CONTROL OF PATTERN FORMATION IN COMPLEX NETWORK BY MULTIPLEXING

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Abstract
In this work we study the influence of mupliplexing on synchronization between unstable chimera patterns in multiplex network of non-locally coupled Kuramoto-Sakaguchi (KS) oscillators. In the framework of current research we analyze the dynamics of the homogeneous network containing identical oscillators. To perform the analysis we have carried numerical simulation of the multiplex KS network and characterized its dynamics in terms of order parameter and averaged frequency difference between interacting layers. We have shown that increasing inter-layer coupling strength leads to stabilization of spatio-temporal chimera pattern.

Key words
Multistability, complex network, control of chaos, synchronization.

1 Introduction

Nowadays, the study coupled oscillators collective behavior excites a great interest from the viewpoint of chimera states analysis [Kuramoto and Battogtokh, 2002; Sethia, Sen, and Johnston, 2013; Omelchenko, et al., 2015]. Such effect occurs in the networks of identical oscillators. Chimera state is a special state of complex oscillator network which represents simultaneous existence of spatial regions of coherent and incoherent subgroups. It has been firstly discovered by Kuramoto and Battogtokh in their work [Kuramoto and Battogtokh, 2002]. This phenomena has also been found in a network of non-locally coupled nonlinear media described by the complex Ginzburg-Landau equation [Kuramoto and Nakao, 1996] and in a network of Kuaramoto-Sakaguchi (KS) phase model [Sakaguchi and Kuramoto, 1986]. Recent studies show that chimera-like states may arise in networks of oscillators with various coupling: global coupling [Schmidt and Krischer, 2015], nearest neighbour local coupling[Bera, Ghosh, and Lakshmanan, 2016] too. Along with theoretical study chimera-state has been observed in experiment on chemical [Tinsley, Nkmo, and Showalter, 2012], electronic [Larger, Penkovsky, and Maistrenko, 2013] and opto-electronic [Hagerstorm et al., 2012] systems.

Among many other effects associated with the emergence of chimera states, important and less studied topic is how different isolated networks individually representing different chimera states may be affected when they interact with each other? Obviously, such a situation can appear in real systems, particularly, relevant to many areas of science (e.g., neuroscience [Levy et al., 2000; Motter, 2010]) and technology. Its consideration along with a theoretical investigation demands due attention for prospective practical use [Panaggio and Abrams, 2015]. In recent paper [Maksimenko et al., 2016] problem of chimera state excitation in multilayer network was studied. Such multilayer network structure exists in real world and has been widely used currently both for the analysis of real data and explaining multilayer character of real-world networks [Kivela, Arenas, Barthelemy, Gleeson, Moreno, and Porter, 2014; Bocaletti et al., 2014]. From the dynamical systems' perspective, the multilayer formulation has been applied to networks whose layers coexist or alternate in time [Bocaletti et al., 2014; Makarov et al., 2016]. In both the cases, the multilayer formulation allowed synchronization regions that arise as a consequence of the interplay between the layers' topologies and their coupling [Sorrentino, 2012; Irving and Sorrentino, 2012; Bogojeska et al., 2013], and defined new type of synchronization based on the coordination between the layers [Gutierrez et al., 2012].

Besides, the other intriguing problem is related with methods for control of chimera patterns. Since chimera pattern is very sensitive to system parameters and initial conditions, slight mismatch of control parameters may destroy entire chimera pattern, devide it into multiple patterns or make it unstable in space (travelling chimera [Bera et al., 2016]). In recent works by Prof. E. Schöll et al. [Gjurchinovski et al., 2017; Zakharova et al., 2017] the time-delayed feedback was used to control complex behavior in FitzHugh-Nagumo and Stuart-Landau networks.

In this paper we address the issue of stabilization and control of chimera patterns by multiplexing. We analyze the conditions under which slightly mismatched chimera networks can be stabilized and synchronized being coupled by multiplex coupling.

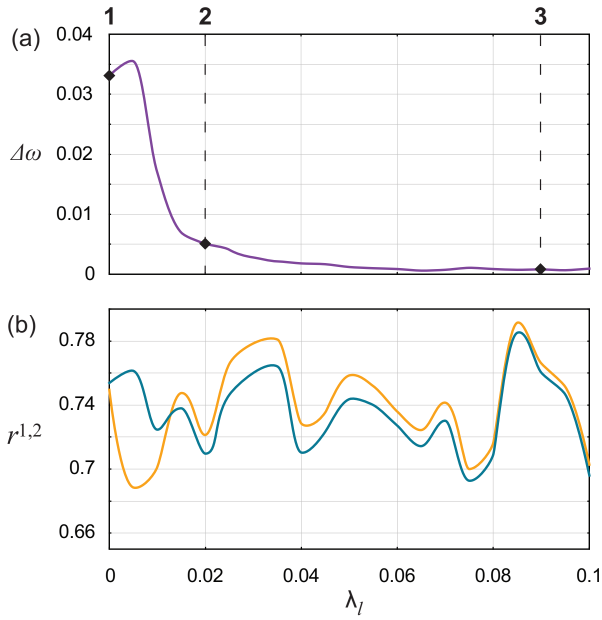

Figure 1. (a) Dependency of frequency difference $\Delta\omega$ on inter-layer coupling strength λ_l. Bold numbers at the top refer to Fig. 2. (b) Dependency of order parameter of layer 1 (blue curve) and layer 2 (yellow curve) on inter-layer coupling.

2 Numerical Model

Generally, the multilayer network model is characterized by nodes that have two types of links. One type establishes intra-coupling interaction between the nodes located in the same layer. The second type determines the inter-coupling of the dynamic elements between the layers. Depending on the specific objectives of the multilayer configuration, the inter-layer relation between the elements of a network may be quite different [Makarov et al., 2016].

Following the recent work [Maksimenko et al., 2016] we observe multilayer network which consists of $L = 2$ layers with $N = 100$ nods on each layer. As it was mentioned above KS phase oscillators play role of nonlinear oscillators located in the nods of the network. Our choice of a dynamical system is motivated by the fact that the nonlocal interaction in a network of KS phase oscillators is a paradigm of chimera states according to [Kuramoto and Battogtokh, 2002]. The state of each KS phase oscillator is denoted by ϕ_i^j, where subscript i corresponds to node index and supersctipt j corresponds to layer index. Evolution of KS phase oscillators coupled in multiplex network is defined by the following equation:

$$\frac{d\phi_i^j}{dt} = \omega - \frac{\lambda_n}{2R}\sum_{n=i-R}^{i+R} \sin\left(\phi_i^j - \phi_n^j + \alpha^j\right) + \frac{\lambda_l}{L}\sum_{l=1}^{L} \sin\left(\phi_i^j - \phi_n^l\right), \quad (1)$$

where ω is a natural frequency of KS phase oscillator, λ_n and λ_l are intra-layer and inter-layers coupling strength, R is non-local coupling radius and α^j is intra-coupling phase delay, which controls pattern formation on each layer of the network. Since we observe two-layer maultiplex homogenous network of KS oscillators with equal natural frequncies, we set $\omega = 0$ and rewrite eq.(1) in the following form:

$$\frac{d\phi_i^j}{dt} = -\frac{\lambda_n}{2R}\sum_{n=i-R}^{i+R} \sin\left(\phi_i^j - \phi_n^j + \alpha^j\right) + \lambda_l \sin\left(\phi_i^1 - \phi_n^2\right). \quad (2)$$

We integrated the eq.(2) numerically using Euler technique with time step $\Delta = 0.01$.

It is clear that in real networks it is very hard to keep control parameters constant in time. Thus, to provide more realistic simulations we introduced control parameter α^j in the following way:

$$\alpha^j = \alpha_0^j + \tilde{\alpha},$$

where α_0^j is a constant value of control parameter, which is set at the start of computation, and $\tilde{\alpha}$ is a noise term, which represents random real number in range [-0.005,0.005]. In addition to the fact that the model becomes more realistic, noise term makes network behavior more unstable in time (see Fig. 2). It helps to demonstrate the effect of control and stabilization of chimera pattern by multiplexing in more evident way.

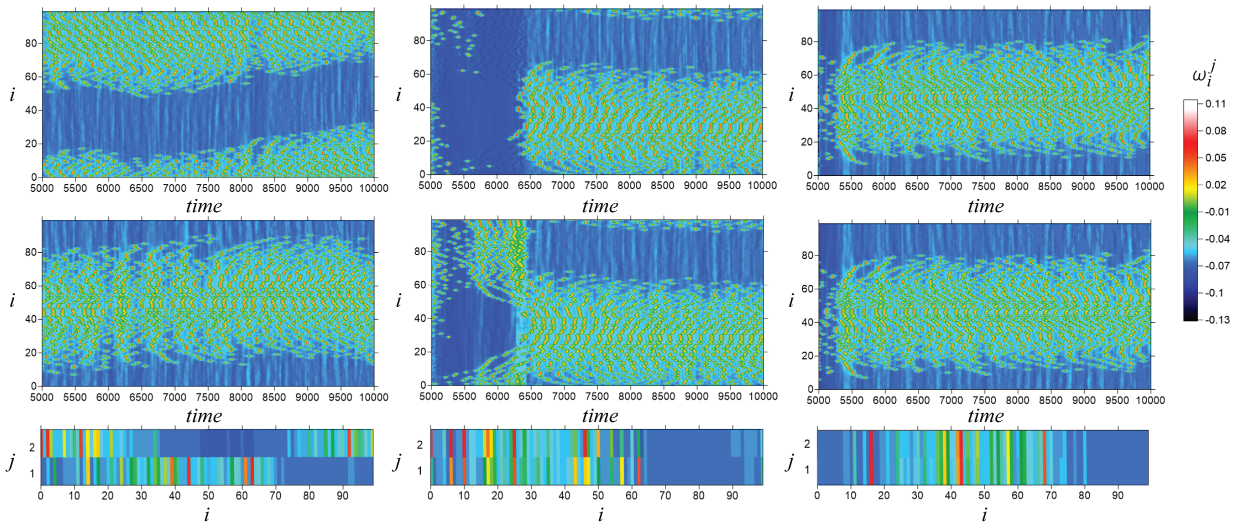

Figure 2. Spatio-temporal behavior of interacting layers of observed multiplex network in dependence of inter-layer coupling strength. Upper row corresponds to the 1st layer, middle row corresponds to the 2nd layer and the lower row corresponds to the final snapshots of both layers. Values of inter-layer coupling: $\lambda_l = 0.0$ (left column); $\lambda_l = 0.02$ (middle column); $\lambda_l = 0.09$ (right column).

Evolution of both leayers of the multiplex network starts with the following initial conditions:

$$\phi_i^{1,2}(0) = \begin{cases} \pi\left(\dfrac{4i}{N} - 1\right), i \in \left[0, \frac{N}{2}\right], \\ \pi\left(3 - \dfrac{4i}{N}\right), i \in \left[\frac{N}{2} + 1, N\right]. \end{cases} \quad (3)$$

Dynamics of each network node is described in terms of its effective frequency:

$$\omega_i^j = \frac{d\phi_i^j}{dt}.$$

To quantify the coherence level on each layer we use typical measure – order parameter:

$$r^j = \frac{1}{N}\sum_{i=1}^{N} \exp\left(\phi_i^j\right).$$

Order parameter r^j takes values in range [0,1], where $r^j = 0$ reflects incoherent dynamics, $r^j = 1$ corresponds to coherent behavior and $0 < r^j < 1$ defines partially coherent states including chimera states and formation of coherent clusters.

Synchronization between layers is defined in terms of frequency lock. Thus, to estimate synchronization we calculated averaged over ensemble difference of effective frequencies between nodes coupled with inter-layer coupling:

$$\Delta\omega = \frac{1}{N}\sum_{i=1}^{N} |\omega_i^1 - \omega_i^2|.$$

Here, $\Delta\omega$ tending to zero means phase synchronization between layers.

3 Results

Following the paper [Maksimenko et al., 2016] we chose the set of control parameters for the multiplex network under study, at which it was possible to study interaction between two spatio-temporal chimera patterns: $R = 35$, $\lambda_n = 0.085$, $\alpha_0^1 = 1.45$, $\alpha_0^2 = 1.50$.

Recall that the main purpose of this paper is to demonstrate the effect of multiplexing on synchronization between interaction layers and to show the possibility to control the unstable spatio-temporal pattern through inter-layer coupling. In Fig. 1(a) the dependency of averaged frequency difference $\Delta\omega$ between coupled layers of multiplex network on inter-coupling strength λ_l is shown. One can see, that increase of coupling strength between layers certainly leads to inter-layer synchronization which is expressed as negligibly small difference between layers behavior. At the same time, weak coupling ($\lambda_l \approx 0.005$) strengthens regime of spatio-temporal turbulence on both layers that leads to more unstable inter-layer behavior of the multiplex network. One can observe the same situation in Fig. 1(b), where the dynamics of layers is shown as a dependency of order paramters $r^{1,2}$ on inter-layer coupling strength λ_l. It is seen that the absence of coupling and the weak coupling are characterized by different dynamics of the layers that reflect different values of order parameter $r^1 \neq r^2$. Strong coupling ($\lambda_l > 0.06$) forces both layers to evolve in the same way, so that finally $r^1 \approx r^2$.

Let us analyze how synchronization infuences on stablization of spatio-temporal patterns in multiplex networks. In Fig. 2 the spatio-temporal behavior of both layers is presented along with visualization of final mutriplex network state in dependence of the coupling strength λ_l. It is seen that in the absennce of coupling individual dynamics of layers is totally different. Also, exccited chimera pattern is not localized and performs drift along the ensemble. Introduction of coupling makes chimera pattern more stable and lozalized in space, but the layers are not fully synchonized – regions of ihcoherent dynamics are characterized by slightly different effective frequencies, while coherent regions are synchronizaed. Finally, strong coupling leads to totally synchonous behavior of interacting layers and chimera pattern is perfectly localized in space.

4 Conclusion

We have demonstrated that effect of multiplexing allows to control and localize unstable pattern formation in the complex networks. It worth notation that weak multiplex coupling increases level of turbulence, but stronger coupling makes network behavior more regular and highly synchonized.

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