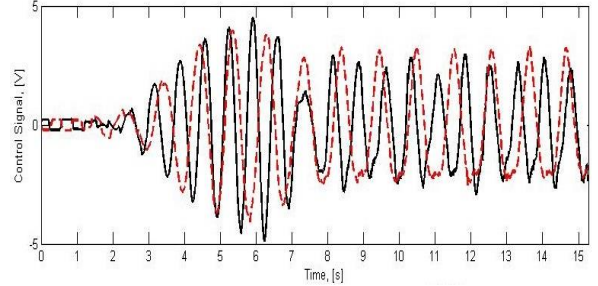


Figure 5. Control signal u_0 with (dashed) and without (solid) body

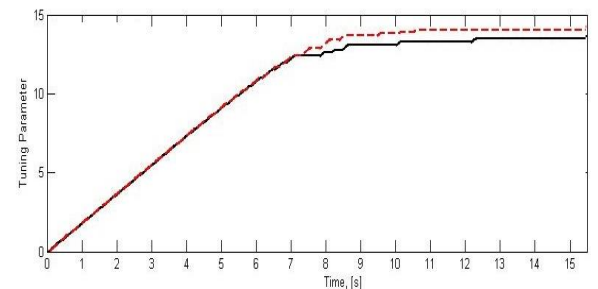


Figure 6. Tuning parameter \tilde{k} complete with filter $W_f(p)$

for the MOI ideal identification simulation using the estimated values and friction parameter $f_1 = 26 \cdot 10^{-5} \text{N} \cdot \text{m} \cdot \text{s}$.

The results of the simulation of the control system

(20) with the parameters $\delta_0 = 0.005$, $\sigma_0 = 0.01$, $\lambda_0 = 150$, $\tilde{k}(0) = 0$ are shown in the Fig. 7 where the angle of the controlled pendulum q_1 is shown in green, the pendulum reference trajectory is shown in blue, a tracking error of e of the control system is shown in red and $0.005\tilde{k}$ in brown. Using the calculation formula for the time moments $t_{10} = 9.023s$, $t_{11} = 9.413s$, $t_{20} = 9.433s$, $t_{21} = 9.823s$ we can get the MOI value \hat{J}_1 with an average identification error 1,4%.

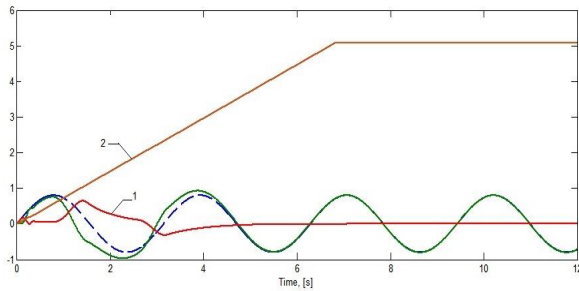


Figure 7. Pendulum angle q_1 of the control system (solid), program trajectory (dashed), tracking error (1) e and tunable parameter (2).

For the comparison, we simulate a controller containing an observer with tunable parameters (21), (23). The numeric parameter values of the control system are: $\gamma_2 = 100$, $\gamma_1 = n_1 = n_2 = 10$ and of the observers $\sigma_{x_2} = \sigma_{x_4} = 10000$, $\rho_{x_2} = \rho_{x_4} = 1$, $\varepsilon_{x_2} = \varepsilon_{x_4} = 1$. The initial values of the tunable parameters are $\hat{\alpha}_2(0) = 1$, $\hat{\alpha}_3(0) = 1$. The tunable parameters are limited to positive values $[10^{-6} \infty]$. The simulation results are shown in Fig. 8. The estimate

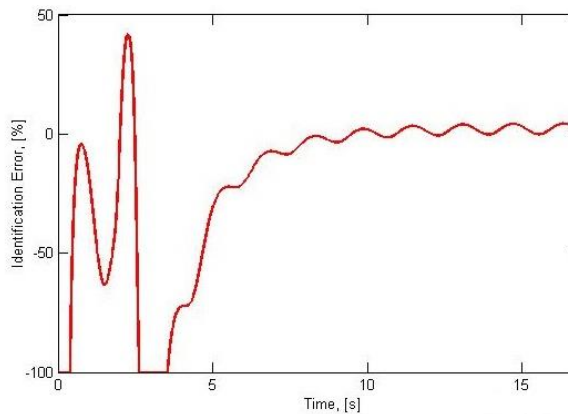


Figure 8. Identification error $J_1 - \hat{J}_1$

of the parameter \hat{J}_1 converges to the 5% identification error in $t = 8s$. The advantage of this method is that the identification occurs full-online. But controller and observers parameters can be difficult to configure and,

due to the possible noise, parameters can be identified incorrectly.

Analysing the experimental and simulation results we conclude that the proposed method with consecutive compensator has several advantages: it does not require measurement of acceleration, cumbersome calculations and linearization; only one two-step symmetric interval is needed to identify the inertia and friction parameters; the dissipative function of any order can be separated from the MOIs; the identification accuracy can be set and estimated by analyzing the tracking error value; the program trajectory can have any convenient symmetric shape with unknown derivatives; proposed method has a fast-time convergence by selecting the right initial values of tuning parameters, can be easily configured with only two controller parameters, has a stable and small amplitude of the control signal, has high accuracy and can provides greater identification performance than the known method. The method can be also used as a discrete-online identification, where the proposed formula is used for each oscillation.

In the future, we plan to develop multidirectional measuring devices and optimize their control structure [Birk and Dudarenko, 2016, Birk and Dudarenko, 2012].

6 Conclusion

This paper presents a new adaptive method for identification of the axial moment of inertia. The developed method is applied to a nonlinear mathematical model of an pendulum system with two degrees of freedom. The method allows to exclude the negative influence of dissipative factors on the accuracy of identification. The proposed method also opens up the possibility of using a convenient and easy-to-install measuring inertial drive for the rapid identification of a wide range of objects. Comparative computer modeling shows that the proposed identification method has higher accuracy and provides greater identification performance. The experimental results are given for illustration.

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