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DESIGN OF OPTIMAL-ROBUST CONTROL OF STEP MOTORS WITH PARTIALLY UNKNOWN PARAMETERS AND DISTURBANCES

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Abstract

The paper describes the design of a novel control law for step motor under participially unknown parameters and bounded external disturbances. The unknown parameters and disturbances are related to the motor torque, the rotor moment of inertia, the viscous friction, the resistance and the load torque. The design of a novel control law is based on feedback linearization and linear robust control. The gain of a linear control law is calculated by solving the linear matrix inequality and some optimization problems. The simulations illustrate the efficiency of the proposed scheme in comparison with classical PID-controller and adaptive control law.

Key words

Step motor, control, linear matrix inequality, optimization.

1 Introduction

At the present moment, step motors are being replaced by AC machines, since the latter ones have a long service life and reliability due to the absence of sliding electrical contacts [Viorel and Lorand, 1998; Gieras et al., 2016]. Electric motors are used to study various physical phenomena [Mihalache et al., 2013; AL-Sabbagh and Mahdi, 2010; Kelemen and Crivii, 1975; Boikov et al., 2016; Tomchina, 2021; Ugalde-Loo et al., 2013]. In aviation and space technology, step motors are actively used in actuating systems, such as drives for opening large structures, guidance and stabilization systems, etc [Sarhan et al., 2009; Fu et al., 2022; Morar, 2007; Acarnley, 2002; Zribi and Chiasson, 1991; Kenjo, 1984]. The most widespread are special synchronous motors, which, compared with other electric motors, have the best indicators of specific power, efficiency, and reliability. These machines include step motors and permanent magnet synchronous motors (PMSM).

In [Furtat et al., 2022] an overview of mathematical models of step motors is given. Based on these models the some effective existing control laws are considered. In [Furtat et al., 2022] it is noted that step motors can be effectively controlled without feedback in the absence of parametric uncertainty and external disturbances. However, under conditions of uncertainty in the motor parameters (for example, changes in resistance, inductance, etc. due to wear and temperature change) and under conditions of external disturbances (for example, changes in the motor load), the open-loop control becomes ineffective. The overview notes that the most effective methods are PID control and adaptive control [Marino et al., 1995].

In this paper, we apply some existing results on robust control [Nazin et al., 2007; Furtat et al., 2020], which allow one to calculate the controller parameters with the solution of some optimization problems. At the end of the paper, we will compare the proposed results and some effective methods, selecting from [Furtat et al., 2022].

2 Step motor model. Problem formulation

Consider a step motor model [Marino et al., 1995] in the form

$$\begin{aligned} \frac{d\theta}{dt} &= \omega, \\ \frac{d\omega}{dt} &= -\frac{K_m}{J} i_a \sin(n\theta) + \frac{K_m}{J} i_b \cos(n\theta) \\ &\quad -\frac{F}{J}\omega - \frac{T_L}{J}, \\ \frac{di_a}{dt} &= -\frac{R}{L} i_a + \frac{K_m}{L}\omega \sin(n\theta) + \frac{u_a}{L}, \\ \frac{di_b}{dt} &= -\frac{R}{L} i_b + \frac{K_m}{L}\omega \cos(n\theta) + \frac{u_b}{L}, \end{aligned} \tag{1}$$

where T_L is load torque, θ and ω are rotor position and speed respectively, i_a, i_b and u_a, u_b are currents and stator voltages of a two-phase motor. Motor parameters are resistance R and self-induction L for each stator phase winding, motor torque K_m, rotor moment of inertia J, viscous friction F, and number of rotor teeth n.

The goal is to design the control law ensuring minimization an influence of the disturbance T_L into the following errors $e_1 = \theta - \theta_m$ and $e_2 = \omega - \omega_m$, where $\theta_m(t)$ is the reference (given, command) position of the rotor of (1), $\omega_m(t) = \dot{\theta}_m(t)$ be the rate of $\theta_m(t)$. The various number of optimization problems will be introduced in Theorem 1.

3 Main results

Forward Park transform converts the input two-coordinate vector to flux and torque components. The Park transform can be used to realize the transformation of the (a, b) currents from the stationary to the moving reference frame and control the spatial relationship between the stator vector current and rotor flux vector. To transform from (a, b) coordinate to (d, q), introduce Park's formula [Marino et al., 1995]

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos(n\theta) & \sin(n\theta) \\ -\sin(n\theta) & \cos(n\theta) \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix}. \tag{2}$$

Considering (2), the model (1) can be rewritten as follows

$$\begin{aligned} \frac{d\theta}{dt} &= \omega, \\ \frac{d\omega}{dt} &= \frac{K_m}{J} i_q - \frac{F}{J}\omega - \frac{T_L}{J}, \\ \frac{di_d}{dt} &= -\frac{R}{L} i_d + n\omega i_q + \frac{u_d}{L}, \\ \frac{di_q}{dt} &= -\frac{R}{L} i_q - n\omega i_d - \frac{K_m}{L}\omega + \frac{u_q}{L}. \end{aligned} \tag{3}$$

Using the deviation errors $e_1 = \theta - \theta_m$ and $e_2 = \omega - \omega_m$, rewrite model (3) as

$$\begin{aligned} \frac{de_1}{dt} &= e_2, \\ \frac{de_2}{dt} &= \frac{K_m}{J} i_q - \frac{F}{J}\omega - \frac{T_L}{J} - \dot{\omega}_m, \\ \frac{di_d}{dt} &= -\frac{R}{L} i_d + n\omega i_q + \frac{u_d}{L}, \\ \frac{di_q}{dt} &= -\frac{R}{L} i_q - n\omega i_d - \frac{K_m}{L} e_2 + \frac{u_q}{L} - \frac{K_m}{L}\omega_m. \end{aligned} \tag{4}$$

Introduce the feedback linearization control law in the form

$$\begin{aligned} u_d &= -Ln\omega i_q, \\ u_q &= Ln\omega i_d + Lv, \end{aligned} \tag{5}$$

where v is an auxiliary control law to reduce the influence of disturbances under the condition of parametric uncertainty of (1).

Substituting (5) into (4), one gets a linear model

$$\begin{aligned} \frac{de_1}{dt} &= e_2, \\ \frac{de_2}{dt} &= \frac{K_m}{J} i_q - \frac{F}{J} e_2 - \frac{T_L}{J} - \dot{\omega}_m - \frac{F}{J}\omega_m, \\ \frac{di_d}{dt} &= -\frac{R}{L} i_d, \\ \frac{di_q}{dt} &= -\frac{R}{L} i_q - \frac{K_m}{L} e_2 + v - \frac{K_m}{L}\omega_m \end{aligned} \tag{6}$$

with bounded perturbations T_L, ω_m and $\dot{\omega}_m$.

Denote by

$$x = \begin{pmatrix} e_1 \\ e_2 \\ i_d \\ i_q \end{pmatrix}, \quad f = \begin{pmatrix} \omega_m \\ \dot{\omega}_m \\ T_L \end{pmatrix},$$

$$\begin{aligned} A &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{F}{J} & 0 & 0 \\ & 0 & -\frac{R}{L} & 0 \\ 0 & -\frac{K_m}{L} & 0 & -\frac{R}{L} \end{pmatrix}, \\ B &= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \end{aligned} \tag{7}$$

$$D = \begin{pmatrix} 0 & 0 & 0 & \\ -\frac{F}{J} & -1 & -\frac{1}{J} & \\ 0 & 0 & 0 & 0 \\ -\frac{K_m}{L} & 0 & 0 & \end{pmatrix}.$$

Using notations (7), rewrite model (6) in the following form

$$\dot{x} = Ax + Bv + Df.$$

Theorem 1. *Consider the closed-loop system consisting of the step motor model* (1) *as well as the feedback linearization control law* (5) *and auxiliary control law*

$$v = Kx \tag{8}$$

with Park's transformation (2)*. Then there exists K such that the following optimization problems hold*

(i) $trace(P) \to \min$,
(ii) $\gamma \to \max$ *and* $P - \gamma I > 0$,
(iii) $trace(Q) + \gamma \to \max$ *and* $Q = P^{-1}$,
(iv) $\|Y\|_c \to \min$,

where

$$\begin{array}{l} A^{\mathrm{T}}P + PA^{\mathrm{T}} + \alpha P \\ +BY + Y^{\mathrm{T}}B^{\mathrm{T}} + \frac{1}{\alpha}DD^{\mathrm{T}} < 0, \\ K = YP^{-1}, \end{array} \quad (9)$$

$P = P^{\mathrm{T}} > 0$, $Y \in \mathbb{R}^{4\times 3}$, $\alpha > 0$ *is the rate of convergence of Lyapunov function* $V = x^{\mathrm{T}}Px$, $\|Y\|_c = \sum_{j=1}^{4} \max_{1\le i\le 4} |x_{ij}|$.

The proof of Theorem 1 is the same as the proofs of theorems in [Nazin et al., 2007; Furtat et al., 2020] by checking the condition $\dot{V} + \alpha V - \alpha f^{\mathrm{T}} f < 0$.

Remark 1. *Let restrictions on the auxiliary control signal* v *be given in the form*

$$|v| \le \mu,$$

where $\mu > 0$ *is a given restriction. Then, in order for the conditions of Theorem 1 hold, it is sufficient that the following additional linear matrix inequality (LMI) is satisfied*

$$\begin{bmatrix} P & Y^{\mathrm{T}} \\ Y & \mu^2 I \end{bmatrix} \ge 0.$$

Remark 2. *In Theorem 1 condition (i) allows one to find the smallest ellipsoid in terms of the sum of the squares of the semiaxes. Condition (ii) allows one to find the smallest value of the error in the steady state. Condition (iii) is a linear combination of conditions (i) and (ii). Condition (iv) makes it possible to design discharged controllers in the sense that if some elements in the matrix* K *are zero or sufficiently close to zero, then the corresponding measurement signals can not be used in the control scheme.*

Remark 3. *Since the pair* (A, B) *is controllable, therefore, there always exists the solution of (i)-(iv) in Theorem 1.*

Remark 4. *Theorem 1 is valid under known parameters of* (1) *and unknown signals* T_L, θ_m, *and* ω_m. *However, problems (i)-(iv) in Theorem 1 can be solved under unknown* R, F, J, *and* K_m *if the following intervals are known*

$$\begin{array}{l} \underline{a}_1 < \frac{F}{J} < \overline{a}_1, \quad \underline{a}_2 < \frac{R}{L} < \overline{a}_2, \\ \underline{a}_3 < \frac{K_m}{L} < \overline{a}_3, \quad \underline{a}_4 < \frac{F}{J} < \overline{a}_4, \\ \underline{a}_5 < \frac{1}{J} < \overline{a}_5. \end{array} \quad (10)$$

In this case LMI (9) *must be solved in the vertexes of polytop* (10).

4 Example

Consider step motor model (1) with known parameters

$$L = 3.9 \cdot 10^{-3} H, \quad n = 50, \quad (11)$$

unknown parameters

$$\begin{array}{l} K_m = 1.19\ N \cdot m \cdot A^{-1}, \ \ J = 10^{-4}\ kg \cdot m^2, \\ F = 0.05\ N \cdot m \cdot s/rad, \ \ R = 1.4\, \Omega, \end{array} \quad (12)$$

and zero initial conditions.

Assume that during operation the parameters can change in the following intervals with known bounds

$$\begin{array}{l} 0.9 < K_m < 1.3, \ \ 10^{-5} < J < 10^{-3}, \\ 0.01 < F < 0.1, \ \ 1 < R < 2, \ \ 0.5 < T_L < 2.5. \end{array} \quad (13)$$

Then, the vertexes of polytop (10) can be rewritten as

$$\begin{array}{l} 10 < \frac{F}{J} < 10^4, \quad 2.6 \cdot 10^2 < \frac{R}{L} < 5.1 \cdot 10^2, \\ 2.3 \cdot 10^2 < \frac{K_m}{L} < 3.3 \cdot 10^2, \quad 10 < \frac{F}{J} < 10^4, \\ 10^3 < \frac{1}{J} < 10^5. \end{array} \quad (14)$$

According to Theorem 1 and Remark 4 as well as intervals (14), the matrices K that ensure the problems (i)-(iv) are equal to

(i) $K = col\{-1.3 \cdot 10^6, -7.2 \cdot 10^3, -7.9, -5.1\}$ without restrictions on u and $K = col\{-0.81 \cdot 10^5, -1.1 \cdot 10^2, -5.4, -6.3\}$ with restriction $|u_a| \le 90$ and $|u_b| \le 90$;

(ii) $K = col\{-0.9 \cdot 10^6, -2.3 \cdot 10^3, 0, 0\}$ without restrictions on u and $K = col\{-8.9 \cdot 10^5, -3.7 \cdot 10^3, 0, 0\}$ with restriction $|u_a| \le 90$ and $|u_b| \le 90$.

Let us compare the proposed result with classical PID-controller and adaptive control law. According to [Bodson and Chiasson, 1989] the PID-controller is given by

$$\begin{array}{l} u_d = -nL\omega i_q - h_4(i_d - i_q) - h_5 \int_0^t [i_d(s) - i_{dm}(s)]ds, \\ u_q = K_m\omega - h_4(i_q - i_{qr}) - h_5 \int_0^t [i_q(s) - i_{qm}(s)]ds, \\ i_{d\tau} = 0, \\ i_{q\tau} = -\frac{J}{K_m}\Big(h_1(\theta - \theta_m) + h_2 \int_0^t [\theta(s) - \theta_m(s)]ds \\ +h_3(\omega - \omega_m)\Big), \end{array}$$

where $h_1 = 6 \cdot 10^5$, $h_2 = 2.7 \cdot 10^7$, $h_3 = 1.5 \cdot 10^3$, $h_4 = \frac{L}{T}$, and $h_5 = \frac{R}{T}$. The structure of adaptive controller [Marino et al., 1995] is complicated and takes up a lot of space, therefore, it is not shown here.

Figs. 1-5 show the transients of θ for the proposed algorithm and the control laws [Marino et al., 1995] for

$$T_L(t) = 1.5 + \sin(20t)\ \mathrm{N \cdot m}.$$

Since the transients for each K in the proposed algorithm are almost the same, we give the worst transient for K calculated from problem (i).

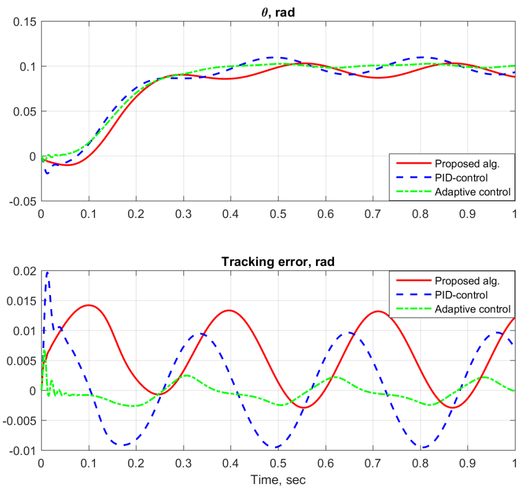

Figure 3. The transients in the proposed algorithm for $K = col\{-0.81 \cdot 10^5, -1.1 \cdot 10^2, -5.4, -6.3\}$ (red solid line), PID-controller (blue dashed line), and adaptive controller (green dot-dashed line) for the largest parameter values from (13).

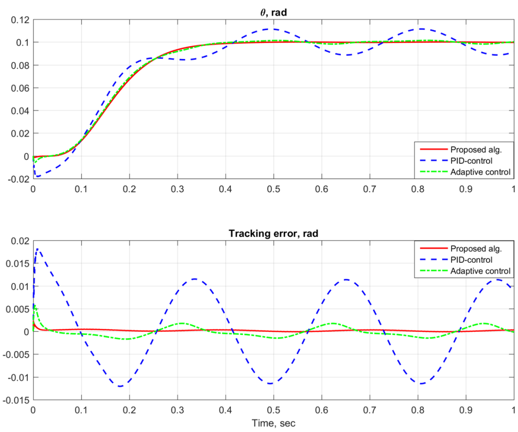

Figure 4. The transients in the proposed algorithm for $K = col\{-1.3 \cdot 10^6, -7.2 \cdot 10^3, -7.9, -5.1\}$ (red solid line), PID-controller (blue dashed line), and adaptive controller (green dot-dashed line) for nominal system parameters (11), (12).

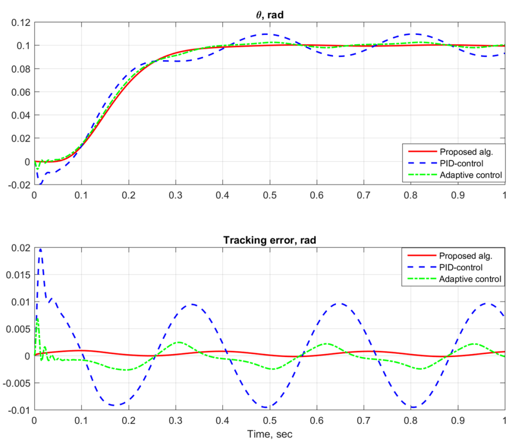

Figure 5. The transients in the proposed algorithm for $K = col\{-1.3 \cdot 10^6, -7.2 \cdot 10^3, -7.9, -5.1\}$ (red solid line), PID-controller (blue dashed line), and adaptive controller (green dot-dashed line) for the largest parameter values from (13).

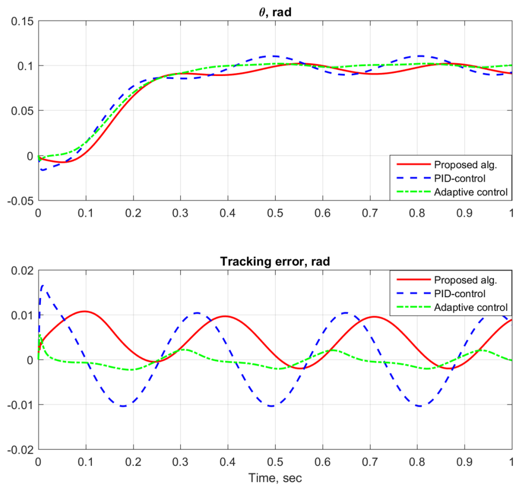

Figure 1. The transients in the proposed algorithm for $K = col\{-0.81 \cdot 10^5, -1.1 \cdot 10^2, -5.4, -6.3\}$ (red solid line), PID-controller (blue dashed line), and adaptive controller (green dot-dashed line) for nominal system parameters (11), (12).

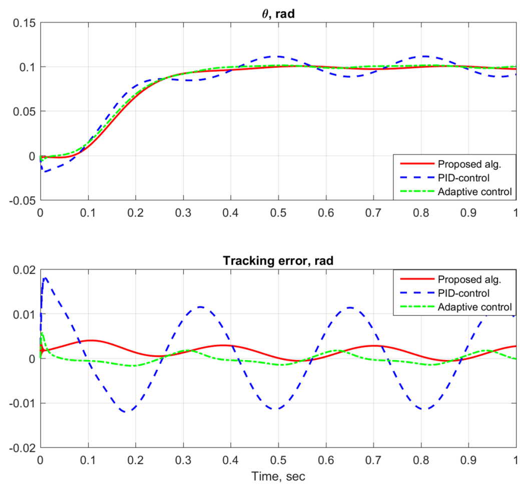

Figure 2. The transients in the proposed algorithm for $K = col\{-0.81 \cdot 10^5, -1.1 \cdot 10^2, -5.4, -6.3\}$ (red solid line), PID-controller (blue dashed line), and adaptive controller (green dot-dashed line) for the smallest parameter values from (13).

The advantages of the proposed algorithm are clearly seen. The proposed algorithm is easy to implement compared to the adaptive algorithm. It allows one to control the motor without measuring currents. The proposed algorithm showed comparable control results with the adaptive approach for $K = col\{-1.3 \cdot 10^6, -7.2 \cdot 10^3, -7.9, -5.1\}$.

However, it should be noted that the adaptive algorithm makes it possible to estimate the values of unknown parameters during operation, therefore, it may provide better results, especially in the steady state.

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5 Conclusions

In this paper, we propose a nonlinear control algorithm based on feedback linearization, linear control law and the technique of linear matrix inequalities. Optimization problems related to the search for parameters of control algorithm to ensure phase responses in the smallest ellipsoid and the smallest error in the steady state are solved. The simulation results illustrate the efficiency of the proposed method in comparison with the PID-controller and the adaptive control law using the backstepping method.

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