CONTROL ON EPILEPTIFORM BEHAVIOR IN THE MESOSCOPIC-SCALE MODEL

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Abstract

We invent a continuous field model for epileptiform dynamics and focus on the spatial evolution of the hypersynchronized ictal phase in the form of a scalar field. We add a control term to its dynamical equation to study the possibility of suppressing the epileptiform regime at the mesoscopic scales. We reproduce the exact analytical solutions for the hypersynchronised phase in two forms: with the separation of variables and in the shape of traveling waves. Then we conclude our results and discuss the possible applications and further developments of our model.

Key words

Epileptic dynamics modeling, diffusion-type PDE, analytical solution, modified Bessel functions, open-loop control.

1 Introduction

Artificial neural networks (ANNs) are a powerful tool for modeling different physical and biological processes, including epilepsy in the human brain [Depannemaecker, Destexhe, et al., 2021]. This modeling covers different aspects of the seizure dynamics, such as the development of epilepsy by coupling of different subsets in ANNs [Naze, Bernard, et al., 2015], and efficient detection and suppression of the epileptiform regime [Stefan and Lopes da Silva, 2013; Song, Deng, et al, 2023], including quantum algorithms [Borisenok, 2022a].

The seizure dynamics are strongly related to the process of hypersynchronization in ANNs [Jiruska, de Curtis, et al., 2012], which can be caused by a variety of factors: inhibitory coupling [Andreev and Maksimenko, 2019], time delay in the system [Furtat and Orlov, 2020], and others.

Modeling of epilepsy with ANNs may cause high computational costs. For that reason, there are different ways to simplify the description of large-scale network structures. In some of them, the cluster evolution organizing the elementary units of the system ('agents') can be implemented [Proskurnikov and Granichin, 2018].

An alternative approach describes the neural population as a smooth neural mass [Deschle, Gossn, et al., 2020; Cooray, Rosch, et al., 2023]. Such models often represent the dynamics in the form of scalar fields [Peters and Stephenson, 2022], and they can be used for the efficient modeling of epilepsy [Hosseini, Yaghmaei, et al., 2025].

The choice of the appropriate mathematical model strongly depends on the scale of the seizure dynamics [Kuhlmann, Grayden, et al., 2015]. Continuous models are based on the phenomenology of the seizure generation in the epileptogenic zones [Saggio and Jirsa, 2023; Bougou, Vanhoyland, et al., 2025]. They are natural for the mesoscopic scales of the neural population (10^5-10^7 neurons), where the collective excitability can be naturally designed as a continuous field [Jedynak, Pons, et al., 2018].

Such a continuous field concept has been developed in [Borisenok, 2022b] for small-scale seizure dynamics, in the form of master equations for the regular, pre-ictal, and ictal phases. Then, in [Borisenok, 2024], it has been extended to the mesoscopic scales of neural populations, representing the progression of epilepsy using partial differential equations for scalar fields. In [Borisenok, 2025], we investigated the spatial spread of the pre-ictal and ictal phases as traveling waves.

The development of spatial longe-range noninvasive brain stimulations, including transcranial magnetic stimulation and transcranial direct current stimulation [Van-Haerents, Chang, et al., 2020;], enriched environment therapy, and olfactory therapy [Li, Chen, et al., 2023; Islam, Starnes, et al., 2025], and others, opens a door for a wide spectrum of control method applications to drive the epileptic dynamics at the mesoscopic level. Thus, the introduction of control algorithms into field models of the mesoscopic level of epilepsy dynamics is long overdue.

In this article, we focus on the spatial evolution of the hyper-synchronized ictal phase in the form of the scalar field S, adding the control term to its dynamical equation to study the possibility of suppressing the epileptiform regime at the mesoscopic scales. In Section 2, we formulate our modified control model, and subsequently derive the exact analytical solutions for the hypersynchronized phase using the separation of variables (Section 3) and in the form of traveling waves (Section 4). In Section 5, we conclude our results and discuss the potential applications and future developments of our model.

2 Model

The epileptiform scalar field for the mesoscopic scale is invented as a dimensionless continuous function $S(\mathbf{r}, t)$ of the 2D position vector $\mathbf{r}$ and the time moment t. The field S describes the rate of hypersynchronized neurons in the population [Borisenok, 2024], and can be evaluated locally with the master equations for the pre-ictal and ictal phases [Borisenok, 2022b].

2.1 Mesoscopic-scale model

Our mesoscopic-scale model [Borisenok, 2025] for the field S was based on the modified diffusion equation with the small time shift τ:

$$\frac{\partial S(\mathbf{r}, t-\tau)}{\partial t} = D\nabla^2 S(\mathbf{r}, t) \ , \tag{1}$$

which was first proposed in [Ahmed 2018; Ahmed 2020]. This time delay may be related to the procedure of the epilepsy phase detection, as well as to the inner dynamics of the seizures [Diamond, Diamond, et al., 2021]. D has the dimension m^2/s, and τ is a positive constant.

In [Ahmed 2018], this delay term in LHS(1) has been presented in the form:

$$S(\mathbf{r}, t-\tau) \simeq S(\mathbf{r}, t) - \tau\frac{\partial S(\mathbf{r}, t)}{\partial t} \ , \tag{2}$$

such that (1) became:

$$-\tau\frac{\partial^2 S(\mathbf{r}, t)}{\partial t^2} + \frac{\partial S(\mathbf{r}, t)}{\partial t} = D\nabla^2 S(\mathbf{r}, t) \ . \tag{3}$$

This model has been used in [Borisenok, 2025] to study the traveling wave solutions for the pre-ictal and ictal phases at the mesoscopic scales.

In this article, we modify our model (3), adding the control term to its RHS.

2.2 Modified control model

Here, we present the diffusion-type constant D explicitly as the combination of two characteristic dimensional parameters (spatial and temporal) in the model: $D = L^2/T$, with the dimensions of m for L and s for T.

We consider the radially symmetric case without the angle variable. Then the Laplace operator in the RHS(3) can be reduced to a one-dimensional radial variable r.

We add the term with the dimensionless control signal $u(t)$ driving the hypersynchonized part of the population S:

$$\begin{aligned} -\tau\frac{\partial^2 S(r,t)}{\partial t^2} + \frac{\partial S(r,t)}{\partial t} &= \\ = \frac{L^2}{T}\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right) S(r,t) &- \frac{1}{T}u(t)S(r,t) \ . \end{aligned} \tag{4}$$

The negative sign in front of the control term in RHS(4) emphasizes that the control target is the suppression of the temporal dynamics of epilepsy.

Eq.(4) can be solved analytically, in two different forms: with separation of variables and as a traveling wave.

3 Solution by Separation of Variables

We are interested in the asymptotic solutions for the large-range r, to investigate the spatial patterns of the ictal phase spread.

Because we study the ictal phase at the mesoscopic regime, we do not apply boundary conditions for the field S.

3.1 Separation of variables

Let's present our solution in the form:

$$S(r,t) = A(r)B(t) \ . \tag{5}$$

Then (4) becomes:

$$\begin{aligned} \frac{T}{B(t)}\left(-\tau\frac{d^2 B(t)}{dt^2} + \frac{dB(t)}{dt}\right) + u(t) &= \\ = \frac{L^2}{A(r)}\left(\frac{d^2 A(r)}{dr^2} + \frac{1}{r}\frac{dA(r)}{dr}\right) &= k^2 \ , \end{aligned} \tag{6}$$

with the dimensionless separation constant k^2. The second power of k emphasizes that we chose the separation constant to be positive to mimic the spatial spread of the epilepsy.

3.2 Spatial part of the solution $A(r)$

The spatial part of the solution satisfies the ODE:

$$\frac{d^2A(r)}{dr^2} + \frac{1}{r}\frac{dA(r)}{dr} - \frac{k^2}{L^2}A(r) = 0 \, , \tag{7}$$

with the solution:

$$A(r) = a_1 I_0\left(\frac{kr}{L}\right) + a_2 K_0\left(\frac{kr}{L}\right) \, , \tag{8}$$

where $a_{1,2}$ are constants, and I_0, K_0 are the modified Bessel functions of the 1st and 2nd types [Abramowitz and Stegun, 1972].

The zero-order modified Bessel functions have the large argument approximations for I_0 [Olivares, Martin, et al., 2018]:

$$I_0(x) \simeq \frac{e^x}{\sqrt{2\pi x}}\left(1 + \frac{1}{8x} + \frac{9}{128x^2} + ...\right) ; \tag{9}$$

and for K_0 [Martin and Maas, 2022; Palade and Pomârjanschi, 2023]:

$$K_0(x) \simeq \sqrt{\frac{\pi}{2x}}e^{-x}\left(1 - \frac{8}{x} + \frac{9}{128x^2} - ...\right) . \tag{10}$$

For the solution decaying in space, we can choose only the K_0 component, taking $a_1 = 0$.

3.3 Temporal part of the solution $B(t)$

The temporal part of the solution corresponds to the ODE:

$$\tau\frac{d^2B(t)}{dt^2} - \frac{dB(t)}{dt} + \left[k^2 - u(t)\right]\frac{B(t)}{T} = 0 \, . \tag{11}$$

Considering the time decay as:

$$B(t) = B_0 \exp\left(-\frac{t}{T}\right) \, , \tag{12}$$

with some initial B_0, and substituting (12) to (11), we get the control field:

$$u = 1 + k^2 + \frac{\tau}{T}. \tag{13}$$

Thus, the open-loop control signal can be chosen as a constant.

3.4 Final shape of the solution

Combining (10) and (12), we finally get for the long-range asymptotics of the field S under the control (13):

$$\begin{aligned} S(r,t) = \; & S_0 \exp\left(-\frac{t}{T}\right)\frac{\exp\left(-kr/L\right)}{\sqrt{kr/L}} \times \\ & \times \left(1 - \frac{8L}{kr} + \frac{9L^2}{128(kr)^2} - ...\right) , \end{aligned} \tag{14}$$

with a constant S_0 and positive k.

4 Traveling Wave Solution

Traveling wave solutions could also be observed in the model networks [Bukh, Nikishina, et al., 2024]. That's why it is especially interesting to investigate such a solution for the continuous model.

Let's define the field S in the form of a traveling wave:

$$\begin{aligned} & S(r,t) = S(\xi) \, ; \\ & \xi = x - vt \, , \end{aligned} \tag{15}$$

where v is the wave speed. Then the control signal u is also represented as $u(\xi)$. After substitution (15) to (4), one gets:

$$\begin{aligned} & \left(\frac{L^2}{T} + \tau v^2\right)\frac{d^2S(\xi)}{d\xi^2} + \\ & + \left(v + \frac{L^2}{T}\xi^{-1}\right)\frac{dS(\xi)}{d\xi} - \frac{1}{T}u(\xi)S(\xi) = 0 \, . \end{aligned} \tag{16}$$

Let's choose the control u in the form providing the spatial exponential decay:

$$S(\xi) = S_0 \exp\left(-\frac{\xi}{L}\right) \; ; \tag{17}$$

with a constant S_0. Then, substituting (17) to (16):

$$u(\xi) = 1 + \frac{vT}{L}\left(-1 + \frac{v\tau}{L}\right) - L\xi^{-1} \, . \tag{18}$$

Finally, substituting the velocity expressed in the typical scales (as it should be for the diffusion equation without the temporal delay τ):

$$v = \frac{L}{T} \, , \tag{19}$$

we get for (18):

$$u(\xi) = \frac{\tau}{T} - L\xi^{-1} \, . \tag{20}$$

In other words,

$$u(r,t) = \frac{\tau}{T} + \left(\frac{t}{T} - \frac{r}{R}\right)^{-1} \, . \tag{21}$$

The control signal (20)-(21) is a traveling wave itself, and it provides the solution for the ictal field S with the decay profile (17):

$$S(r,t) = S_0 \exp\left(\frac{t}{T} - \frac{r}{L}\right) . \tag{22}$$

We remind that we focus here on the large-range (not microscopic) asymptotics to represent the shapes of the control signals and the fields $S(r,t)$.

5 Conclusions and Discussion

The toy model proposed here, despite its simplification, has many advantages: it describes controlled suppression of the ictal phase at the mesoscopic level and allows for an explicit analytical solution.

The form of the open-loop control field is capable of providing the decay of the ictal phase at the mesoscopic scale of the epileptiform evolution. Moreover, the control signal profile itself is trivial (a constant or a power function), which makes it possible to easily convert the model into specific forms of non-invasive external fields that suppress epilepsy.

Further development of the model requires clarification of the typical spatial and temporal scales of L and T for different types of epilepsy, as well as the characteristic delay time τ.

It also seems fruitful to study the model on larger scales, where individual epileptiform zones are described by equations like (4), coupling each to the other to form larger clusters. The key question here will be how local suppression of the ictal phase in one zone will affect the overall dynamics of epilepsy in the entire cluster.

A feasible development of our model can cover the classification of spatio-temporal seizure propagation patterns to compare with experimental data [Vattikonda, Hashemi, et al., 2021; Gromov, Smirnov, et al., 2024].

Another possible extension of the model is embedding the mechanism of sequential restarts of the spike-wave discharges, similar to the one described in [Dolinina, Sysoeva, et al., 2022].

Finally, by combining local control equations for the pre-ictal and ictal phases at the microscopic level [Borisenok, 2022b] with the mesoscopic control model presented here, we can trace the dynamics of ictal phase formation at different levels and stages of its development.

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