CONTROL OF PATTERN FORMATION IN BISTABLE NETWORKS

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Abstract
Control and design of stationary localized patterns is studied in large random bistable networks. The formation mechanism of these patterns is controlled by a negative feedback which depends on the total activation of the system. The active nodes in such a pattern form a subnetwork, whose size decreases as the feedback intensity is increased following a power law.

Key words
Pattern formation, feedback control, bistability, complex networks.

1 Introduction

Control and design of patterns is an essential investigation topic in complex systems [1; 2]. Global feedback control schemes serve as standard methods used for this purpose. Typically, global feedback requires a common control signal, generated by the entire system and applied back to all its elements. Various feedback schemes have been used in theoretical studies [3; 4] and in the experiments [5; 6; 7], either for stabilizing existing unstable patterns or for inducing new kinds of patterns that do not existing in the absence of feedback.

Self-organization phenomena, such as epidemic spreading [8], clustering [9] and synchronization [10] of oscillators, Turing patterns [11; 12] or traveling and pinned fronts [13; 14; 15], have been studied in reaction-diffusion systems organized in complex networks. Moreover, some effects of control by global feedbacks have been previously investigated for networks systems. It has been demonstrated, for example, that turbulence in oscillator networks can be suppressed [16] and hysteresis of Turing network patterns can be prevented [17] when such feedbacks are applied. Moreover the deliberate design of stationary localized patterns have been studied in bistable networks [18].

Here we show that feedback control may also suppress spreading of activation in networks of bistable elements, and leads to the formation of localized stationary patterns. We undertake a systematic numerical simulations for random Erdös-Rényi and scale-free networks and we present the statistical properties of the developed stationary patterns.

2 Complex bistable networks and negative feedback

A simple bistable system organized on a complex network, is given by the one-component equation

$$\dot{u}_i = f(u_i, h) + D \sum_{j=1}^{N} A_{ij} \left(u_j - u_i\right) , \qquad (1)$$

where u_i is the amount of activator in network node i and $f(u_i, h)$ describes the bistable dynamics of the activator in the nodes i ($i = 1, ..., N$). The summation term in Eq. (1) takes into account diffusive coupling between the nodes. Parameter D characterizes the rate of diffusive transport of the activator over the network links. The connectivity structure of the network can be described in terms of its adjacency matrix whose elements are $A_{ij} = 1$, if there is a link connecting the nodes i and j, and $A_{ij} = 0$ otherwise. Here we consider processes in bidirected networks, where the adjacency matrix is symmetric ($A_{ij} = A_{ji}$). The local bistable dynamics is described by the cubic polynomial

$$f(u, h) = u(h - u)(u - a) , \qquad (2)$$

where $f(u, h)$ has one unstable ($u_1^{(\text{unst})} = h$) and two stable ($u_0^{(\text{st})} = 0$ and $u_2^{(\text{st})} = a$) fixed points; $0 < h < a$. Depending on the particular context, the activator variable u may represent concentration of a chemical or biological species which amplifies (i.e auto-catalyzes) its own production.

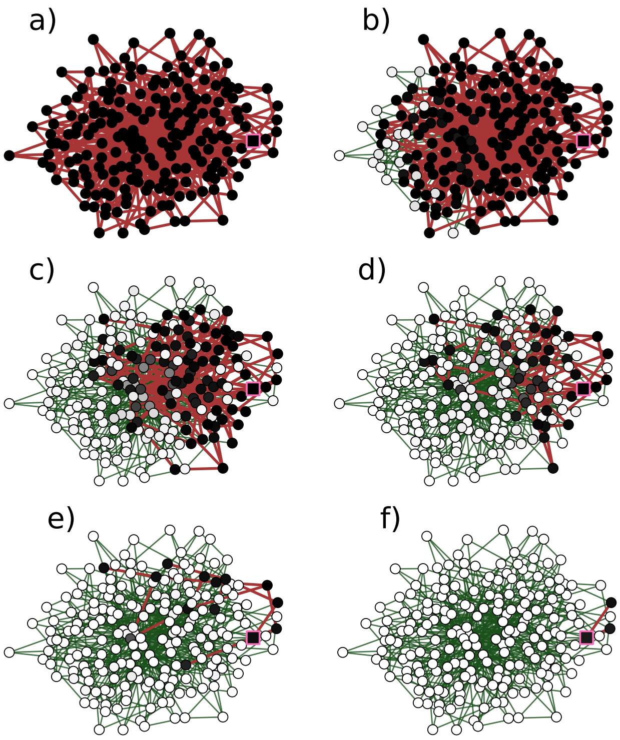

Figure 1. Stationary patterns in a scale-free network with $\langle k \rangle = 6$ and $N = 200$ for a) $\mu = 0.001$, b) $\mu = 0.002$, c) $\mu = 0.003$, d) $\mu = 0.004$, e) $\mu = 0.006$ and f) $\mu = 0.05$. The square indicates the node at which the activation was initially applied. White nodes correspond to inactive states whereas black nodes represent the active ones. The red links connect the active nodes. Other parameters are $h_0 = 0.1$, $a = 1$ and $D = 0.02$.

In continuous bistable media stationary localized patterns can be established if a global coupling is introduced, so that the parameter h depends on the total activation of the system. In the same fashion we can control the formation of stationary localized patterns in the networks by introducing a global negative feedback. We assuming that the parameter h depends on the total concentration of the activator u on the network according to the formula

$$h = h_0 + \mu(S - S_0)\,, \quad (3)$$

where h_0 is a positive constant, $\mu > 0$ is the intensity of the feedback, $S = \sum_{i=1}^{N} u_i$ is the total activation in the networks and S_0 is a parameter defining the size of localized patterns. In our simulations it was taken equal to the number of the nodes which were initially activated. Hence, the control parameter h depends now on the total activation. It increases when more nodes are activated, so that a negative feedback is realized.

Effects of global negative feedback have been numerically studied for large Erdös-Rényi and scale-free networks. Figure 1 presents an illustrative example of the deliberate design of stationary patterns when the system (1) is subjected to the control with the feedback signal described in Eq. (3). The activation that is applied initially to one node (marked by a square in Figure 1) starts to spread. This results in a growing subnetwork of activated nodes which is accombined with an increase of the negative feedback. Consequently, the control parameter h increases with the size of the subnetwork making the activation more difficult. As a result, the growth of the active subnetwork slows down and finally stops. Therefore, a stationary pattern, representing a small subnetwork of activated nodes embedded in the entire network (see Figure 1) becomes formed. By retaining only the nodes with high activation level $u > 0.8$, active subnetworks can be identified. A sequence of such subnetworks, obtained under an increase of the global feedback intensity, is shown in Figure 1 for a scale-free network.

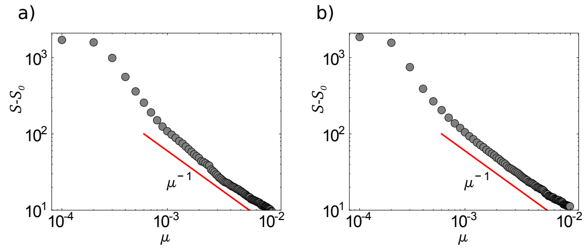

Figure 2. Dependence of the number of activated nodes on the feedback intensity μ for random a) Erdös-Rényi and b) scale-free networks with $\langle k \rangle = 6$ and $N = 2000$ nodes. Other parameters are $h_0 = 0.1$, $a = 1$ and $D = 0.02$.

Our statistical analysis has revealed that, in both Erdös-Rényi and scale-free networks networks, the average size of active subnetworks decreases as the feedback becomes stronger following, approximately, the power law $S \propto \mu^{-1}$ (see Figure2). This power-law breaks for sufficiently weak feedback, where the entire network becomes activated (cf. Figure1(a)). Interestingly, the numerically found power law has the exponent -1. The same exponent can be derived by means of some simple analytical calculations.

Let us assume that the size S of a stationary pattern grows like,

$$\dot{S} = c(h) - \frac{D}{S - S_0}\,, \quad (4)$$

where $c(h)$ is the propagation velocity of a bistable front without feedback control. When a stationary pattern is established, its size should be constant in time and thus $\dot{S} = 0$, namely

$$c(h) = \frac{D}{S - S_0} \Rightarrow$$

$$c\left[h_0 + \mu(S - S_0)\right] = \frac{D}{S - S_0} \quad (5)$$

Assuming a linear function for the $c(h)$ given by $c(h) = c_0 - kh$, where k denotes the degree of a node, we find that the intensity of the feedback can be given by

$$\mu = \frac{c_0 - kh_0}{k(S - S_0)} - \frac{D}{k(S - S_0)^2} \,. \tag{6}$$

When S is large, the later equation can take the form $\mu \propto 1/(S - S_0)$. Then by taking into account that $\langle u \rangle \propto (S - S_0)$ we conclude on the relation

$$\langle u \rangle \propto \mu^{-1} \,, \tag{7}$$

which was also found in the numerical simulations for both, the Erdös-Rényi and scale-free networks networks.

3 Conclusion

We have analyzed some effects of a global negative feedback control scheme on the pattern formation mechanisms of bistable networks. In large random networks feedback-induced stationary patterns are localized on subnetworks of the entire system. The structure and the size of such subnetworks can be controlled by tuning the feedback intensity. Their size decreases as the feedback becomes stronger and follows a power law distribution with the exponent -1. Simple analytical derivation have revealed the origins of this particular exponent.

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References

Schöll E. and Schuster H. G. (Editors) Handbook of Chaos Control 2nd Edition (Wiley-VCH, Winheim) 2007.

Mikhailov A. S. and Showalter K. Physics Reports425200679.

Krischer K. and Mikhailov A. S Physical Review Letters 7319943165.

Battogtokb D., Hildebrand M., Krischer K. and Mikhailov A. S. Physics Reports2881997435.

Kim M., Bertram M., Pollmann M., von Oertzen A., Mikhailov A. S., Rotermund H. H. and Ertl G. Science (New York, N.Y.)29220011357.

Wang W., Kiss I. and Hudson J. Physical Review Letters8620014954.

Sakurai T., Mihaliuk E., Chirila F. and Showalter K. Science (New York, N.Y.)29620022009.

Barrat A., Barthelemy M. and Vespignani A. Dynamical Processes on Complex Networks (Cambridge University Press) 2008.

Nakao H. and Mikhailov A. S. Physical Review E792009036214.

Arenas A., Díaz-Guilera A., Kurths J., Moreno Y. and Zhou C. Physics Reports469200893.

Nakao H. and Mikhailov A. S. Nature Physics62010544.

Wolfrum M. Physica D: Nonlinear Phenomena24120121351.

N. E. Kouvaris, H. Kori and A. S. Mikhailov, PLoS ONE **7**, e45029 (2012).

N. E. Kouvaris, M. Sebek, A. S. Mikhailov and I. Z. Kiss, Angew. Chem. Int. Ed.,**55**, 13267–13270 (2016); Angew. Chem.,**128**, 13461–13464 (2016).

N. E. Kouvaris, M. Sebek, A. Iribarne, A. Díaz-Guilera and I. Z. Kiss, Physical Review E **95** (4), 042203 (2017).

Gil S. and Mikhailov A. S. Physical Review E792009026219.

Hata S., Nakao H. and Mikhailov A. S. EPL98201264004.

N. E. Kouvaris and A. S. Mikhailov, Europhys. Lett. **102**, 16003 (2013).