ROBUST NONLINEAR OBSERVER DESIGN APPLIED TO A CLASS OF OSCILLATORY CONTINUOUS CHEMOSTAT

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Abstract
The aim of this work is to apply a new nonlinear observer in order to estimate the trajectories of an oscillatory continuous chemostat model. The local observability test and the observer convergence proof are presented as well as numerical simulations shows the observer performance.

Key words
Nonlinear observer, oscillatory behavior, biological system, convergence Lyapunov proof, chemostat, model uncertainties.

1 Introduction

The biological systems (commonly nonlinear) are well known for its complexity and variety, a large number of these systems are bounded in bio-reactors for biotechnology purposes such as the production of antibiotics, vaccines, food, contaminant removal, etc (Krylov & Zaikov, 2006; Liese, Seelbach, & Wandrey, 2006). By which the monitoring and process control is indispensable in order to achieve a high and safely production, even more when the system exhibits oscillatory dynamic, hysteresis or chaos (Ajbar, 2001; Angeli, Ferrell, & Sontag, 2004; D. Dochain, 2008).

One of the main tasks in bio-reactor control is the biomass monitoring, which is the biological catalyst of the system; this is due to the lack of reliable sensors for measurement the concentration of biomass (Bastin & Dochain, 1990; Denis Dochain, 2010). A solution to the previous problem is the estimation of the biomass concentration by state observers, the development of new observers has been the focus in many researches because the nonlinear estimation theory is still incomplete, so a variety of approaches and methods have been proposed to tackle the nonlinear observer design (Aguilar-Lopez, Mata-Machuca, & Martinez-Guerra, 2010; Ahrens & Khalil, 2009; Alvarez-Ramirez & Morales, 2000; Biagiola & Figueroa, 2004; Ciccarella, Dallamora, & Germani, 1993; Edwards, Spurgeon, Tan, & Patel, 2007; Gauthier, Hammouri, & Othman, 1992; Ibrir, 2006; MartinezGuerra & deLeonMorales, 1996; Tornambe, 1992; Weiwen & Zhiqiang, 2003; Zhu, 2012).

The simplest model of bio-reactor is the chemostat, is an important laboratory apparatus used to culture microorganisms(Fu & Ma, 2006). It is an open system where the substrate is input continuously for the biomass growth, the residual substrate and a fraction of the biomass generated are drawn off continuously, the inlet and outlet flow is the same. The chemostat model only is constituted for a general biomass and substrate balance (Dong & Ma, 2013). This model has been very helpful to explain systems stability, observability, control and bifurcation theory in bioprocesses (Ajbar & Alhumaizi, 2012).

For the above reasons, the chemostat model is employed in this work to prove the performance of a new nonlinear observer where the biomass concentration is difficult, slow or expensive to measure or at worst unmeasurable.

2 Biological model study case

The biological model taken as study case is the classical chemostat, and is as follows:

-Substrate mass balance

$$\frac{dS}{dt} = D(S_{in} - S) - \frac{\mu(S)X}{Yxs(S)} \tag{1}$$

-Biomass mass balance

$$\frac{dX}{dt} = -DX + \mu(S)X \tag{2}$$

Where: D is the dilution rate (h^{-1}); $Yxs(S)$ is the biomass yield ($Kg_{biomass}\ Kg_{CMC}^{-1}$); S_{in} is the feed concentration, in this case Carboxymethylcellulose

(CMC) (Kg m^{-3}), S is the substrate concentration in the reaction mixture (Kg m^{-3}); X is the biomass concentration (Kg m^{-3}); the kinetic reaction $\mu(S)$ was taken from a previous work developed by (Agarwal, Mahanty, & Dasu, 2009); this deals with the hydrolysis of CMC by *Cellulomonas cellulans* in a culture presenting substrate inhibition.

$$\mu(S) = \mu max \frac{S}{Ks+S} e^{-S/Ki} \quad (3)$$

$$Yxs(S) = \frac{1}{a+e^{-bS}} - c \quad (4)$$

Equation (3) presents the kinetic reaction (Aiba´s model), where μ_{max}, is a constant that means the maximal specific growth rate (h^{-1}); Ks describes the overall affinity of an organism for its growth limiting substrate (Kg m^{-3}) and Ki is the substrate concentration at which the microorganism growth is inhibited (Kg m^{-3}). Equation (4) shows the biomass-substrate yield as a nonlinear function of the CMC (logistic model). Table 1 shows the values of parameters used in the chemostat model (Agarwal, et al., 2009).

Table 1. Model parameters

Parameter	Value
μmax	0.383
Ks	3.69
Ki	6.569
a	0.965
b	2.83
c	0.514

3 Framework

As a theoretical framework, let us consider an affine nonlinear system configuration, given as follows:

$$\dot{x} = f(x) + g(x)u \quad (1)$$
$$y = h(x) \quad (2)$$

where $x \in \mathbb{R}^n$ denotes the state vector, taking values in X as a connected manifold of dimension n, $u \in \mathbb{R}^q$ denotes the vector of known external inputs, taking values in some open subset U, and $y \in \mathbb{R}^m$ denotes the vector of measured outputs taking values in some open subset Y. Function f will be assumed to be C^∞ of their arguments, and input functions $u(^o)$ to be locally and essentially bounded and measurable functions in a set U.

Let us consider the following specific nonlinear representation of the system (1-2) with linear measured output:

$$\dot{x} = Ax + \Psi(x) + (g_0 + \Delta g)u + j(x)d \quad (3)$$
$$y = h(x) = Cx \quad (4)$$

Where $x \in \mathbb{R}^n$ is the state vector, $\Psi(x) := \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear smooth vector function, $u \in \mathbb{R}^q; with\ q \le n$ is the control input, $j(x) := \mathbb{R}^n \to \mathbb{R}^m$; with $m \le n$, g_0 is the control input coefficient, Δg is the additive uncertainty or the control input and $d \in \mathbb{R}^l$; $l \le n$ is the input disturbance vector.

Now, considering that $\Psi(x)$, Δg and $j(x)d$ are unknown, the following change of variable is proposed:

$$\omega = \Psi(x) + \Delta g u + j(x)d \quad (5)$$

Therefore the following extended system is considered:

$$\dot{x} = Ax + g_0 u + \omega \quad (6)$$
$$\dot{\omega} = \wp(x, u, d) \quad (7)$$

Where $\wp := \mathbb{R}^{n+q+l} \to \mathbb{R}^n$ is an unknown vector field which is assumed that satisfies a Lipschitz condition, respect to the vector x, i. e.

$$\|\omega(x, u, d) - \hat{\omega}(\hat{x}, u, d)\| \le L\|x - \hat{x}\| \quad (8)$$

And considering that $|\wp| \le \Omega < \infty$

As above mentioned, for control purposes, it is needed an estimation of the uncertain term ω in order to made the considered control realizable, therefore the following uncertainty observer is proposed.

Proposition 1:

The following dynamical system is an asymptotic observer for the system (6-7)

$$\dot{\hat{x}} = A\hat{x} + g_0 u + \sum_{i=0}^{m} K_i (y - C\hat{x})^{2i-1} \int \sum_{i=0}^{m} K_i (y - C\hat{x})^{2i-1}\ dt \quad (9)$$

Considering the following assumptions:

ω is observable on $\mathbb{R}\langle y, u\rangle$

K_i is selected in accordance with the following Ricatti algebraic equation, which has a symmetric and positive definite solution P for some $\delta > 0$.

$$(L-K_1C)^T P + P(L-K_1C) + L^2 PPI + \delta I = 0 \quad (10)$$

K_i is selected such that $\lambda_{min}(PK_iC) \ge 0$

Where an alternative representation of the proposed observer is given by the following extended state system:

$$\dot{\hat{x}} = A\hat{x} + g_0 u + \hat{\omega} + \sum_{i=0}^{m} K_i (y - C\hat{x})^{2i-1} \quad (11)$$
$$\dot{\hat{\omega}} = \sum_{i=0}^{m} \bar{K}_i (y - C\hat{x})^{2i-1} \quad (12)$$

Defining the estimation error as:

$$\xi^T = (\xi_x, \xi_\omega) \quad (13)$$

Where:

$$\xi_x = x - \hat{x} \quad (14)$$
$$\xi_\omega = \omega - \hat{\omega} \quad (15)$$

Therefore the corresponding dynamic equation of the estimation error is:

$$\dot{\xi}_x = (A - K_1 C)\xi_x - \sum_{i=2}^{m} K_i (C\xi_x)^{2i-1} + \omega - \hat{\omega} \quad (16)$$
$$\dot{\xi}_\omega = \wp - K_1(y - C\hat{x}) - \sum_{i=2}^{m} K_i (y - C\hat{x})^{2i-1} \quad (17)$$

Sketch of proof of Proposition 1

Now, let us to consider the following Lyapunov function candidates:

$$V = V_1 + V_2 \quad (18)$$
$$V_1 = \xi_x^T P \xi_x \quad (19)$$
$$V_2 = \frac{1}{2}\xi_\omega^2 \quad (20)$$

Where $0 < P = P^T$

Now, $\dot{V}_1 = \dot{\xi}_x^T P \xi_x + \xi_x^T P \dot{\xi}_x$ (21)

$$\dot{V}_1 = \xi_x^T[(A - K_1 C)^T P + P(A - K_1 C)]\xi^T + 2\xi_x^T P[\omega - \hat{\omega}] - 2\sum_{i=2}^{m}(C\xi_x)^{2i-2} P K_i C \xi_x \quad (20)$$

Considering the Lipschitz condition:

$$2\xi_x^T P[\omega - \hat{\omega}] \le L^2 \xi_x^T P P \xi_x + \xi_x^T \xi_x \quad (21)$$

Applying the Rayleigh inequality and considering H_3

$$-\xi_x^T P K_i C \xi_x \le -\lambda_{min}(PK_iC)\|\xi_x\|^2 \quad (22)$$

Therefore:

$$\dot{V}_1 \le [(A - K_1C)^T P + P(A - K_1 C) + L^2 PP + I]\xi^T - 2\sum_{i=2}^{m}(C\xi_x)^{2i-2}\lambda_{min}(PK_iC)\|\xi_x\|^2 \quad (23)$$

$$\dot{V}_1 \le -\delta\|\xi_x\|^2 - 2\sum_{i=2}^{m}(C\xi_x)^{2i-2}\lambda_{min}(PK_iC)\|\xi_x\|^2 \le 0 \quad (24)$$

Taking into account that:

$$\sum_{i=2}^{m}(C\xi_x)^{2i-2}\lambda_{min}(PK_iC)\|\xi_x\|^2 > 0 \quad (25)$$

Furthermore, if the state estimation error remains small enough, then:

$$\lim_{\xi_x \to 0} inf\left(\sum_{i=2}^{m}(C\xi_x)^{2i-2}\lambda_{min}(PK_iC)\|\xi_x\|^2\right) = 0 \quad (26)$$

And:

$$\dot{V}_1 \le -\delta\|\xi_x\|^2 \le 0 \quad (27)$$

Now:

$$\dot{V}_2 = \xi_\omega \dot{\xi}_\omega = \xi_\omega\left(\wp - K_1(y - C\hat{x}) - \sum_{i=2}^{m} K_i(y - C\hat{x})^{2i-1}\right) \quad (28)$$
$$\dot{V}_2 = \xi_\omega \wp - \bar{K}_1 C \xi_\omega^2 - \sum_{i=2}^{m} \bar{K}_i (C\xi_\omega)^{2i} \quad (29)$$
$$\dot{V}_2 \le \Omega|\xi_\omega| - \bar{K}_1 C|\xi_\omega|^2 - \sum_{i=2}^{m} \bar{K}_i (C\xi_\omega)^{2i} \quad (30)$$

$$\dot{V}_2 \le -(\bar{K}_1 C|\xi_\omega| - \Omega)|\xi_\omega| - \sum_{i=2}^{m} \bar{K}_i (C\xi_\omega)^{2i} \le 0 \quad (31)$$

Considering that: $\sum_{i=2}^{m} \bar{K}_i (C\xi_\omega)^{2i} > 0$; and:

$$\lim_{\xi_{\omega \to 0}} inf\left(\sum_{i=2}^{m} \bar{K}_i (C\xi_\omega)^{2i}\right) = 0 \quad (32)$$

Therefore:

$$\dot{V}_2 = -(\bar{K}_1 C|\xi_\omega| - \Omega)|\xi_\omega| \le 0 \quad (33)$$

Such that $\dot{V}_2$ is negative on the set $\left\{|\xi_\omega| \le \lim_{t\to\infty} sup \frac{\Omega}{\bar{K}_1}\right\}$

From the above, can be concluded that:

$$\dot{V} \le 0 \quad (34)$$

4 Numerical Results

The numerical simulations were made with the Matlab® ode23s routine. A previously observability matrix rank test (see (Aguilar-Lopez, et al., 2010; Hespanha, 2009) was applied to the chemostat model, the results are showed below:

Firstly, a linearized version of the chemostat system (Equations (1-2)) is presented in Equation (35):

$$J = \begin{bmatrix} -D - \frac{x(Yxs(S)\mu(S)' - Yxs(S)'\mu(S))}{Yxs(S)^2}, & -\frac{\mu(s)}{Yxs(S)} \\ \mu(s)'x & \mu(s) - D \end{bmatrix} \quad (35)$$

Where:

$$\mu(S)' = \mu max \frac{\left(1 - \frac{S}{Ki} - \frac{S}{Ks+S}\right)e^{-S/Ki}}{Ks+S} \quad (36)$$

$$Yxs(S)' = \frac{b}{e^{-bS}\left(a + e^{-bS}\right)^2} \quad (37)$$

Equations (36) and (37) represent the first derivate respect to substrate for the kinetic growth model and the biomass-substrate yield model, respectively.

Considering the CMC as the measurable output (C),

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

The observability matrix (Obs) is presented in Equation (38)

$$Obs = \begin{bmatrix} 1, & 0 \\ -D - \frac{x(Y(S)\mu(S)' - Yxs(S)'\mu(S))}{Yxs(S)^2}, & -\frac{\mu(s)}{Yxs(S)} \end{bmatrix} \quad (38)$$

The Observability matrix rank is 2, hence the system is observable, addition to this, the determinant (Det) of Equation (38) was calculated in Equation (39):

$$Det\ (Ob) = -\frac{\mu(s)}{Y} \quad (39)$$

The $Det\,(Ob)$, reveals that the system is observable for any operation condition in which the growth rate is different to zero, it means whereas there are living cells.

The performance of proposed observer was compared with a Luenberger observer. It was considered the CMC concentration as the measurable output. The initial conditions were [S_{in}=8 kg m^{-3}, D=0.08 h^{-1}, X_i= 1.44 kg m^{-3}, S_i= 0.1 kg m^{-3}] and [S_{in}=8 kg m^{-3}, D=0.08 h^{-1}, X_i = 3 kg m^{-3}, S_i = 1 kg m^{-3}] for the plant (real dynamic) and both observers respectively. The gains employed were: $g1_{\text{Luenberger}}$=[0.1; -0.035]; $g1s_{\text{Proposed}}$=[0.1; -0.035]; $g2s_{\text{Proposed}}$=[0.001; -0.00035]; $g3s_{\text{Proposed}}$=[2; -0.7]. It was considered a modeling error of 20% in the kinetic constants of Aiba´s model.

Figure 1 shows the time series of CMC for the plant and both observers. The observers began at the same initial conditions that are different to that used in the nominal model. The estimated dynamic given by the proposed observer is synchronized rapidly with the real dynamic, in contrast to the Luenberger observer that is incapable of achieve the real dynamic.

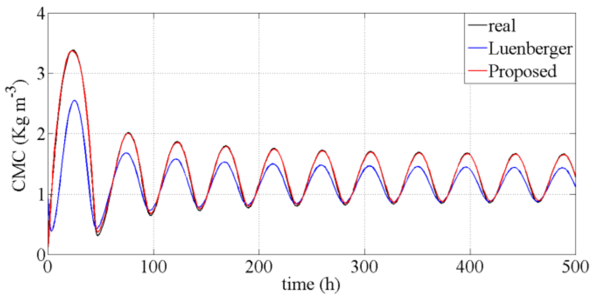

Figure 1. Real and estimated dynamics for CMC concentration.

Figure 2 shows the real and estimated CMC dynamics, it can be noted that the proposed observer presents an overshot that diminishes gradually and about the 50 h it overlapped with the real trajectory, in contrast, the Luenberger observer shows a slight overshot and in less time achieves the real trajectory (25 h approximately) but it is only temporally because near the 40 h this moves away from the real trajectory and never converge again.

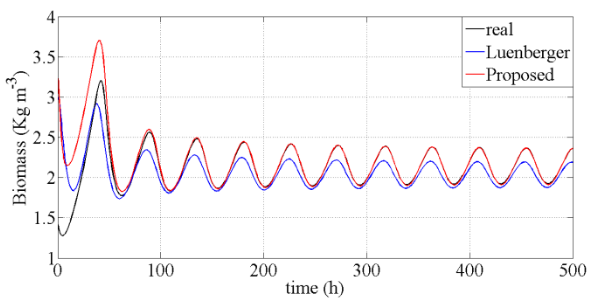

Figure 2. Real and estimated dynamics for biomass concentration.

Figure 3 shows a visualization of the real and estimated trajectories as a 3D phase portrait, where the differences between the proposed and the Luenberger observers are more evident, in particular it should be noted the fast and accurate convergence of the proposed observer.

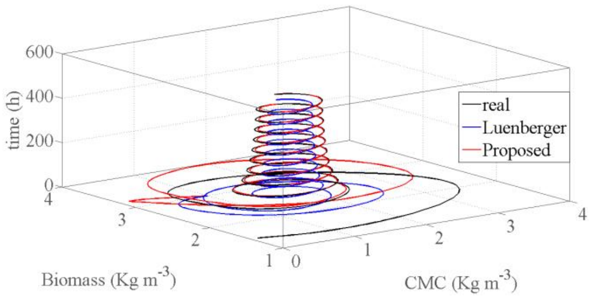

Figure 3. Phase portrait-time with the real an estimated trajectories of the chemostat model.

Finally, the Figure 4 shows the dynamic estimation error for biomass and CMC, as expected the error is closer to zero with the proposed observer compared to the Luenberger observer.

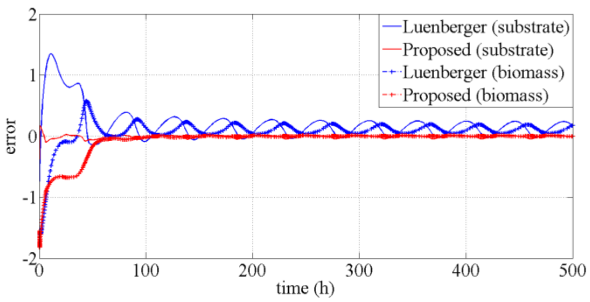

Figure 4. Biomass and substrate estimation errors for the proposed and Luenberger observers.

5 Conclusions

The proposed observer is successful in estimating the dynamics of the chemostat model in spite of being considered with modeling errors that commonly occurs in biological systems, as demonstrated by numerical simulation, furthermore, it was compared with a Luenberger observer in order to demonstrate that a better performance can be achieved.

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