# OPTIMIZATION OF THE INITIAL CONDITIONS OF THE PLASMA DISCHARGE IN THE ITER TOKAMAK

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**Abstract**

Mathematical model and optimization of control at the initial stage of discharge in the ITER tokamak. Results and computer realization.

**Key words**

ITER, tokamak, plasma, modeling, optimization, control

## 1 Introduction

Modern approach to plasma modeling and control is based on modeling and control of the electro-magnetic fields governing plasma in the tokamak vessel. This approach seems very attractive, because in this case the whole case can be put into a form of a system of differential equations. Still how these equations should solved is another important question.

The electro-magnetic system of the tokamak has an extremely complicated structure, numerous plasma parameters are to be controlled in real-time on every stage of discharge so the mathematical modeling of this system demands a lot of effort.

In this paper the modeling and optimization of plasma on the initial stage of discharge will be considered.

Plasma discharge in tokamak is only possible under certain conditions. To deliver these conditions an effective automatic control system should be designed.

## 2 Mathematical model

The whole system of the tokamak to some extend can be represented by a set of conducting contours. At the fig.1 the sketch picture of the electro-magnetic system of the ITER tokamak is represented.

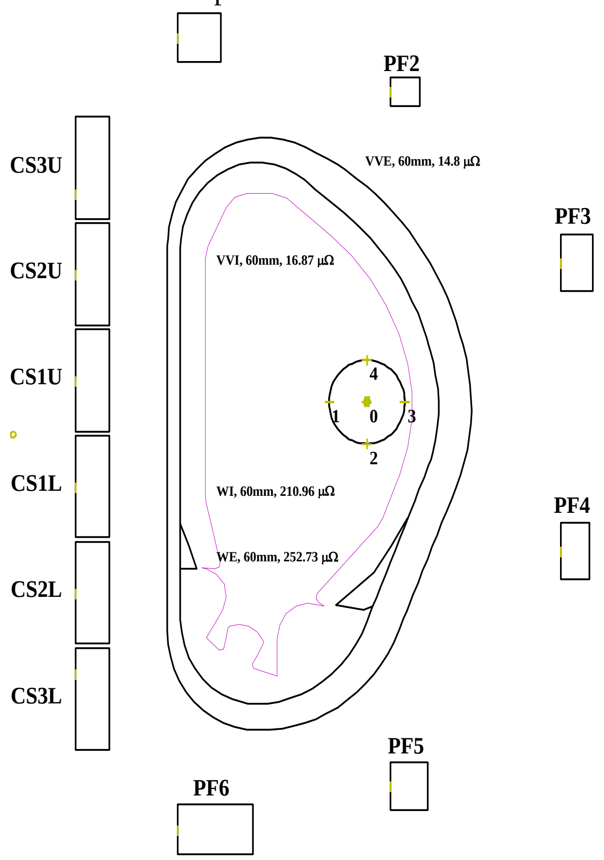


*Fig. 1*
*0-4 are model points – the supposed breakdown area, CS – central solenoid coils, PF – poloidal coils.*

To build an effective control system the conducting parts of the tokamak (i.e coils, plasma column) are considered as independent current contours for which a system of the Kirchghoff equations can be solved.

In this case there are 4 active poloidal coils pf2-pf5, 5 control points and upper and lower CS coils.

So we get a system of 11 differential equations:

$$\frac{d(LI)}{dt} + RI = U, \qquad (1)$$

Where L stands for the matrix of inductivities, R is a diagonal matrix of conductivities, I is a vector of currents, U is a vector of voltages which are none zero for coils PF2-PF5. In this case U is a vector of time-dependent parameters and represents itself programmed control.

The dimension of the system is a rational balance between the accuracy of the model and the volume of computations.

## 3 Limitations and conditions

To get the plasma discharge in the ITER tokamak started it is necessary to reach a certain state of the electro-magnetic field in the tokamak vessel – that puts respective terms and limitations on the major parameters. Some of conditions are due to the physics of plasma which determine the configuration of the electro-magnetic field under which avalanche ionization of plasma is possible. Some of the contingencies are determined by the parameters of the device and materials it is made of.

All of the conditions considered can be divided into two major groups: integral and terminal. The first group should be satisfied alongside the trajectories (i.e. solutions) of the system and the second group of conditions should be met at the very end of the beam of trajectories – which corresponds to the moment of the breakdown.

## 3.2 Terminal conditions

Firstly let us have a look at the so called terminal contingencies, which are conditions which should be delivered at the end point of the time-interval of modeling – that is the starting point of the discharge.

1) By the moment of the breakdown the loop voltage on the contour, which passes the ”0-point” should be equal to 14,1 V:

$$U_{loop}(T) = U_T = 14{,}1V$$

$$U_{loop}(t) = 2\pi R_0 E_0 = \sum_{k=1}^{K} \frac{dI_k L_k}{dt} = \frac{d\Psi(R_0, Z_0)}{dt},$$

where $R_0, Z_0$ – corresponds to the 0-point – center of the breakdown, $E_0$ – voltage of the electric field in the 0-point, $K$-number of conducting contours, $I_k$ – current in the contours, $L_k$ – it’s co-inductivity with the contour passing via the 0-point.

2) There are contingencies, which are determined by the physical processes during the discharge. In particular the magnitude of the magnetic induction in the breakdown area should be less than 2 mlT:

$$B(R, Z, T) \le 2mlT$$

3) There is a demand to maximize the magnetic flow reserve in the breakdown area by the time when the discharge starts with the given starting flow in the central solenoid of $\Psi(t_0) = 90\,W$ .

This condition is due to the fact that the plasma discharge goes while the flow of the CS is rising.

## 3.2 Intergal conditions

These are conditions that should be delivered from the start to the end of the modeling interval.

1) The following correlation should be maintained:

$$\frac{dU_{loop}(t)}{dt} > 0, t \in [t_0, T],$$

That condition demands the constant rise of the current which allows to avoid pre-breakdown.

2) The alternating current s in the active coils are limited by amplitude by some maximum value:

$$| I_k(t) | < I_k^{\max}, t \in [t_0, T],$$

Where $I_k(t)$ stands for the current in the coil and $I_k^{\max}$ is a maximum magnitude for each coil.

3) The magnitude of the magnetic fields in the coils limited from above – that is because under strong magnetic fields the super-conducting material of the coils loses its qualities. Magnetic field reaches its maximum on the edge on the conductor, so the following correlation should be maintained:

$$|\max_{m\in M}\sum_{k\in K} I_k(t)\sum_{l\in L(k)}\sqrt{\left(b_r\,(R_m,Z_m,R_{kl},Z_{kl})\right)^2+\left(b_z\,(R_m,Z_m,R_{kl},Z_{kl})\right)^2}\;| < B^{\max}(I)$$

4) The maximum voltage at the active coil is also limited due to the design of the device. So alongside the trajectories the voltage is limited from above:

$$|U_k(t)|\le U_k^{\max}, t\in[t_0,T], k\in K,$$

## 4 Functional

To build an adequate and effective control system all of these conditions should be considered. It can be done in a form of a functional – an integral characteristic of a dynamic system. The most general form of it can be written down as:

$$J(u)=\int_{t_0}^{\hat{L}}\int_{M_{t,u}}\varphi(t,x_t)dx_t dt+\int_{M_{T,u}} g(x_T)dx_T.$$

Usually condition in any system can be put into a quite simple arithmetical form. Using this functional approach an estimation of the parameters of the system can be made. And thus the task of the creating of the optimal control can be replaced by minimizing of the proper functional of the dynamic system.

Functional represents itself a function of many variables, which are control parameters. Its' minimum can be found In particular using the method of gradient descend. So the task of optimization consists of three steps: writing down a functional, finding components of its gradient by control parameters, descend to minimum.

1) Maximization of the poloidal magnetic flow in the 0-point by the moment of breakdown:

$$g_1(I(T))=\begin{cases}\dfrac{1}{\alpha_\psi^* I(T)}, \alpha_\psi^* I(T)\neq 0\\ 0, \alpha_\psi^* I(T)=0\end{cases},$$

Where T – stands for the moment of breakdown, $\alpha_\psi^* I(t)$ - instant poloidal magnetic flow in the 0-point.

2) Delivering the loop voltage U=14,1 V in the 0-point at the start of the discharge:

$$g_2(I(T),UK,R)=\left(\alpha_\psi^*\dot{I}(T)-14.1\right)^2= \\ =(\alpha_\psi^*(L^{-1}U(T)-L^{-1}RI(T))-14{,}1)^2.$$

3) Limitation of the magnitude of the poloidal field in the control points at the start of the discharge:

$$g_3(I(T))=\sum_{i=1}^{5} g_{3_i}(I(T)).$$

where

$$g_{3_i}(I(T))= \\ =\left\{\left(\alpha_{r_i}^* I(T)\right)^2+\left(\alpha_{z_i}^* I(T)\right)^2-4*10^{-3}, \right. \\ \sqrt{\left(\alpha_{r_i}^* I(T)\right)^2+\left(\alpha_{z_i}^* I(T)\right)^2}>2*10^{-3}.$$

Where $\alpha^*{}_{r_i} I(t)$ and $\alpha^*{}_{z_i} I(t)$ are respectively radial and vertical components of the poloidal magnetic field in the control points.

4) Limitatition of the maximum magnitude of the current in coils:

$$\varphi_1(t,I,Uk,tk,R)=\sum_{n=1}^{11}\left(\varphi_{1n}(t,I,Uk,tk,R)\right),$$

Where:

$$\varphi_{1n}=\begin{cases}(I_n(t)-I_{\max n})^2, I_n(t)>I_{\max n}\\ (I_n(t)+I_{\max n})^2, I_n(t)<-I_{\max n}\\ 0, -I_{\max n}\le I_n(t)\le I_{\max n}\end{cases}.$$

5) Limitation on the magnitude of the fields in the coils:

$$\varphi_2(t,I,Uk,tk,R)=\sum_{n=1}^{11}\left(\varphi_{2n}(t,I,Uk,tk,R)\right),$$

where

$$\varphi_{2n}=\left\{\left(\alpha_{r_n}^* I(t)\right)^2+\left(\alpha_{z_n}^* I(t)\right)^2-4*10^{-3},\right. \\ \sqrt{\left(\alpha_{r_n}^* I(T)\right)^2+\left(\alpha_{z_n}^* I(T)\right)^2}>B_{n_{\max}}, \\ =0,\quad \sqrt{\left(\alpha_{r_n}^* I(T)\right)^2+\left(\alpha_{z_n}^* I(T)\right)^2}\le B_{n_{\max}}$$

Here $B_{i_{\max}}$ - stands for the maximum magnetic field in each coil, $\alpha_{r_i}$ and $\alpha_{z_i}$ are coefficients corresponding to the radial and vertical component of the magnetic field.

The resulting functional can be obtained as a sum of the above.

## 5 Functional

Another way to describe a dynamical system is a variational approach. System (1) in variation can be written as follows:

$$\frac{d}{dt}\delta I = \frac{\partial f}{\partial I}\delta I + \Delta_{Uk} f + \Delta_{tk} f + \Delta_R f,$$

Where $f = f(t, x(t), u(t))$,

With the initial condition:

$$\delta I(0) = \Delta I_0 .$$

Using extra $\psi(t)$ vector-functions the variation of the functional will be:

$$\delta J = \delta J + \int_0^T \psi^*(t)(\frac{d}{dt}\delta I - \frac{\partial f(t,I(t),Uk,tk,R)}{\partial I})\delta I - \\ -\frac{\partial f(t,I,Uk,tk,R)}{\partial Uk}\delta Uk - \frac{\partial f(t,I,Uk,tk,R)}{\partial tk}\delta tk - \\ -\frac{\partial f(t,I,Uk,tk,R)}{\partial R}\delta R)dt =$$

$$= \left.\frac{\partial g(I(T),UK,R)}{\partial x}\right|_{t=T}\delta x(T) + \left.\frac{\partial g(I(T),UK,R)}{\partial R}\right|_{t=T}\delta R + \\ + \left.\frac{\partial g}{\partial Uk}\right|_{x=X_T}\delta Uk + \int_0^T \frac{\partial \varphi}{\partial I} dt\delta I + \\ + \int_0^T \frac{\partial \varphi}{\partial Uk} dt\delta Uk + \int_0^T \frac{\partial \varphi}{\partial tk} dt\delta tk + \int_0^T \frac{\partial \varphi}{\partial R} dt\delta R + \\ + \psi^*(T)\delta I(T) - \psi^*(0)\delta I(0) + \int_0^T \Big(-\dot{\psi}^*(t)\delta I(t) - \\ -\psi^*(t)\left(\frac{\partial f}{\partial I}\delta I + \frac{\partial f}{\partial Uk}\delta Uk + \frac{\partial f}{\partial tk}\delta tk + \frac{\partial f}{\partial R}\delta R\right)dt.$$

If

$$\dot{\psi}^*(t) = -\psi^*(t)\frac{\partial f(t,x(t),u(t))}{\partial I} + \frac{\partial \varphi(t,I(t),Uk,tk,R)}{\partial I}$$

$$\psi^*(T) = -\left.\frac{\partial g(I(t),UK,R)}{\partial I}\right|_{t=T}$$

Then the variation of the functional will be:

$$\delta J = \left.\frac{\partial g(I(t),Uk,R)}{\partial R}\right|_{t=T}\delta R + \left.\frac{\partial g(I(t),Uk,R)}{\partial Uk}\right|_{t=T}\delta UK + \\ + \int_{t_0}^T (-\psi^*(t)\frac{\partial f(t,I,Uk,tk,R)}{\partial Uk} + \frac{\partial \varphi(t,I(t),Uk,tk,R)}{\partial Uk})dt\delta Uk + \\ + \int_{t_0}^T (-\psi^*(t)\frac{\partial f(t,I,Uk,tk,R)}{\partial tk} + \frac{\partial \varphi(t,I(t),Uk,tk,R)}{\partial tk})dt\delta tk + \\ + \int_{t_0}^T (-\psi^*(t)\frac{\partial f(t,I,Uk,tk,R)}{\partial R} + \frac{\partial \varphi(t,I(t),Uk,tk,R)}{\partial R})dt\delta R - \\ -\psi^*(t_0)\delta I(t_0)$$

The expression above for the variation of the functional actually contains the components of the gradient. After that one of the methods of directional descend can be applied.

## 6 Computer realization

The results presented above were realized in a program Discharge Initial Stage developed at the Faculty of Applied Mathematcs and Control Processes. This program has the initial conditions (i.e. currents at t=0), resistances and voltages as model parameters. After those are entered the program executes modeling of the process. The output in this case are curves for such characteristics as loop voltage, magnetic flow etc. To execute the optimization scheme in addition optimization parameters should be given. Optimisation parameters represent themselves weight coefficient with which different parts of the functional are included into the resulting expression. Usually already after 5$^{th}$ iteration a noticeable decrease of the value of the functional is observed, after a hundred of iterations that value is almost zero. As result of optimization a set of values of control parameters is obtained, after what new “better” trajectories of the system are gained.

## 7 Conclusions

Thus approach and its computer realization, described above, are very useful though not too complicated model of the initial stage of the discharge in the ITER tokamak.

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