DYNAMICAL DAMPING OF PARAMETRIC OSCILLATIONS OF A FLEXIBLE ROD

Alexander Gouskov, Evgeniya Myalo, Gregory Panovko
Laboratory of Vibromechanics
Blagonravov Mechanical Engineering Research Institute of the RAS
Russia
gouskov_am@mail.ru, jenny99@inbox.ru, gpanovko@yandex.ru

Abstract
A flexible rod with constant cross section which buckles under gravity in static state is considered. Nonlinear lateral parametric oscillations of this rod when its base vibrates vertically and periodically are described. Possibility and conditions of stabilization of rod's axis rectilinear form are determined. Decay of lateral oscillations of the rod by freely sliding rigid body (disc) are investigated. It is shown that for certain parameters of the system under consideration the disc can have stable position on the rod that minimizes amplitudes of its lateral oscillations.

Keywords
Parametric oscillations, flexible rod, dynamical damping, stabilization.

1 Introduction

Dynamics of different mechanical structures (antennas, pylons, columns, blades, cable systems, hoses) is described as an elastic rod with the rectilinear axis mounted on a vibrating base.

Lateral parametric oscillations occur if the direction of the base vibration coincides with the vertical rectilinear axis of the rod.

The main subject of this paper represents some means of amplitude oscillation decay caused by additional sliding mass.

Equations of parametric oscillations of the system under consideration depend on applicable assumptions: type of possible nonlinearities, displacement values, flexural stiffness of the rod, types of friction and so on.

It is assumed, the rod has constant continuous circle cross section along length, the axis of the rod is inextensible but rectilinear and vertical in an unstrained state, material of the rod is linear elastic, and its internal friction is described by Voigt model. Whereas amplitudes of lateral oscillations of this rod can be extremely significant under real condition, we take into account finite rotations of the rod's axis. That results to geometrical nonlinearity.

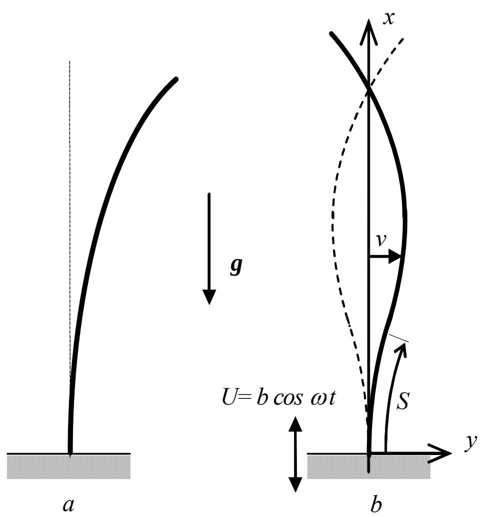

Fig. 1. Scheme of the flexible rod (*a*) and the second form of its vibration under parametric excitation (*b*).

2 Supercritical rod

First of all we consider the case of the flexible rod for which $mgL > 7.839\,EI/L^2$ (m - linear density, L - length of the rod, EI - bending stiffness of the cross section, g - acceleration of gravity); this rod we call "supercritical", otherwise - "subcritical". The supercritical rod buckles and its axis becomes bent in the absence of the base vibration, only under gravity (Fig. 1, *a*).

Natural oscillations of the rod distinguish themselves by large amplitudes, and they can be both near to the static state of the deflected axis and near to the vertical axis. If the rod base is subjected to the periodic vibration, whose direction coincides with unstrained axis of the rod, parametric oscillations occur. At that, the rod vibrates near vertical axis as shown at Fig. 1, *b* [Hamdan, Al-Qaisia and Al-Bedoor, 2001; Blekhman, 1994].

Differential equation, which describes oscillations of the flexible rod in a dimensionless form, is given by (with neglect of fourth infinitesimal order sums) [Gouskov and Panovko, 2006]:

$$\begin{aligned}&\ddot{\xi}+2\psi\dot{\xi}+\xi^{IV}+p_\Sigma(\tau)\left[(1-\zeta)\xi'\right]'=\\&=-2\psi_I\,\dot{\xi}^{IV}-2\psi_I\,\varepsilon\frac{\partial}{\partial\tau}\left[\xi'(\xi''\xi')'\right]'+\\&+\varepsilon\left\{\begin{matrix}\xi'\int\limits_\zeta^1 d\zeta_1\int\limits_0^{\zeta_1}\left(\dot{\xi}'^2+\xi'\ddot{\xi}'+2\psi\xi'\dot{\xi}'\right)d\zeta_2-\\ \xi'(\xi''\xi')'-p_\Sigma(\tau)(1-\zeta)\xi'^3/2\end{matrix}\right\}',\end{aligned}\qquad(1)$$

where $p_\Sigma(\tau)=\gamma-\beta\Omega^2\cos\Omega\tau-2\psi\beta\Omega\sin\Omega\tau$. Points denote dimensionless time differentiation τ, strokes – dimensionless natural coordinate differentiation ζ.

The next dimensionless parameters and complexes are used here:

$\tau=\frac{t}{L^2}\sqrt{\frac{EI}{m}}$ - dimensionless time;

$\zeta=\frac{S}{L}$ - dimensionless natural coordinate;

$\xi=\frac{\nu}{h}$ - dimensionless lateral displacement;

$$\Omega=\omega L^2\sqrt{\frac{m}{EI}},\quad \beta=\frac{b}{L},\quad \varepsilon=\left(\frac{h}{L}\right)^2,\quad \gamma=\frac{mgL^3}{EI},$$

$$\psi=dL^2\big/\left(2\sqrt{mEI}\right)<1,\quad \psi_I=\frac{d_I}{2L^2}\sqrt{\frac{EI}{m}},$$

where t - time, S - natural coordinate of rod section taken from base (Fig. 1, *b*), ω - circular frequency of excitation, b - amplitude of excitation , ν - lateral displacement of the rod's axis, h - diameter of the cross-section, d - coefficient of external linear friction, proportionate to absolute velocity, d_I - coefficient of internal friction.

Eq. (1) differs from one obtained in [Hamdan, Al-Qaisia and Al-Bedoor, 2001] that it has sum with damping coefficient ψ and sum $p_\Sigma(\tau)\left[(1-\zeta)\xi'^3/2\right]'$, which reflects the nonlinear parametric excitation. Taking the last item into account has influence on the behavior of the rod vibration. Moreover, difference in sums, which have effect of rod axial inertia on lateral oscillation, is presented in Eq. 1.

Solution of Eq. (1) was derived numerically on the base of Galerkin's technique in [Gouskov and Panovko, 2006].

According to the linear theory of parametric stabilization, the axis of the supercritical rod tends to take up the vertical position if excitation parameters satisfy to the next inequality (the critical value of excitation parameters follows from one-rate approximation when equality is complied) [Gouskov, Myalo, Panovko and Tretyakova, 2007]:

$$\beta\Omega>\sqrt{\frac{2}{|J_3|}(\gamma-\gamma_{Crit})}\qquad(2)$$

Modeling shows that stabilization does not occur, and rod vibrates close to the static deflected equilibrium position, if an excitation level smaller than critical value and in the presence of friction. If the excitation level is high enough (over critical), the rod becomes stabilize near to the vertical position.

Numeric calculation results of oscillation process of the supercritical rod ($\Omega=10;\ \varepsilon=0,02;\ \gamma=8,2>\gamma_{Crit}$) for different friction types are shown at Fig. 2. In the absence of friction (curve 1), lateral oscillations of the rod occur near to the vertical axis. Taking friction into account leads to the decay of the lateral oscillation amplitude almost to the null, and internal friction exerts greater influence on stabilization than external one (curves 2 and 4).

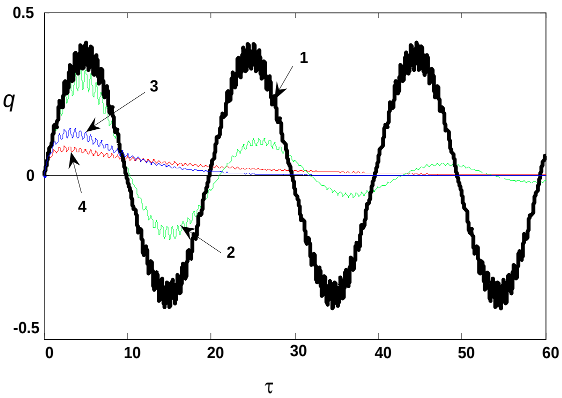

Fig. 2. Oscillation process of the supercritical rod for different types of friction:
1 - friction is absent;
2 - external friction $(\psi=0,05;\ \psi_I=0)$;
3 - external and internal friction $(\psi=0,025;\ \psi_I=0,025)$;
4 - internal friction $(\psi=0,\ \psi_I=0,05)$.

Thus, stabilization of the vertical position for the supercritical rod is reached due to vertical vibrations of the base, and the amplitude of the lateral oscillation can be decayed significantly by suitable selection of the excitation and damping parameters (for example, additional damping coating). Danger to get the main parametric resonance zone is remains for subcritical rod, and this oscillation can be stabilized by additional devices.

3 Subcritical rod

For the parametric oscillation decay the dynamical damper can be used. It represents a rigid body freely sliding with friction along rod.

The disc with hole stringed on the rod without tightness can perform function of this body (Fig. 3). Also, in case of hollow rod, it can be an entire disc or a ball put into the rod without tightness. It is known, that under certain conditions the disc moves up along the rod [Chelomei, 1983], and if the rod have flexural oscillations the disc tends to the antinode [Blekhman, 1994; Thomsen and Tcherniak, 2001]. Carried out experiments show, that mass of the disc and friction conditions influence on lateral oscillation amplitude.

Let's consider consistent equations of the rod oscillations and disc movement along the rod. Equation for the rod now have additional sum and it is given by:

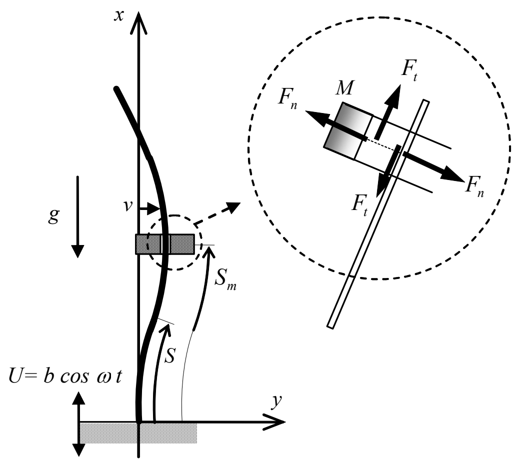

Fig. 3. Scheme of rod parametric oscillations with the disc

$$\ddot{\xi}+2\psi\dot{\xi}+2\psi_I\,\dot{\xi}^{IV}+\xi^{IV}+p_\Sigma(\tau)\left[(1-\zeta)\xi'\right]'=$$

$$=\varepsilon^2\left\{\begin{array}{l}\xi'\int\limits_{\zeta}^{1}d\zeta_1\int\limits_{0}^{\zeta_1}\left(\dot{\xi}'^2+\xi'\ddot{\xi}'+2\psi\xi'\dot{\xi}'\right)d\zeta_2-\\ -\xi'\left(\xi''\xi'\right)'-2\psi_I\left[\xi'\left(\xi''\xi'\right)'\right]^{\bullet}-\\ -p_\Sigma(\tau)(1-\zeta)\xi'^3/2\end{array}\right\}'+$$

$$+\varepsilon^2\bar{q}_n(\tau,\zeta)-\varepsilon^3\left\{\bar{q}_t(\tau,\zeta)\xi'+\left[\xi'\int\limits_{\zeta}^{1}\bar{q}_t(\tau,\zeta)\,d\tilde{\zeta}\right]'\right\}-\tag{3}$$

$$-\varepsilon^4\left\{\left[\xi'\int\limits_{\zeta}^{1}\bar{q}_n(\tau,\zeta)\xi'\,d\tilde{\zeta}\right]'+\bar{q}_n(\tau,\zeta)\xi'^2/2\right\}-$$

$$-\varepsilon^5\left\{\left[\xi'^3/2\int\limits_{\zeta}^{1}\bar{q}_t(\tau,\zeta)d\zeta\right]'-\left[\xi'\int\limits_{\zeta}^{1}\bar{q}_t(\tau,\zeta)\xi'^2/2\,d\zeta\right]'\right\},$$

where $\varepsilon=\dfrac{h}{L}$, dimensionless forces acting on the rod from the disc:

$$\bar{q}_n(\tau,\zeta)=\bar{F}_n(\tau,\zeta)\delta(\zeta-\zeta_m)=\frac{L^3}{EI}F_n(t,S)\delta(S-S_m),$$

$$\bar{q}_t(\tau,\zeta)=\bar{F}_t(\tau,\zeta)\delta(\zeta-\zeta_m)=\frac{L^3}{EI}F_t(t,S)\delta(S-S_m);$$

$$\bar{F}_t=-f\left|\bar{F}_n\right|\mathrm{sign}\left(\dot{\zeta}_m\right)-\psi_v\dot{\zeta}_m\left(1+c\,L^2\dot{\zeta}_m{}^2\right)$$

($\psi_v=d_i\dfrac{L^3}{EI\varepsilon^3}$ is nonlinear viscous friction acting on the disc).

Dimensionless equations, which describe disc behavior, are given by:

$$\left\{\begin{array}{l}\dot{\zeta}_m=V,\\ \dot{V}=\Omega^2\beta\cos\Omega\tau-\gamma-2\psi_e^{disk}V+\varepsilon^3\dfrac{1}{\mu}\bar{F}_t+\\ +\varepsilon^2\left[\begin{array}{l}2\psi_e^{disk}\Omega\beta\sin\Omega\tau+\int\limits_0^{\zeta_m}\left(\xi'\ddot{\xi}'+\dot{\xi}'^2\right)d\zeta_m-\ddot{\xi}\xi'+\\ +\dfrac{1}{2}\left(\gamma-\Omega^2\beta\cos\Omega\tau\right)\xi'^2+\\ +2\varepsilon\psi_e^{disk}\left(\dfrac{1}{2}\varepsilon\Omega\beta\sin\Omega\tau\xi'^2+\int\limits_0^{\zeta_m}\xi'\dot{\xi}'\,d\zeta_m-\dot{\xi}\xi'\right)\end{array}\right]\end{array}\right.\tag{4}$$

where

$$\bar{F}_t=-f\left|\bar{F}_n\right|\mathrm{sign}\left(\dot{\zeta}_m\right)-d_i\frac{L^3}{EI\varepsilon^3}\dot{\zeta}_m\left(1+c\,L^2\dot{\zeta}_m{}^2\right),$$

$$\bar{F}_n=-\mu\frac{1}{\varepsilon^2}\left[\begin{array}{l}\ddot{\xi}+2\dot{\xi}'V+\xi''V^2-\gamma\xi'+\\ +\xi'\,\Omega^2\beta\cos\Omega\tau+2\psi_e^{disk}\dot{\xi}\end{array}\right]+$$

$$+\frac{\mu}{\varepsilon}\left\{\begin{array}{l}2\psi_e^{disk}\xi'\,\Omega\beta\sin\Omega\tau-\xi'\int\limits_0^{\zeta_m}\left(\xi'\ddot{\xi}'+\dot{\xi}'^2\right)d\zeta_m+\\ +\dfrac{1}{2}\xi'^2\left(\ddot{\xi}-2\dot{\xi}'V-\xi''V^2\right)-\\ -2\psi_e^{disk}\left(\xi'\int\limits_0^{\zeta_m}\xi'\dot{\xi}'\,d\zeta_m-\dfrac{1}{2}\xi'^2\dot{\xi}\right)\end{array}\right\},$$

$\psi_e^{disc}=\dfrac{d\,L^2}{2\sqrt{mEI}}$ - dimensionless external friction acting on the disc with mass M, dimensionless parameter $\mu=\dfrac{mL}{M}$.

Thus, we have system of two nonlinear differential Eq. (3), (4) of third infinitesimal order with regards to lateral displacement of the rod with unknown parameters: $\xi(\tau,\zeta)$ - rod lateral displacement, $\zeta_m(\tau)$ - disc position on the rod. Forces of interaction $\bar{F}_n(\tau,\zeta_m)$ and $\bar{F}_t(\tau,\zeta_m)$ are connection for Eq. (3) and (4).

System integration is made numerically with use of Galerkin's technique. Solution for the rod is given by single mode approximation:

$$\xi(\tau,\zeta)\approx q_2(\tau)\,\varphi_2(\zeta),$$

where $q_2(\tau)$ - amplitude function, $\varphi_2(\zeta)$ - coordinate function corresponding to second form of the rod natural oscillations, normalized as

$$\int\limits_0^1\varphi_2{}^2(\zeta)\,d\zeta=1.$$

Integration of this system has certain difficult concerned with it is impossible to resolve system relative to higher derivatives and obtain it in Cauchy form, in particular due to taking dry friction forces into account. Therefore, the method of successive approximations is applicable here.

Graphs of the amplitude function q and disc position on the rod ζ_m in time are shown at the Fig. 5 for dimensionless parameters $\gamma = 0.12$, $\beta = 0.0022$, $\Omega = 43.5$, $\psi_I = 0.00084$, $\varepsilon = 1$, $\psi = 0$, $\psi_v = 0$, $c = 0$, $f = 0.1$, $\mu = 0.012$. Initial value of the amplitude function for the rod and its derivative correspond to onset of periodic solution with the period $4\pi/\Omega$ without disc (Fig. 4), initial disc position and velocity $\zeta_m(0) = 0.1$, $\dot{\zeta}_m(0) = 0$.

From the graph analysis it is follows that disc on the vibrating rod rise up with monotonically varying average velocity over a period of time and become stabilize on the rod close to antinode while rod oscillations are decay approximately in to times (Fig. 5). Disc oscillates near dynamical equilibrium position with the frequency of excitation, which two times more than parametric oscillations of the rod. This effect describes the function of the disc as a dynamical damper of the rod oscillations.

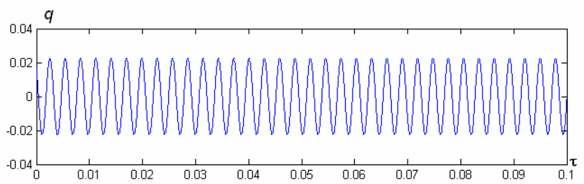

Fig.4. The amplitude function for the rod q without disc in time

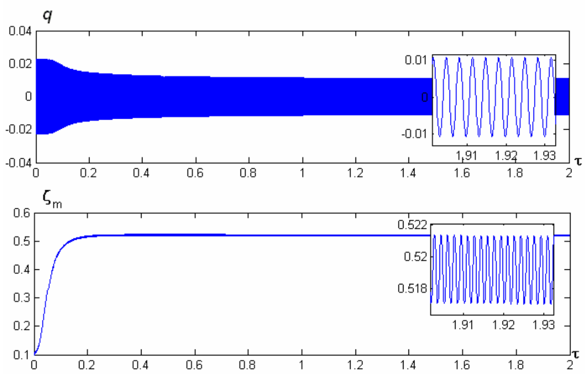

Fig.5. The amplitude function for the rod q and disc position ζ_m in time

Behavior of the disc on the rod depends on initial conditions, excitation and mass of the disc. It is possible that disc can fly out of the rod or decline.

8 Conclusion

Two possible variants of the dynamical damping of rod parametric oscillations were represented. System of equations, which describes interaction of the rod and the disc by means not only inertia forces but dry friction, and fact that disc can rise up, take stationary position on the rod and decay amplitude of rod oscillations were achieved.

Acknowledgements

The paper was supported by RFFI grants № 07-08-00253-a and № 07-08-00592-a and grant CRDF REC-018.

References

Blekhman I.I. *Vibrational mechanics*. M.: Fizmatlit, 1994. 394 p.

Chelomei V.N. Paradoxes in mechanics caused by vibrations// *DAN USSR*.-1983.-T.270, №1.

Gouskov A.M., Myalo E.V., Panovko G.Y., Tretyakova V.G. Dynamical stability of a flexible rod under parametric oscillations. *Journal of Vibroengineering*. V9, 2007, p. 16-20

Gouskov A.M., Panovko G.Y. Vibrational stabilization of a vertical axis of a flexible rod. *Problemy prochnosti i nadezhnosti mashin*. №3, 2006, p. 13-19

Hamdan M.N., Al-Qaisia A.A, Al-Bedoor B.O. Comparison of analytical techniques for nonlinear vibrations of a parametrically excited cantilever // *Int. J. of Mechanical Sciences*, V. 43 (2001), pp. 1521-1542.

Thomsen J.J., Tcherniak D.M. Chelomei's pendulum explained. *Proc. R. Soc. Lond. A* (2001) 457, 1889-1913.