CYBERNETICS AND PHYSICS, VOL. 12, NO. 1, 2023, 93–96

LEV MEEROVICH BREGMAN

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Article history:
Received 05.05.2023, Accepted 25.06.2023

Abstract

In the paper, a brief history of life and work of celebrated Russian mathematician Lev Meerovich Bregman (the author of Bregman divergence and Bregman method) is exposed. The list of Bregman's main publications is given.

Lev Meerovich Bregman was born on January 31, 1941 in Leningrad. A significant part of his conscious life was connected with the Faculty of Mathematics and Mechanics of the Leningrad State University. In 1958-1963 he studied at this faculty, graduating with honors. In 1963-1966 studied in graduate school, and in 1967 he defended his Ph.D. thesis on the topic "Relaxation method for finding a common point of convex sets and its application" under the guidance of I.V. Romanovsky. After defending his dissertation, he worked as a researcher, and then as a senior researcher in the Operations Research Laboratory of the NIIMM (Research Institute of Mathematics and Mechanics within the Faculty of Mathematics and Mechanics) until 1991. During all these years he combined research work with teaching. L.M. Bregman was the first to develop and teach the course "Extremal Problems" for students of the Department of Operations Research, which is now an important course for most departments of the Faculty. It was during this period that L.M. Bregman obtained mathematical results that immortalized his name in mathematics: "Bregman's theorem" on the evaluation of the matrix permanent, "Bregman's method" "Bregman's divergence" and a number of others. He emigrated to Israel in 1991.

Lev Bregman is the author and co-author of several textbooks and about 50 publications in leading Russian and international journals. In 1967, he was awarded the VDNKh (All-Union Exhibition of Industry and Agriculture Achievements) silver medal for success in the national economy development and the creation of automated control systems. Since 1971 he was a member of the Leningrad (later St. Petersburg) Mathematical Society. In 1980-1986, he was a member of the All-Union Commission on the Application of Optimal Planning and Computers in the National Economy, which was headed by Academician L.V. Kantorovich, future Nobel prize winner.

The best known are the results of L.M.Bregman on the method of finding a common point of convex sets, based on sequential projections. The related terms *Bregman divergence*, *Bregman method*, *Bregman splitting, etc.* are now widely used: number of articles in Scopus-indexed journals containing the terms in their titles named after Bregman exceeds 960 as of May 2023. As for the main article [Bregman, 1967b], it has over 1700 citations in Scopus.

L.M. Bregman was always interested in applied problems, and the above results were born when justifying the iterative algorithm of the architect G.V. Sheleikhovsky in the problem of calculating passenger traffic. The first version of the method was published in 1965 in Doklady AN SSSR [Bregman, 1965] (the paper was presented by L.V. Kantorovich). Further, it was shown that this iterative process converges to the nearest (in the entropy pseudometric) of admissible points to the starting point of the iterative process. Thus, the result of the iterative method can be interpreted as a solution to the vector optimization problem with the target point as the starting point of the iterative process. In [Bregman, 1967b], instead of the entropy pseudometric, its generalization was introduced, later called the Bregman divergence, and in joint works with I.V. Romanovsky and N.I. Naumova, axiomatic justifications for the solutions of

these problems were proposed (see, for example, [Bregman and Naumova, 1984]).

The concept of divergence was introduced in [Bregman, 1966; Bregman, 1967b] as follows. Let $f(x),\ x \in R^n$ be a strictly convex twice differentiable function. Define the function $D(x,y) = f(x) - f(y) - (gradf(y), x-y)$, where $gradf(y)$ is the gradient of the function $f(y)$. The function $D(x,y)$,, which is analogous to the distance between two points and is called the *divergence*. It turned out to be convenient for proving convergence as a candidate for the Lyapunov function.

Since the 1990s the method based on using divergence as the substitute for a distance has been widely used in machine learning, clustering, denoising images, image segmentation, data reconstruction, etc. Thus, today the method of L.M. Bregman and related concepts proposed in the 1960s and bearing his name have become widely known throughout the world and have entered the gold fund of scientific results obtained by scientists of the Faculty of Mathematics and Mechanics of St. Petersburg State University. His publications are listed below.

In September 1991 L.M. Bregman emigrated to Israel, where until the last days he did not stop doing his favorite mathematics. In 1992-1993 he worked at Ben-Gurion University in Beer-Sheba, and then at the Institute for Industrial Mathematics in the same city. Since 1992 he has been a member of the Israel Mathematical Society. At the same time, he maintained a friendly and scientific relationship with colleagues from Russia.

Colleagues in the laboratory and faculty highly appreciated Lev Meerovich's modesty, kindness and responsiveness, readiness to help.

L.M. Bregman passed away on February 23, 2023 in Beer-Sheba at the age of 82.

The main published works of Lev Bregman are: [Bregman and Rakhman, 1970; Bregman and Surin, 1985; Bregman and Gribov, 1964; Bregman, 1965; Bregman, 1966; Bregman, 1967a; Bregman, 1967b; Bregman and Muraviev, 1967; Bregman, 1969; Bregman and Fokin, 1969; Bregman, 1970; Bregman, 1972; Bregman et al., 1972a; Bregman et al., 1972b; Bregman and Fokin, 1972; Bregman and Fokin, 1973b; Bregman, 1973; Bregman and Naumova, 1984; Bregman and Fokin, 1973a; Bregman and Fokin, 1974a; Bregman and Fokin, 1974b; Bregman et al., 1975; Bregman and Romanovsky, 1975; Bregman, 1974; Bregman et al., 1976; Bregman et al., 1977; Bregman, 1979; Bregman and Surin, 1979; Bregman et al., 1981; Bregman, 1982b; Bregman, 1982a; Bregman et al., 1983; Bregman et al., 1984; Bregman and Fokin, 1987; Brégman, 1988; Bregman and Fokin, 1988; Bregman and Fokin, 1988; Bregman, 1989; Bregman, 1989; Brégman, 1990; Bregman et al., 1990; Bregman and Fokin, 1991; Baranov and Bregman, 1993; Baranov and Bregman, 1994; Bregman and Fokin, 1998; Bregman et al., 1999; Berger et al., 1999; Bregman and Fokin, 2001; Bregman and Naumova, 2002; Bregman and Fokin, 2002; Bregman and Bregman, 2002; Bregman et al., 2003; Kalir et al., 2013; Bregman, 2014].

He coauthored also two patents: [Pridor et al., 1998; Barel et al., 2000].

The main published works of Lev Bregman

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