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# SOLVING DISTURBANCE DECOUPLING FOR SINGULAR SYSTEMS BY P&D-FEEDBACK AND P&D-OUTPUT INJECTION

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**Abstract**
Singular systems are an important class of systems from both point of view theoretical and practical. In this paper we analyze the problem of constructing feedbacks and/or output injections that suppress this disturbance in the sense that it does not affect the input-output behavior of the system and makes the resulting closed-loop system regular and of index at most one. All results are based on the canonical reduced forms that they can be computed using a complete system of invariants and can be implemented in a numerically stable way.

**Key words**
Singular Systems, equivalence relation, disturbance decoupling.

## 1 Introduction

Singular systems (also referred to as differential-algebraic, descriptor, generalized, or semistate systems) constitute an important class of systems of both theoretical interest and practical significance. Mechanical multibody systems (see [A.M. Bloch, M. Reyhanoglu, N.H. McClamroch, (1992), L.S. You, B.S. Chen, (1993), M. Hou, (1995)], for example), are modeled naturally by singular systems. Singular systems are also known to arise as dynamic models in power systems [D.J. Hill, I.M. Mareels, (1990)], chemical processes [A. Kumar, P. Daoutidis, (1995)] and electrical circuits [R.W. Newcomb, (1981), M. Günther M, U. Feldmann, (1995)], for example, and they have been studied under different points of view.

We consider linear and time-invariant continuous singular systems of the form

$$\begin{cases} E\dot{x}(t)=Ax(t)+Bu(t)+Gg(t),\ x(t_0)=x_0,\ t\geq 0\\ y(t)=Cx(t), \end{cases} \tag{1}$$

where $E, A \in M_n(C)$, $B \in M_{n\times m}(C)$, $C \in M_{p\times n}(C)$, $G \in M_{n\times q}(C)$ and $\dot{x} = dx/dt$. The term $g(t)$, $t \geq 0$, represents a disturbance, which may represent modeling or measuring errors, noise, or higher order terms in linearization.

The problem of constructing feedbacks and/or output injections that suppress this disturbance in the sense that $g(t)$ does not affect the input-output behavior of the system is analyzed. In the case of standard state space systems the disturbance decoupling problem has been largely studied (see [A. Ailon, (1993),A. S. Morse and W. M. Wonham, (1970), M. Rakowski, (1994)]for example). This problem for singular systems has also been studied (see [D. Chu and V. Mehrmann, (2000), L. R. Fletcher and A. Asaraai, (1989)] for example). In this paper we study the disturbance decoupling problem for singular systems that can be stated as follows: Find necessary and sufficient conditions under which we can choose proportional and derivative feedback as well proportional and derivative output injection such that, the matrix pencil $(E+BF_E^B+F_E^CC, A+BF_A^B+F_A^CC)$ is regular of index at most one and

$$C(s(E+BF_E^B+F_E^CC)-(A+BF_A^B+F_A^CC))^{-1}G=0.$$

In the case where $C(sE-A)^{-1}G=0$ we say that the system is trivially decoupled.

We remember that a system $(E, A, B, C)$ is regular if and only if there exist a couple of complex numbers $(\lambda, \mu)$ such that $\det(\lambda E+\mu A)\neq 0$. If the system is not regular but there exist proportional and derivative feedback $F_A^B$, $F_E^B$ as well proportional and derivative output injection $F_A^C$, $F_E^C$ such that $\det(\lambda(E+BF_E^B+F_E^CC)+\mu(A+BF_A^B+F_A^CC))\neq 0$ we say that the system is regularisable.

We assume without loss of generality that matrices $B$, $G$ are full column rank and C is full row rank, i.e., $\operatorname{rank} B = m$, $\operatorname{rank} G = q$, $\operatorname{rank} C = p$. If this is not the case, then this can be easily achieved, by removing the nullspaces and appropriate renaming of variables.

## 2 Notations

In the sequel we will use the following notations.
- $I_n$ denotes the $n$-order identity matrix,
- $N$ denotes a nilpotent matrix in its reduced form $N = \text{diag}\,(N_1, \ldots, N_t)$, $N_i = \begin{pmatrix} 0 & I_{n_i-1} \\ 0 & 0 \end{pmatrix} \in M_{n_i}(C)$,
- $J$ denotes the Jordan matrix $J = \text{diag}\,(J_1, \ldots, J_t)$, $J_i = \text{diag}(J_{i_1}, \ldots, J_{i_s})$, $J_{i_j} = \lambda_i I_{i_j} + N$.

We represent systems of the form (1) as quadruples of matrices $(E, A, B, C)$ in the case of disturbance does not appear or it is not considered and quintuples of matrices $(E, A, B, C, G)$ otherwise.

## 3 Reduced Form

We recall that, given a regularisable singular system using standard transformations in state, input and output spaces $x(t) = Px_1(t)$, $u(t) = Ru_1(t)$, $y_1(t) = Sy(t)$, premultiplication by an invertible matrix $QE\dot{x}(t) = QAx(t) + Qu(t)$ making proportional feedback $u(t) = u_1(t) - Vx(t)$ and derivative feedback $u(t) = u_1(t) - U\dot{x}(t)$ as well as proportional output injection $u(t) = u_1(t) - Wy(t)$ and derivative output injection $u(t) = u_1(t) - Z\dot{y}(t)$, it is possible to reduce to $E_r\dot{x}_1(t) = A_r x_1(t) + B_r u_1(t) + G_1$, $y_1 = C_r x(t)$ where

$$E_r = \begin{pmatrix} I_1 & & & & \\ & I_2 & & & \\ & & I_3 & & \\ & & & I_4 & \\ & & & & N_1 \end{pmatrix}, \quad A_r = \begin{pmatrix} N_2 & & & & \\ & N_3 & & & \\ & & N_4 & & \\ & & & J & \\ & & & & I_5 \end{pmatrix}$$

$$B_r = \begin{pmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad C_r = \begin{pmatrix} C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_2 & 0 & 0 \end{pmatrix}$$

and

i) $(I_1, N_2, B_1, C_1)$ is a $n_1$ size completely controllable and observable system in its canonical reduced form.
ii) $(I_2, N_3, B_2)$ is a $n_2$ size completely controllable non observable system in its canonical reduced form.
iii) $(I_3, N_4, C_2)$ is a $n_3$ size completely observable non controllable system in its canonical reduced form.
iv) $(I_4, J)$ is a $n_4$ size system having only finite zeroes.
v) $(N_1, I_5)$ is a $n_5$ size system having only non transferable infinite zeroes.

$(\sum_{i=1}^{7} n_i = n)$.

The proof is based in the following proposition.

**Proposition 3.1.** *Two quadruples of matrices $(E_i, A_i, B_i, C_i)$ are equivalent under equivalence relation considered if and only if the matrix pencils $\lambda \begin{pmatrix} E_i & B_i & 0 \\ C_i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} A_i & 0 & B_i \\ 0 & 0 & 0 \\ C_i & 0 & 0 \end{pmatrix}$ are strictly equivalent.*

**Remark 3.1.** *Not all parts i),..., v), necessarily appear in the decomposition of a system.*

**Remark 3.2.** *The reduced form of a regularisable system is a regular system and in this case there exists a couple $(s, -1)$ such that $\det(sE_r - A_r) \neq 0$.*

## 4 The disturbance decoupling problem

In this section we will use the reduced form for the system in order to analyze the disturbance decoupling problem.

**Proposition 4.1 ([M. I. García-Planas, (2010)]).** *Consider a system of the form (1). The system can be regularized by means a state and derivative feedback as well state a derivative output injection with index at most one if and only if the reduced form does not contain parts vi), vii), and viii), and if it contains v), the nilpotent matrix $N_1$ is the zero matrix.*

**Theorem 4.1 ([M. I. García-Planas, (2010)]).** *Consider a system of the form (1). The system can be regularized by means a proportional and derivative feedback as well as proportional and derivative output injection with index at most one if and only if*

*i)* $r_1 - r_0 \geq n$,
*ii)* $s_k \leq 2(r_B - t)$.
*iii)* $l_k \leq 2(r_C - t)$,

*where*

- $r_0 = \text{rank} \begin{pmatrix} E & B \\ C & 0 \end{pmatrix}$
- $r_1 = \text{rank} \begin{pmatrix} E & B & & \\ C & 0 & & \\ A & 0 & E & B \\ & & C & 0 \end{pmatrix}$
- $s_k$ *is the number of column minimal indices of the pencil* $\lambda \begin{pmatrix} E & B & 0 \\ C & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} A & 0 & B \\ 0 & 0 & 0 \\ C & 0 & 0 \end{pmatrix}$
- $r_B = \text{rank}\, B$
- $l_k$ *is the number of row minimal indices of the pencil* $\lambda \begin{pmatrix} E & B & 0 \\ C & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} A & 0 & B \\ 0 & 0 & 0 \\ C & 0 & 0 \end{pmatrix}$
- $r_C = \text{rank}\, C$
- $t = r_n - r_{n-1} - n$
- $r_\ell = \text{rank}\, M_\ell$

$$M_\ell = \begin{pmatrix} E & B & & & & & & & \\ C & 0 & & & & & & & \\ A & 0 & E & B & & & & & \\ & & C & 0 & & & & & \\ & & A & 0 & & & & & \\ & & & & \ddots & & & & \\ & & & & & E & B & & \\ & & & & & C & 0 & & \\ & & & & & A & 0 & E & B \\ & & & & & & & C & 0 \end{pmatrix}$$

$\in M_{(\ell+1)(n+p)\times(\ell+1)(n+m)(C)}$.

Based on reduced form and extending the equivalence to the quintuples of matrices (i.e. $QG = \overline{G}$), the system (1), is reduced to the following independent subsystems.

$$\begin{cases} \dot{x}_1 = N_2 x_1 + B_1 u_1 + G_1 g_1 \\ y_1 = C_1 x_1 \end{cases} \quad (2)$$

$$\{\dot{x}_2 = N_3 x_2 + B_2 u_2 + G_2 g_2 \quad (3)$$

$$\begin{cases} \dot{x}_3 = N_4 x_3 + G_3 g_3 \\ y_3 = C_2 x_3 \end{cases} \quad (4)$$

$$\{\dot{x}_4 = J x_4 + G_4 g_4 \quad (5)$$

$$\{N_1 \dot{x}_5 = x_5 + G_5 g_5 \quad (6)$$

So, we can study disturbance decoupling problem for each subsystem separately. Taking into account [D. Chu and V. Mehrmann, (2000)], lemma 2.4, we have the following proposition.

**Proposition 4.2.** *Assume* $\overline{G} = \begin{pmatrix} G_1 \\ \vdots \\ G_5 \end{pmatrix}$ *according to the subsystems (2), …, (6). Let* $s \in C$ *such that* $\det(sI_{n_1} - N_2) \neq 0$, $\det(sI_{n_2} - N_3) \neq 0$, $\det(sI_{n_3} - N_4) \neq 0$, $\det(sI_n - J) \neq 0$ *and* $\det(sN_1 - I_{n_5}) \neq 0$, *(it exists because of regularity of the subsystems (2),..., (6)). Then*

*i)* $C_1(sI_{n_1} - N_2)^{-1}G_1 = 0$ *if and only if* $\operatorname{rank}\begin{pmatrix} sI_{n_1} - N_2 & G_1 \\ C_1 & 0 \end{pmatrix} = n_1$,

*ii)* $(sI_{n_2} - N_3)^{-1}G_2 = 0$ *if and only if* $G_2 = 0$

*iii)* $C_1(sI_{n_3} - N_4)^{-1}G_3 = 0$ *if and only if* $\operatorname{rank}\begin{pmatrix} sI_{n_3} - N_4 & G_3 \\ C_2 & 0 \end{pmatrix} = n_3$

*iv)* $(sI_{n_4} - J)^{-1}G_4 = 0$ *if and only if* $G_4 = 0$

*v)* $(sN_1 - I_{n_5})^{-1}G_5 = 0$ *if and only if* $G_5 = 0$

*Proof.* Calling $P(s) = \begin{pmatrix} (sI_{n_1} - N_2)^{-1} & 0 \\ -C_1(sI_{n_1} - N_2)^{-1} & I_{p_1} \end{pmatrix}$ and $Q(s) = \begin{pmatrix} I_{n_1} & (sI_{n_1} - N_2)^{-1}G_1 \\ 0 & -I_q \end{pmatrix}$ we have

$$\begin{aligned} &\operatorname{rank} P(s)\begin{pmatrix} sI_{n_1} - N_2 & G_1 \\ C_1 & 0 \end{pmatrix} Q(s) \\ &= \operatorname{rank}\begin{pmatrix} I_{n_1} & 0 \\ 0 & C_1(sI_{n_1} - N_2)^{-1}G_1 \end{pmatrix} \\ &= n_1 + \operatorname{rank} C_1(sI_{n_1} - N_2)^{-1}G_1 = r \end{aligned}$$

$r = n_1$ if and only if $C_1(sI_{n_1} - N_2)^{-1}G_1 = 0$.
We compute analogously for the other cases.

As a consequence we have.

**Corollary 4.1.** *Let* $(E, A, B, C, G)$ *be a quintuple of matrices in its reduced form, and we assume* $\overline{G} = \begin{pmatrix} G_1 \\ \vdots \\ G_5 \end{pmatrix}$ *according to the decomposition of the system. If* $\operatorname{rank}\begin{pmatrix} sI_{n_1} - N_2 & G_1 \\ C_1 & 0 \end{pmatrix} = n_1$ *and* $\operatorname{rank}\begin{pmatrix} sI_{n_3} - N_2 & G_3 \\ C_2 & 0 \end{pmatrix} = n_3$, *then the given system is disturbance decoupled.*

**Corollary 4.2.** *Let* $(E, A, B, C, G)$ *be a quintuple of matrices in its reduced form, and we assume* $\overline{G} = \begin{pmatrix} G_1 \\ \vdots \\ G_5 \end{pmatrix}$ *according to the decomposition of the system. If* $G_1 = 0$, $G_3 = 0$, *then the given system is trivially disturbance decoupled.*

**Theorem 4.2.** *Let* $(E, A, B, C, Q)$ *be a system such that its equivalent canonical reduced form* $(E_r, A_r, B_r, C_r, G_r)$ *is trivially decoupled. Then* $(E, A, B, C, Q)$ *may be decoupled.*

*Proof.* If $(E_r, A_r, B_r, C_r, G_r)$ is the equivalent canonical reduced form of $(E, A, B, C)$ then $E_r = Q(E + BF_E^B + F_E^C C)P$, $A_r = Q(A + BF_A^B + F_A^C C)P$, $B_r = QBR$, $C_r = SCP$ and $G_r = QG$.
By hypothesis $C_r(sE_r - A_r)^{-1}G_r = 0$. So,

$$\begin{aligned} &SCP(s(Q(E + BF_E^B + F_E^C C)P) - \\ &\quad (Q(A + BF_A^B + F_A^C C)P))^{-1}QG = \\ &SCPP^{-1}(s(E + BF_E^B + F_E^C C) - \\ &\quad (A + BF_A^B + F_A^C C))^{-1}Q^{-1}QG = \\ &SC(s(E + BF_E^B + F_E^C C) - \\ &\quad (A + BF_A^B + F_A^C C))^{-1}G = 0 \end{aligned}$$

equivalently

$$C(s(E+BF_E^B+F_E^CC)-(A+BF_A^B+F_A^CC))^{-1}G=0.$$

Now, we are going to try to obtain conditions to ensure existence of solution for disturbance decoupling problem depending only, on matrices $E, A, B, C, G$.

Following notations of theorem 4.2, we will call $\bar{E} = E + BF_E^B + F_E^C C$ and $\bar{A} = A + BF_A^B + F_A^C C$.

**Lemma 4.1.** *$C(s\bar{E} - \bar{A})^{-1}G = 0$ if and only if* $\operatorname{rank}\begin{pmatrix} s\bar{E}-\bar{A} & G \\ C & 0 \end{pmatrix} = n.$

*Proof.*

$$\begin{pmatrix} (s\bar{E}-\bar{A})^{-1} & \\ -C(s\bar{E}-\bar{A})^{-1} & I \end{pmatrix}\begin{pmatrix} s\bar{E}-\bar{A} & G \\ C & 0 \end{pmatrix}\begin{pmatrix} I & (s\bar{E}-\bar{A})^{-1}G \\ 0 & -I \end{pmatrix} = \begin{pmatrix} I & \\ & C(s\bar{E}-\bar{A})^{-1}G \end{pmatrix}.$$

**Lemma 4.2.** $C(s\bar{E} - \bar{A})^{-1}G = 0$ *if and only if*

$$\operatorname{rank}\begin{pmatrix} s(E+BF_E^B)-(A+BF_A^B) & G \\ C & 0 \end{pmatrix} = n.$$

*Proof.*

$$\operatorname{rank}\begin{pmatrix} s\bar{E}-\bar{A} & G \\ C & 0 \end{pmatrix} = \operatorname{rank}\begin{pmatrix} I & sF_E^C C - F_A^C C \\ & I \end{pmatrix}\begin{pmatrix} s(E+BF_E^B)-(A+BF_A^B) & G \\ C & 0 \end{pmatrix} = \operatorname{rank}\begin{pmatrix} s(E+BF_E^B)-(A+BF_A^B) & G \\ C & 0 \end{pmatrix}.$$

**Lemma 4.3.**

$$\operatorname{rank}\begin{pmatrix} sE-A & B & G \\ C & 0 & 0 \end{pmatrix} = \operatorname{rank}\begin{pmatrix} s(E+BF_E^B)-(A+BF_A^B) & B & G \\ C & 0 & 0 \end{pmatrix}.$$

*Proof.*

$$\operatorname{rank}\begin{pmatrix} sE-A & B & G \\ C & 0 & 0 \end{pmatrix}\begin{pmatrix} I & 0 & 0 \\ sF_E^B - F_A^B & I & 0 \\ 0 & 0 & I \end{pmatrix} = \operatorname{rank}\begin{pmatrix} s(E+BF_E^B)-(A+BF_A^B) & B & G \\ C & 0 & 0 \end{pmatrix}.$$

**Theorem 4.3.** *A necessary condition for $C(s\bar{E} - \bar{A})^{-1}G = 0$ is*

$$\mathit{rank}\begin{pmatrix} sE-A & B & G \\ C & 0 & 0 \end{pmatrix} = n+t$$

*Proof.* If $C(s\bar{E} - \bar{A})^{-1}G = 0$, then

$$\operatorname{rank}\begin{pmatrix} sE-A & B & G \\ C & 0 & 0 \end{pmatrix} = \operatorname{rank}\begin{pmatrix} s(E+BF_E^B)-(A+BF_A^B) & B & G \\ C & 0 & 0 \end{pmatrix} \geq \operatorname{rank}\begin{pmatrix} s(E+BF_E^B)-(A+BF_A^B) & G \\ C & 0 \end{pmatrix} = n.$$

On the other hand

$$\begin{aligned}
&\operatorname{rank}\begin{pmatrix} sE-A & B & G \\ C & 0 & 0 \end{pmatrix} = \\
&\operatorname{rank}\begin{pmatrix} Q & sF_E^C - F_A^C \\ 0 & S \end{pmatrix}\begin{pmatrix} sE-A & B & G \\ C & 0 & 0 \end{pmatrix}\begin{pmatrix} P & 0 & 0 \\ sF_E^B - F_A^B & R & 0 \\ 0 & 0 & I \end{pmatrix} = \\
&\operatorname{rank}\begin{pmatrix} sE_c - A_c & B_c & \bar{G} \\ C_c & 0 & 0 \end{pmatrix} = \\
&\operatorname{rank}\begin{pmatrix} I & 0 & 0 \\ 0 & C_1(sI-N_2)^{-1}B_1 & C_1(sI-N_2)^{-1}\bar{G}_1 \\ 0 & 0 & C_2(sI-N_4)^{-1}\bar{G}_3 \end{pmatrix} = \\
&n + t + \operatorname{rank} C_2(sI-N_4)^{-1}\bar{G}_3.
\end{aligned}$$

**Proposition 4.3.** *A sufficient condition for $C(s\bar{E} - \bar{A})^{-1}G = 0$ is that*

$$\operatorname{rank}\begin{pmatrix} sE-A & B & G \\ C & 0 & 0 \end{pmatrix} = n.$$

In the case we also study in addition the use of a possible changes in control of form $u(t) = F_G^B q(t)$. We can consider the following matrices

$$\bar{M}_\ell = \begin{pmatrix}
E & B & G_1 & & & & & & & & & \\
C & 0 & 0 & & & & & & & & & \\
A & 0 & 0 & E & B & G_1 & & & & & & \\
 & & & C & 0 & 0 & & & & & & \\
 & & & A & 0 & 0 & & & & & & \\
 & & & & & & \ddots & & & & & \\
 & & & & & & & E & B & G_1 & & \\
 & & & & & & & C & 0 & 0 & & \\
 & & & & & & & A & 0 & 0 & E & B & G_1 \\
 & & & & & & & & & & C & 0 & 0
\end{pmatrix} \in M_{x\times y}(C).$$

with $x = (\ell+1)(n+p)$, $y = (\ell+1)(n+m+q)$ and $G_1 = G + BH$.
and we have the following result

**Theorem 4.4.** *For some $F_G^B$, a sufficient condition for $C(s\bar{E} - \bar{A})^{-1}\bar{G} = 0$ is*

$$\operatorname{rank}\bar{M}_\ell = \operatorname{rank} M_\ell$$

*Proof.* It suffices to observe that

$$\operatorname{rank}\begin{pmatrix}
E & B & G & & & & & & & & & \\
C & 0 & 0 & & & & & & & & & \\
A & 0 & 0 & E & B & G & & & & & & \\
 & & & C & 0 & 0 & & & & & & \\
 & & & A & 0 & 0 & & & & & & \\
 & & & & & & \ddots & & & & & \\
 & & & & & & & E & B & G & & \\
 & & & & & & & C & 0 & 0 & & \\
 & & & & & & & A & 0 & 0 & E & B & G \\
 & & & & & & & & & & C & 0 & 0
\end{pmatrix} =$$

$$\operatorname{rank}\begin{pmatrix} E_r & B & G \\ C_r & 0 & 0 \\ A_r & 0 & 0 & E_r & B_r & \bar{G} \\ & & & C_r & 0 & 0 \\ & & & A_r & 0 & 0 \\ & & & & & & \ddots \\ & & & & & & & E_r & B_r & \bar{G} \\ & & & & & & & C_r & 0 & 0 \\ & & & & & & & A_r & 0 & 0 & E_r & B_r & \bar{G} \\ & & & & & & & & & & C_r & 0 & 0 \end{pmatrix}$$

where $(E_r, A_r, B_r, C_r, \bar{G})$ is the reduced form of $(E, A, B, C)$ extended to the quintuple $(E, A, B, C, G + BF_G^B)$.

Thus, for example for $\ell = 0$

$$\operatorname{rank}\begin{pmatrix} E & B & G \\ C & 0 & 0 \end{pmatrix} =$$
$$\operatorname{rank}\begin{pmatrix} Q & F_E^C \\ 0 & S \end{pmatrix}\begin{pmatrix} E & B & G \\ C & 0 & 0 \end{pmatrix}\begin{pmatrix} P & 0 & 0 \\ F_E^B & R & H \\ 0 & 0 & I_p \end{pmatrix} =$$
$$\operatorname{rank}\begin{pmatrix} E_r & B_r & \bar{G}_1 \\ C_r & 0 & 0 \end{pmatrix}.$$

where $\bar{G}_1 = QG_1 = Q(G + BH)$.

In a analogous manner we can test for all values of $\ell$

If the system $(\bar{E}, \bar{A}, \bar{B}, \bar{C})$ is of index 0 (that is to say the system is equivalent to a standard one), the condition before is also necessary.

**Theorem 4.5.** *Let $(E, A, B, C)$ a standardizable system. For some $F_G^B$, a necessary and sufficient condition for $C(s\bar{E} - \bar{A})^{-1}\bar{G} = 0$ is*

$$\operatorname{rank} \bar{M}_\ell = \operatorname{rank} M_\ell.$$

*Proof.* It suffices to observe that in this case the submatrix $G_5$ do not appears in the decomposition of $QG$.

### 4.1 Disturbance decoupling problem with stability

The disturbance decoupling problem is called with stability if one imposes the additional constraint that the close-loop $(E + BF_E^B + F_E^C C)\dot{x}(t) = (A + BF_A^B + F_A^C C)x(t) + Bu(t) + Gg(t)$, $y(t) = Cx(t)$ system is stable. Remember that a singular system is stable if and only if the spectrum of the system lies in $C^{-1}$.

**Proposition 4.4.** *Given a singular system $(E, A, B, C)$. There exist a proportional and derivative feedback as well a proportional and derivative output injection such that the close-loop system $(E + BF_E^B + F_E^C C, A + BF_A^B + F_A^C C, B, C)$ is stable (and we call stable under proportional and derivative feedback and proportional and derivative output injection) if and only if*

$$\operatorname{rank}\begin{pmatrix} sE - A & B \\ C & 0 \end{pmatrix} = n,$$

$\forall s \in C^+$.

*Proof.* The spectrum of a system coincides with the spectrum of the associate pencil, and the spectrum is invariant under equivalence relation.

As a consequence we have.

**Corollary 4.3.** *Let $(E, A, B, C, G)$ a quintuple of matrices in its reduced form, and we assume $\overline{G} = \begin{pmatrix} G_1 \\ \vdots \\ G_5 \end{pmatrix}$ according to the decomposition of the system. If $\operatorname{rank}\begin{pmatrix} sI_{n_1} - N_2 & G_1 \\ C_1 & 0 \end{pmatrix} = n_1$, $\operatorname{rank}\begin{pmatrix} sI_{n_3} - N_4 & G_3 \\ C_1 & 0 \end{pmatrix} = n_3$ and $\sigma(J) \subset C^{-1}$. Then the given system is trivially disturbance decoupled with stability.*

**Corollary 4.4.** *Let $(E, A, B, C, G)$ a quintuple of matrices in its reduced form, and we assume $\overline{G} = \begin{pmatrix} G_1 \\ \vdots \\ G_5 \end{pmatrix}$ according to the decomposition of the system. If $G_1 = 0$, $G_3 = 0$, and $\sigma(J) \subset C^{-1}$. Then the given system is trivially disturbance decoupled with stability.*

## 5 Conclusions

In this paper a qualitative description of the disturbance decoupling problem is considered. A necessary and sufficient condition for the existence of a proportional and derivative feedback, as well as, a proportional and derivative output injection, such that the close-loop system is regular with index at most one is obtained and for systems in its reduced form a condition for decoupling is presented.

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