QUANTUM AGENTS THAT OBEY BOSE DISTRIBUTION

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Abstract

We study social systems of agents without special ad-hoc rules in time development. Only random exchange of resource is done among agents. Bose distribution in units of resource holds when we cannot identify characteristic or individuality of each agent in our systems. The Bose distribution holds even for systems with only a few agents. Rules regarding what is exchanged among agents are also explicitly given.

Key words

Quantum agents, Bose distribution, Principle of equal *a priori* probabilities, Non ad-hoc rules, Social systems

1 Introduction

We have so far various studies[Deguchi and Kijima, 2009; Axelrod and Cohen, 1999; Epstein and Axtell, 1996] on social systems using method of multi-agent that was pioneered by [Schelling, 1971]. Simulation by specified rules on element agents leads to corresponding results. There are possibilities that someone can purposely make specified rules to guide our society to some political aim based on his simulation results. To avoid the possible situation, we should take rules that govern agent behavior as natural as possible. In the literature[Oosawa, 2011], systems of agents have been studied where they only exchange their resources by equal chance without any ad-hoc specified rule. Distribution of resources was shown to become statistics that we fit as Boltzmann distribution. As we also reexamine in the paper, even among a few agents statistics met that of Boltzmann. Oosawa's method is applicable also to social systems, although he is interested mainly in biochemistry[Kawamura and Maruyama, 1970]. In various social statistical research, it is tacitly assumed that we can identify **who** contributes to each point in the background data. But is this identification absolutely necessary? For example, in distributing a profit obtained especially among unskilled laborers, managements are interested in numbers of them corresponding to a level of salary rather than a detailed information who takes how much salaries. In this paper, we take social systems of agents where we cannot identify characteristic or individuality of each agent. We call such an agent as *quantum agent* while the ordinary one as *classical agent*. We examine an exchange rule that is not ad-hoc and that we can fit as "Bose distribution" well-known in quantum statistics. This is done by extending Oosawa's method. We show appropriate dependence of results on agent number X and resource number Y. As identifying character of each agent is considered as meaningless, we anticipate resource distributing under Bose or Fermi statistics. We have no maximum units of resource and the resulting statistics will be Bose type.

First in **2**, we review the problem[Oosawa, 2011] of sharing finite resources among finite number of agents. After showing a rule of exchanging resource in **2.1**, we restrict ourselves to systems with resources of $Y = 3$ units among $X = 3$ agents to clarify specified numerical values as easily as possible. We give explicit calculation of trends in **2.2** according to transition matrix and quantitative fitting of resource distribution in **2.3** as our contribution to the problem. To individuality of each agent, we put out of consideration in **3**. We characterize an agent system only as an ensemble with specified number of agents and resources. We consider in **3.1** fair exchange of resource. According to the results we examine dynamics in **3.2** and show appropriate fitting to "Bose distribution" in **3.3**. In the studies of quantum agents, we take a number of agents X and that of resources Y as $10 \sim 100$. We also present dependence of how appropriately the results fit to the Bose distribution on these X and Y. Summary and discussion are given in **4**. This article shows contents those presented in a conference[Itami, 2012] with slight modifications.

2 Statistics in exchanging resource among agents

According to the literature[Oosawa, 2011], we study a system of 3 agents where fair exchange of 3 units of resources is done. After we show in **2.1** a rule of exchange, system dynamics is explicitly calculated by transition matrix in **2.2**. Even for only $X = 3$ agents and $Y = 3$ units of resources, the distribution of resources is shown in **2.3** to become Boltzmann's one.

2.1 Exchanging resource

Let us share 3 units of resource among 3 agents according to the following rules

1. Any one of 9 combinations $T \to T$,$T \to E$,$T \to K$, $E \to T$,$E \to E$,$E \to K$, $K \to T$,$K \to E$ and $K \to K$ of agent giving to another one being given, is assumed to occur in an equal probability $\frac{1}{9}$. Fig.**1** shows an exchange of $T \to E$.

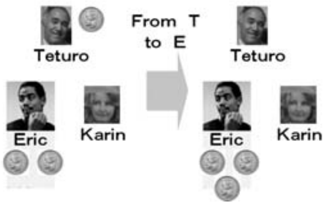

Figure 1. When Mr.T has a turn to give one unit of his resource to Mr.E, we have a pattern where both Miss.K and Mr.T have no resource while Mr.E monopolizes three units of resource.

When T gives his resource to E, E monopolizes all resource while T and K become without any unit of resource. Among the same agent, $T \to T$, $E \to E$ or $K \to K$, we also assume resource exchange that occurs also in a probability $\frac{1}{9}$. After such exchange, obviously no change of state takes place.

2. To an agent without any resource, we do not require anyone to lend a unit of resource. When K has no unit of resource, as shown in Fig.**2**, an request of $K \to E$ makes the present state unchanged, as K has no unit.

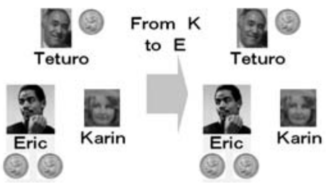

Figure 2. When Miss.K has to give one unit of her resource, a pattern of Mr.E with two units and Mr.T with one unit does not change as she has no unit of resource.

Let us examine the following 10 patterns, from $a1$ to $d4$, of resource distribution to study statistics of the distribution. In the following formula (N_T, N_E, N_K), N_X shows units of resource that Mr./Miss.X holds.

$a1 = (3, 0, 0)$
$b1 = (2, 1, 0)$
$b2 = (2, 0, 1)$
$c1 = (1, 2, 0)$
$c2 = (1, 1, 1)$
$c3 = (1, 0, 2)$
$d1 = (0, 3, 0)$
$d2 = (0, 2, 1)$
$d3 = (0, 1, 2)$
$d4 = (0, 0, 3)$

According to the above symbols, Fig.1 means a transition $c1 \to d1$ and Fig.2 a transition of $c1 \to c1$. When we ignore individuality of each agent, only 3 patterns will be considered instead of 10, $a1$ to $d4$, as shown in **3.1**.

2.2 Representation of exchanging resource by transition matrix

Transition according to the foregoing rules is expressed by a matrix among 10 patterns

$$\mathsf{P}_{Cl} = \begin{bmatrix} \frac{7}{9} & \frac{1}{9} & 0 & 0 & \frac{1}{9} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{9} & \frac{5}{9} & \frac{1}{9} & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{9} & \frac{5}{9} & \frac{1}{9} & 0 & \frac{1}{9} & \frac{1}{9} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{7}{9} & 0 & 0 & \frac{1}{9} & 0 & 0 & 0 \\ \frac{1}{9} & \frac{1}{9} & 0 & 0 & \frac{5}{9} & \frac{1}{9} & 0 & \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{9} & \frac{1}{9} & 0 & \frac{1}{9} & \frac{3}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & \frac{1}{9} & \frac{5}{9} & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{9} & \frac{1}{9} & 0 & \frac{5}{9} & \frac{1}{9} & \frac{1}{9} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{5}{9} & \frac{1}{9} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9} & \frac{1}{9} & \frac{7}{9} \end{bmatrix} \quad (1)$$

This matrix P_{Cl} enables us to calculate a probability p_x that the agent system takes a state x at time $n + 1$ as

$$\begin{bmatrix} p_{a1} \\ p_{b1} \\ \cdots \\ p_{d4} \end{bmatrix}_{n+1} = \mathsf{P}_{Cl} \begin{bmatrix} p_{a1} \\ p_{b1} \\ \cdots \\ p_{d4} \end{bmatrix}_{n} \quad (2)$$

Fig.**3** shows a time trend that starts an initial state generated using appropriate uniform random numbers. We see that a weight of any state approaches to $0.1 = \frac{1}{10}$. Starting with any initial state gives the same result as the transition matrix defined by (1) satisfies

$$(\mathsf{P}_{Cl})^n \underset{n\to\infty}{\longrightarrow} \frac{1}{10} \times \mathsf{1}_{10} \quad (3)$$

where $\mathsf{1}_N$ is a N dimensional square matrix with every element being 1.

2.3 Boltzmann distribution

As a result that every state $a1$,$b1$,$\cdots$,$d4$ occurs in an equal probability, distribution for units of resource k is fitted by the following one by Boltzmann

$$\rho_{Bltz}(k) \equiv C_{Bltz} \times e^{-\beta_{Bltz} k} \quad (4)$$

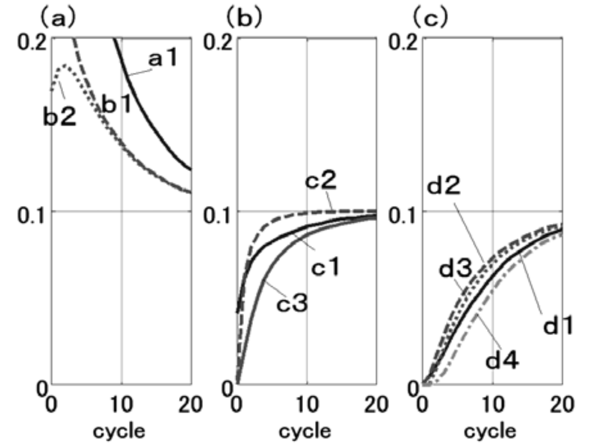

Figure 3. Probabilities of each state $a1$ to $d4$ vary according to the transition matrix (1). All probabilities approach to $0.1 = \frac{1}{10}$, where 10 is the number of these states.

3 Agent that has no individuality

Studies above discussed showed that by fair exchange of resource among agents

1. every pattern of resource distribution occurs in an equal probability: principle of *a priori* probabilities
2. as a result of the equality, resource distribution fits Boltzmann distribution

It is the point that the literature[Oosawa, 2011] emphasizes that at first distribution of resource unit follows Boltzmann distribution while principle of equal *a priori* probabilities comes last. In our fair exchange, Boltzmann distribution does work without assuming principle of equal *a priori* probabilities.

After we characterize our system by "occupation number" in **3.1**, exchanging resource is described as state transition in **3.2**. We see in **3.3** that statistics of resource distribution is approximated by Bose distribution for Y units of resource among X agents.

3.1 Resource exchange among agent without individuality

Let us assume that agent has no individuality. How many units which agent has does not appeal us. We are interested only in how many agents have their own units of resource. This situation reduces a number 10 of states given in **2.1** to the following 3 states. In the following $|N_0, N_1, N_2, N_3 >$ indicates the state where we have N_0 agents without any unit of resource, N_1 ones with 1 unit, N_2 ones with 2 units and N_3 agents who monopolize 3 units of resource.

$$\alpha = |2, 0, 0, 1 >$$
$$\beta = |1, 1, 1, 0 >$$
$$\gamma = |0, 3, 0, 0 >$$

For example, Fig.1 shows a transition of $\beta \to \alpha$ when we paint any face of Teturo, Eric and Karin that characterize his/her individuality into black. A transition corresponding to Fig.2 is equal to $\beta \to \beta$.

3.2 State transition

We examine dynamics that governs state transitions similarly as in **2.2**.We take into account that our agents have no individuality. Exchange of resource only by 1 unit allows the following 10 patterns. For example, $0 \to 1$ shows an operation ($N_0 \to N_0 - 1$,$N_1 \to N_1 +1$,$N_2 \to N_2$ and $N_3 \to N_3$) that we reduce a number N_0 of agents without any resource by 1, while a number N_1 of agents who have 1 unit is increased by 1.

$0 \to 0$
$0 \to 1$
$1 \to 0$
$1 \to 1$
$1 \to 2$
$2 \to 1$
$2 \to 2$
$2 \to 3$
$3 \to 2$
$3 \to 3$

We calculate how each state α, β and γ changes. Regarding a method of calculation Fig.**4** shows an example how the state α changes by a transition $0 \to 1$. According to (a) in this Fig.4 we must take

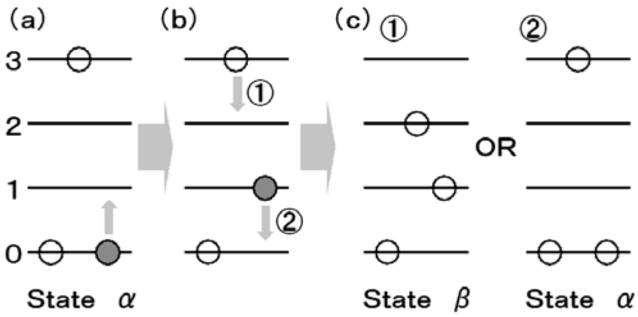

Figure 4. The transition of 0th level to 1st level for the state α is shown in (a). In the transition, first one of the agents in the 0th level goes up to 1st level as shown in (b). To conserve total units of resource, we have two options of ① and ②, which lead to the state β or α as shown in (c), respectively.

one(encircled gray mass) of $N_0 = 2$ to upper state resulting (b). As an agent system denoted in (b) has 4 units of resource, we must reduce, by 1 level, recource by any one of agents. Candidates that we can lower his level are only ① or ②. Probability of choosing one of these two agents is equally $\frac{1}{2}$. (c) shows the results when 1 unit of ① or ②is lowered. A state α changes in an equal probability to β or α according to a transition $0 \to 1$. Calculation for all transitions for all states gives the following transition matrix

$$\mathsf{P}_{Qm} = \begin{bmatrix} \frac{9}{10} & \frac{1}{10} & 0 \\ \frac{1}{10} & \frac{8}{10} & \frac{1}{10} \\ 0 & \frac{1}{10} & \frac{9}{10} \end{bmatrix} \qquad (5)$$

Use of this transition matrix allows us to calculate a probability p_x where the agent system takes a state x as follows

$$\begin{bmatrix} p_\alpha \\ p_\beta \\ p_\gamma \end{bmatrix}_{n+1} = \mathsf{P}_{Qm} \begin{bmatrix} p_\alpha \\ p_\beta \\ p_\gamma \end{bmatrix}_{n} \quad (6)$$

Time trends are calculated as thin solid line(a state α),dashed line (β) and dotted line(γ) in Fig.**5**.In the Fig. we overwrite by thick lines "direct" simulation by uniform random number starting with an appropriate initial condition. We easily see that both thick and thin lines converge to a common value of $\frac{1}{3}$. Similarly to (3), that thin lines converge to $\frac{1}{3}$ is a result of

$$(\mathsf{P}_{Qm})^n \underset{n\to\infty}{\longrightarrow} \frac{1}{3} \times \mathsf{1}_3 \quad (7)$$

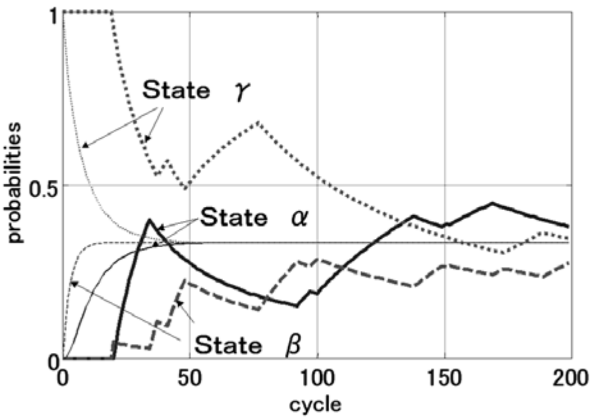

Figure 5. Trends according to transition matrix and probabilistic simulation are simultaneously drawn. Approaching to the equal probability $\frac{1}{3}$ are seen in both trends.

3.3 Bose distribution

In generalizing these results above, we assume principle of equal *a priori* probabilities even when a number X of agents or resource units Y increase. When we restrict ourselves to processes where Y units of resource are exchanged among X agents we believe that lengthy but straightforward calculation in larger values of X or Y directly proves the principle. Under an assumption that the principle of *a priori* equal probabilities holds both in quantum agents without individuality and ordinary classical agents, we calculate distributions of resource units. The results are shown in Fig.**6**, where (a) corresponds to quantum agents while (b) to classical agents. We set agent number as

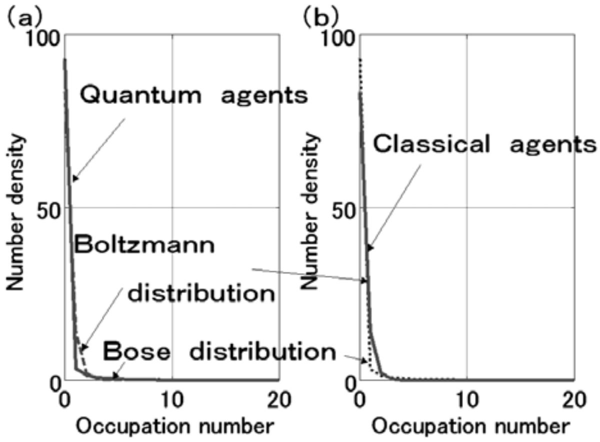

Figure 6. Number density of systems of quantm agents are given in (a) that is approximated by Bose distribution, while in (b) that of classical agents is approximated by Boltzmann distribution. We set here $Y = 20$ tips distributed over $X = 100$ agents.

$X = 100$, and total units of resource $Y = 20$. In these Figures (a) and (b), we set Bose distribution as

$$\rho_{Bose}(k) \equiv \frac{1}{\frac{1}{C_{Bose}} e^{\beta_{Bose} k} - 1} \quad (8)$$

In both (a) and (b), we show Boltzmann distribution defined by (4) as dashed line and Bose distribution of (8) as dotted line. These overwriting show that in (a) resource distribution among quantum agents is close to Bose distribution, while the distribution close to Boltzmann's one in (b). How close these are can

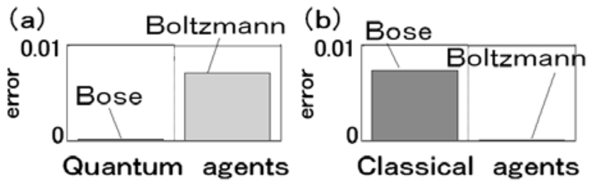

Figure 7. Sum of squared error in number distribution of quantum agents from Bose distribution and Boltzmann distribution are given in (a). Figure (b) shows those for classical agents.

be expressed quantitatively by square residual sum that are given in Fig.**7**. This Fig.7 explicitly presents that quantum agents can be expressed by Bose distribution while cannot be expressed as Boltzmann distribution. Oppositely classical agents can be represented by Boltzmann distribution, while it is difficult to see this as Bose distribution. These calculations above restrict us to take agent number $X = 100$ and resource units $Y = 20$. Now let us examine how the residuals depends on these values of X and Y. These are written in Fig.**8**, where in (a) $Y = 5$, while in (b) residual error for $Y = 20$ are plotted for each agent number

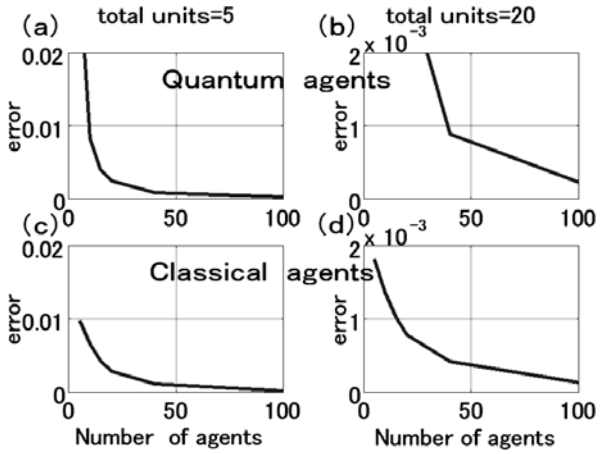

Figure 8. Dependence of error on the number of agents are shown in (a) and (b) for quantum agents. For classical agents we show (c) and (d). Two systems of total energy $= 5$ and $= 20$ are calculated.

$X \leq 100$. Residual error monotonically decrease as increase of agent numbers. Also this decrease in comparing (a) with (b) is seen for more units of total resource. These observation of the data is consistent with intuition.

4 Summary and discussion

Among agents without any individuality, we examined a fair share of resource in a similar way adopted in the literature[Oosawa, 2011]. We cannot identify an agent, who has a specified number of units of resource. The fair sharing led the result that resource distributes among agents according to "Bose" distribution. The results for quantum agents were obtained as various simulation showed the principle of *a priori* equal probabilities. Error of fitting to Bose distributions was shown to decrease as a number X of agents of total units Y of resource increase. In various social statistical research, it is tacitly assumed that we can identify **who** contributes to each point in the background data. It is interesting that what can be claimed when individuality is completely eliminated. Furthermore we have an interesting argument[Tanji, 1977] that even "rice grain" that we tacitly believe their individuality can probably obey Bose or "Fermi" statistics. It is necessary to examine what relation is laid between such argument and our present research.

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