RESONANT NEGATIVE FEEDBACK FOR CONTROLLING CHAOS IN THE TWO-WELL NONAUTONOMOUS OSCILLATOR

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Abstract

We consider second order linear resonant filter inserted in the negative feedback loop of the chaotic two-well oscillator for stabilizing unstable periodic orbit. Experiments have been performed with an electrical circuit, imitating dynamical behaviour of the two-well nonautonomous chaotic oscillator. Stabilization of periodic oscillations can be achieved with small control force. Mathematical model based on a two-well piecewise parabolic potential is discussed and numerical simulations are presented.

Key words
Dynamical chaos, chaos control, electronic circuits.

1 Introduction

One of the most successful chaos control techniques is the time-continuous delayed feedback control (DFC) method [Pyragas, 1992; Pyragas and Tamaševičius, 1993], also known as Pyragas' method. The DFC method and his extensions have been applied to many dynamical systems, including electrical circuits [Pyragas and Tamaševičius, 1993; Kittel *et al*. 1994; Gauthier *et al.*, 1994; Celka, 1994], laser [Bielawski, Derozier and Glorieux, 1994], mechanical [Hikihara and Kawagoshi, 1996], and chemical [Parmananda *et al.*, 1999] systems. The method has been introduced in the review papers [Namajūnas, Pyragas and Tamaševičius, 1996; Pyragas, 2006], journal theme issue [Lenci and Rega, 2006], and handbooks [Schuster, 1999; Schöll and Schuster, 2008]. Practical implementation of the DFC method requires a delay unit in the feedback loop. At high frequencies (gigahertz range) the delay unit is simply a short segment of either coaxial transmission cable or a micro-strip line. However at lower frequencies (megahertz and kilohertz ranges) the delay unit appears to be rather complicated and inconvenient device.

In this paper, we describe an extremely simple technique for controlling unstable periodic orbits embedded in chaotic systems. The basic idea behind the method is to use a resonant negative feedback (RNF) with the resonator tuned to the main frequency of the unstable orbit. As a result the negative feedback damps all the oscillations except the desired periodic orbit. We consider a two-well nonautonomous chaotic oscillator and employ a second order LC filter somewhat similarly to the recently described technique for synchronization of simple periodic oscillators [Pyragienė *et al.*, 2007].

2 Experimental setup

The idea of the technique is sketched in Fig. 1.

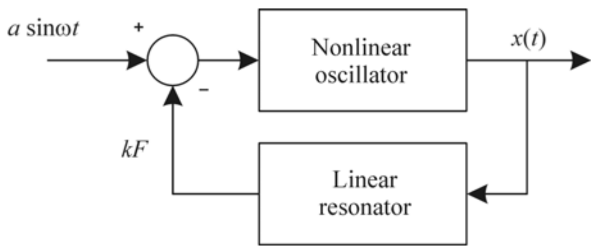

Figure 1. Block diagram of the control technique.

The full circuit diagram is shown in Fig. 2. The subcircuit, composed of the operational amplifier OA1, the R-L-C tank, the resistors R1-R3 and the diodes D1-D2, is a two-well nonautonomous chaotic oscillator, already employed to test the unstable version of the DFC method developed for stabilizing torsion-free periodic orbit [Tamaševičius *et al.*, 2007a]. Actually, it is a simplified version of the Young-Silva chaotic circuit [Lai *et al.*, 2005], described and characterized in more details elsewhere [Tamaševičiūtė *et al.*, 2008a].

The rest of the circuit in Fig. 2 is the controller. The OA2 based stage is a differentiator. Given the input of the differentiator is the voltage across the capacitor C, i.e. $V_{in} = V_C$,

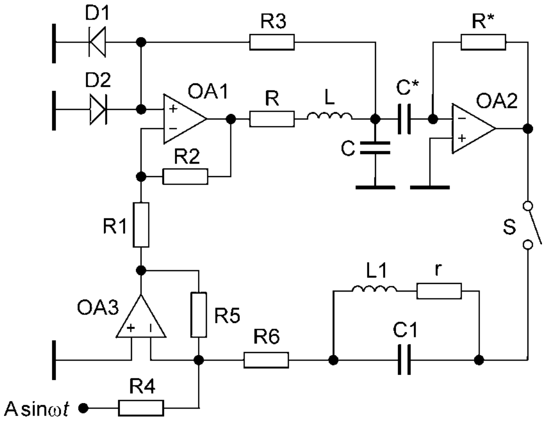

Figure 2. Circuit diagram of the two-well oscillator with the RNF controller in the feedback loop. $\omega = 2\pi f$. The S is an electronic switch.

the output of the differentiator is

$$V_0 = -R^* C^* \frac{dV_C}{dt} \tag{1}$$

On the other hand the current through capacitor C is just the current through inductor L:

$$C \frac{dV_C}{dt} = I_L \tag{2}$$

Combining (1) and (2) we obtain

$$V_0 = -\frac{R^* C^*}{\sqrt{LC}} \rho I_L , \tag{3}$$

where $\rho = \sqrt{L/C}$. It is convenient to set the values of R* and/or C* so, that $R^* C^* = \sqrt{LC}$.

The basic element of the controller is the linear resonator r-L1-C1. It is a notch filter with the frequency $f_1 = 1/2\pi\sqrt{L_1 C_1}$ (Fig. 3) tuned to the main harmonic of the unstable periodic orbit. We note, that the latter value is defined not by the fundamental frequency of the chaotic oscillator $f_0 = 1/2\pi\sqrt{LC}$, but by the external drive frequency $f = \omega/2\pi$. Finally, the OA3 stage is an adder.

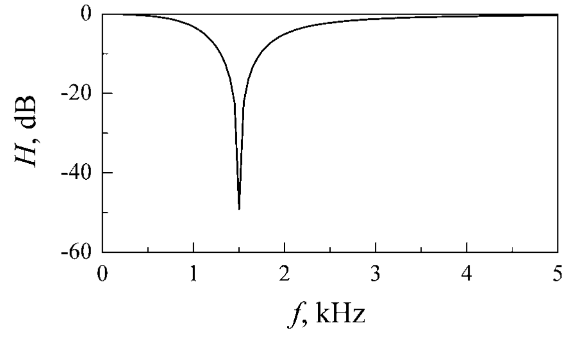

Figure 3. Transfer function H of the linear resonator. L_1 = 47 mH, C_1 = 240 nF, r = 1 Ω (f_1 = 1.5 kHz).

3 Experimental results

The following circuit values have been used in the experiment: L = 19 mH, C = 470 nF (f_0 = 1.7 kHz), R = 20 Ω, r = 1 Ω, $R_1 = R_2 = R_3$ = 10 kΩ, $R_4 = R_5$ = 1 kΩ, R_6 = 510 Ω. The below photos demonstrate the free running and the controlled oscillator.

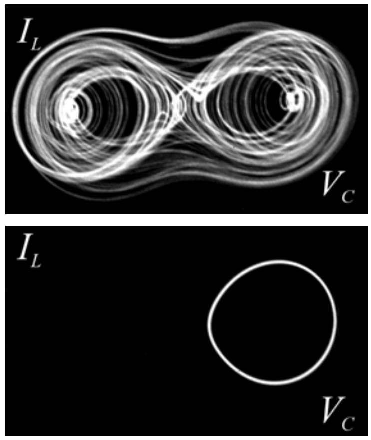

Figure 4. Phase portraits. (top) without control, (bottom) under control. A = 200 mV, $f_1 = f$ = 1.5 kHz.

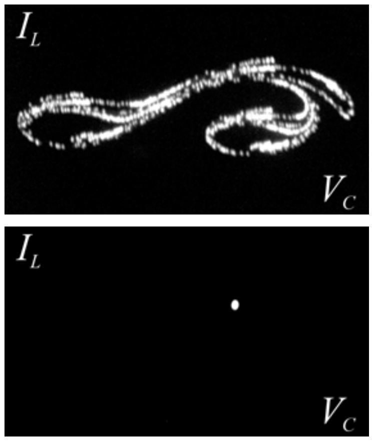

Figure 5. Stroboscopic maps (Poincaré sections). (top without control, (bottom) under control. Parameters are in Fig. 3.

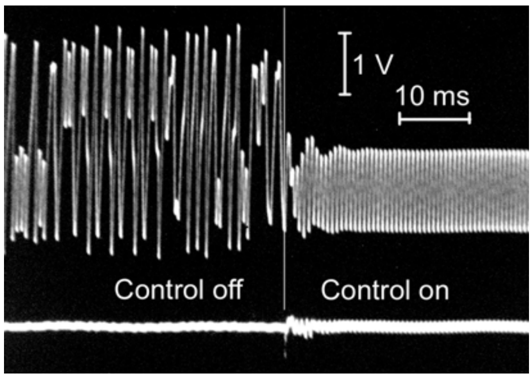

Figure 6. Experimental waveforms. Upper trace is the output signal $V_C(t)$ and lower trace is the control signal taken across resistor R6. Fine vertical line in the photo divides the regions, where control is off and control is on. Parameters are in Fig. 3.

The closed loop in Fig. 4(bottom) and the single dot in Fig. 5(bottom) do confirm that the RNF method successfully stabilizes period-1 orbit. A snapshot of the typical waveform including transient process is presented in Fig. 6.

4 Mathematical model

Using the Kirchhoff's laws the following equations are obtained for the nonautonomous two-well oscillator

$$C\frac{dV_C}{dt} = I_L, \quad L\frac{dI_L}{dt} = -RI_L - V_C + f(V_C) + \widetilde{E}, \tag{4}$$

where $\widetilde{E} = (R_2/R_1)(R_5/R_4)A\sin\omega t$. If $R_2 = R_1$, $R_5 = R_4$, then $\widetilde{E} = A\sin\omega t$. The term $f(V_C)$ can be approximated by a piecewise linear function

$$f(V_C) = \begin{cases} -k_0V^*, & V_C \le -V^* \\ k_0V_C, & -V^* < V_C < V^* \\ k_0V^*, & V_C \ge V^* \end{cases} \tag{5}$$

Here $k_0 = R_2/R_1+1$, and V^* is the forward voltage drop across the opened diodes (about 0.5 V for silicon diodes at 0.1 mA). It is convenient to set $R_2 = R_1$ in order to have $k_0 = 2$. Then merging in Eq. (4) the terms '$-V_C$' and $f(V_C)$ we obtain the full restoring nonlinear force:

$$N(V_C) = \begin{cases} -V_C - 2V^*, & V_C < -V^*, \\ V_C, & -V^* \le V_C \le V^*, \\ -V_C + 2V^*, & V_C > V^*. \end{cases} \tag{6}$$

The differentiator has been discussed in the previous section; a simple algebraic relation gives the output $V_0 = -\rho I_L$. The r-L1-C1 resonator is described by the following second order linear set:

$$C_1\frac{dV_{C1}}{dt} = I_{L1}, \quad L_1\frac{dI_{L1}}{dt} = -rI_{L1} - V_{C1}, \tag{7}$$

When the feedback loop is closed the system is given by

$$\begin{aligned} C\frac{dV_C}{dt} &= I_L, \\ L\frac{dI_L}{dt} &= -RI_L + N(V_C) + \widetilde{E} + \frac{R_5}{R_6}F, \\ C_1\frac{dV_{C1}}{dt} &= I_{L1} + \frac{F}{R_6}, \\ L_1\frac{dI_{L1}}{dt} &= -rI_{L1} - V_{C1}, \end{aligned} \tag{8}$$

where the feedback force $F = V_0 - V_{C1} = -\rho I_L - V_{C1}$. By introducing the following dimensionless variables and parameters

$$\begin{aligned} &x = \frac{V_C}{V^*},\ y = \frac{\rho I_L}{V^*},\ x_1 = \frac{V_{C1}}{V^*},\ y_1 = \frac{\rho_1 I_{L1}}{V^*}, \\ &t \to \frac{t}{\sqrt{LC}},\ \Omega = \omega\sqrt{LC},\ \Omega_1 = \sqrt{\frac{LC}{L_1C_1}}, \\ &a = \frac{A}{V^*},\ b = \frac{R}{\rho},\ b_1 = \frac{r}{\rho_1}, \\ &\rho = \sqrt{\frac{L}{C}},\ \rho_1 = \sqrt{\frac{L_1}{C_1}},\ k = \frac{R_5}{R_6},\ k_1 = \frac{\rho_1}{R_6} \end{aligned} \tag{9}$$

we come to the following compact set of equations, convenient for numerical integration:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -by + N(x) + a\sin\Omega t - k(y + x_1), \\ \dot{x}_1 &= \Omega_1[y_1 - k_1(y + x_1)], \\ \dot{y}_1 &= \Omega_1(-b_1y_1 - x_1). \end{aligned} \tag{10}$$

Here the coupling coefficients, k and k_1 may be different. The term $(y+x_1) \propto F$, the dimensionless nonlinear function $N(x)$ and the corresponding to $N(x) = -dW/dx$ potential $W(x)$ are presented by

$$N(x) = \begin{cases} -x - 2, & x < -1, \\ x, & -1 \le x \le 1, \\ -x + 2, & x > 1, \end{cases} \tag{11}$$

$$W(x) = \frac{1}{2}\begin{cases} (x+2)^2 - 2, & x < -1, \\ -x^2, & -1 \le x \le 1, \\ (x-2)^2 - 2, & x > 1. \end{cases} \tag{12}$$

It is a two-well potential with maximum $W_{max} = 0$ at $x = 0$ and two minima $W_{min} = -1$ at $x_0 = \pm 2$.

5 Numerical results

Numerical results obtained from Eq. (10) using the program *Mathematica* are shown in Figs. 7-9.

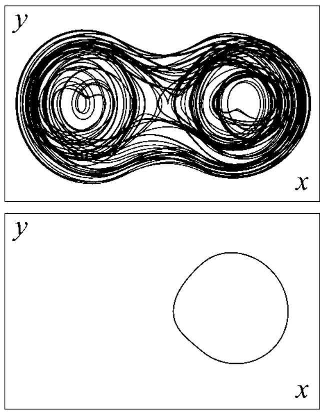

Figure 7. Phase portraits from Eq. (10). (top) without control, $k = k_1 = 0$, (bottom) under control, $k = 1$, $k_1 = 0.5$. Other parameter values: $a = 0.4$, $b = 0.1$, $b_1 = 0.01$, $\Omega = \Omega_1 = 0.9$.

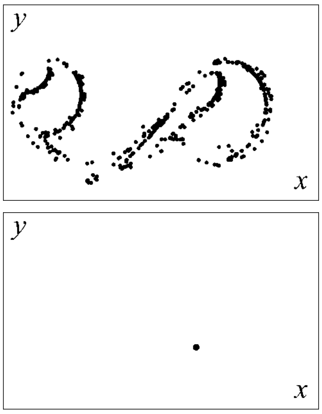

Figure 8. Poincaré sections from Eq. (10). (top) without control, (bottom) under control. Parameters are the same as in Fig. 7.

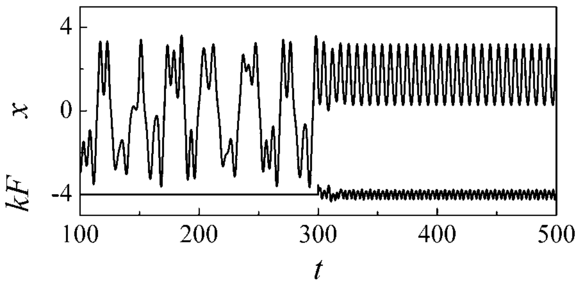

Figure 9. Waveforms from Eq. (10). Parameters are the same as in Fig. 7. Control is activated at the time moment $t = 300$. The lower trace for $kF = k(y + x_1)$ is intentionally shifted by value –4.

In addition, to characterize the performance of the RNF control method quantitatively the Lyapunov exponents have been calculated as a function of the control coefficient k from Eq. (10) and the leading one is presented in Fig. 10. While stabilization is observed at $k > 0.7$ the fastest control of period-1 orbit is achieved at $k \approx 1$.

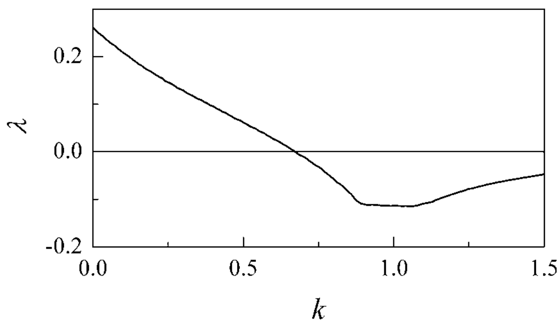

Figure 10. Leading Lyapunov exponent λ versus the control coefficient k from Eq. (10). $k_1 = 0.5k$. Other parameters are the same as in Fig. 7.

6 Concluding remarks

As mentioned in the Introduction the RNF method has been recently successfully applied to extend the region of synchronization of two simple periodic oscillators [Pyragienė *et al.*, 2007]. In the present paper, we have described a similar RNF analogue controller and have applied it to stabilize unstable periodic orbit in a chaotic electronic circuit. Since the controller includes the second order filter we can specify the technique as the RNF2 method. In contrast to the DFC method the residual control force kF in the RNF2 method does not vanish. However it appears to be rather small (about 10% compared to the main signal). In the RNF2 method only the first harmonic of the stabilized periodic orbit remains unchanged, but its higher harmonics may be slightly affected. Detailed numerical and experimental analysis (not discussed in the previous sections) shows that the main component of the control force is just the second harmonic. Therefore the RNF2 controller can be easily upgraded to RNF4 or RNF6 controller by inserting in the feedback loop an additional second order resonator(s) with the resonance frequency twice (and thrice) higher as of the main resonator [Tamaševičiūtė *et al.*, 2008b].

A related mathematical model, specifically the classical nonautonomous Duffing-Holmes equation with continuous nonlinearity

$$N_{DH}(x) = x - x^3$$

has been considered in [Tamaševičius *et al.*, 2007b]. The corresponding continuous potential is

$$W_{DH}(x) = -\frac{x^2}{2} + \frac{x^4}{4}.$$

The model in [Tamaševičius *et al.*, 2007b] exhibits similar results, however for the experimental system in Fig. 2 the piecewise linear and the piecewise parabolic functions given by (11) and (12) are more suitable.

The RNF2 technique is similar in a sense to the notch filter feedback (NFF) method [Ahlborn and Parlitz, 2006]. The notch filter described by Ahlborn and Parlitz is a Wien-bridge. The circuit is an active one and can be implemented by means of two RC chains and an amplifier. Moreover, hardware implementation of the NFF technique requires an additional inverter and an adder to construct the control signal as the difference of the input and the output signals of the Wien-bridge. Therefore, practical circuits appear rather complicated. In contrast, the RNF method uses simply a passive second order LC filter inserted in the negative feedback loop. In addition, the LC filter is easier to tune, than the Wien-bridge based controller.

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References

Ahlborn, A. and Parlitz, U. (2006). Chaos control using notch filter feedback. *Phys. Rev. Lett.*, **96**, pp. 034102-1-4.

Bielawski, S., Derozier, D. and Glorieux, P. (1994). Controlling unstable periodic orbits by a delayed continuous feedback. *Phys. Rev. E*, **49**, pp. R971-R974.

Celka, P. (1994). Experimental verification of Pyragas' chaos control method applied to Chua's circuit. *Int. J. Bifurc. Chaos*, **4**, pp. 1703-1706.

Gauthier, D. J., Sukow, D. W., Concannon, H. M. and Socolar, J. E. S. (1994). Stabilizing unstable periodic orbits in fast diode resonator using continuous time-delay autosynchronization. *Phys. Rev. E*, **50**, pp. 2343-2346.

Hikihara, T. and Kawagoshi, T. (1996). An experimental study on stabilization of unstable periodic motion in magnetoelastic chaos. *Phys. Lett. A*, **211**, pp. 29-36.

Kittel, A., Parisi, J., Pyragas, K. and Richter, R. (1994). Delayed feedback control of chaos in an electronic double-scroll oscillator. *Z. Naturforsch.*, **49A**, pp. 843-846.

Lai, Y.-Ch., Kandangath, A., Krishnamoorthy, S., Gaudet, J. A. and de Moura, A. P. S. (2005). Inducing chaos by resonant perturbations: theory and experiment. *Phys. Rev. Lett.*, **94**, pp. 214101-4.

Lenci, S. and Rega, G. (eds.). (2006). *Exploiting Chaotic Properties of Dynamical Systems for their Control: Suppressing, Enhancing, Using Chaos (theme issue)*, *Phil. Trans. R. Soc. A*, **364**(1846), pp. 2267-2563.

Namajūnas, A., Pyragas, K. and Tamaševičius, A. (1996). Continuous control of chaos. *Lithuanian J. Phys.*, **36**, pp. 346-350.

Parmananda, P., Madrigal, R., Rivera, M., Nyikos, L., Kiss, I. Z. and Gaspar, V. (1999). Stabilization of unstable steady states and periodic orbits in an electrochemical system using delayed-feedback control. *Phys. Rev. E*, **59**, pp. 5266-5271.

Pyragas, K. (1992). Continuous control of chaos by self-controlling feedback. *Phys. Lett. A*, **170**, pp. 421-427.

Pyragas, K. and Tamaševičius, A. (1993). Experimental control of chaos by delayed self-controlling feedback. *Phys. Lett. A*, **180**, pp. 99-102.

Pyragas, K. (2006). Delayed feedback control of chaos. *Phil. Trans. R. Soc. A*, **364**(1846), pp. 2309-2334.

Pyragienė, T., Tamaševičius, A., Mykolaitis, G. and Pyragas, K. (2007). Non-invasive control of synchronisation region of a forced self-oscillator via a second order filter. *Phys. Lett. A*, **361**, pp. 323-331.

Schöll, E. and Schuster, H. G. (eds.). (2008). *Handbook of chaos control*, 2nd edition. Wiley-VCH. Weinheim.

Schuster, H. G. (ed.). (1999). *Handbook of Chaos Control*. Wiley-VCH. Weinheim.

Tamaševičius, A., Mykolaitis, G., Pyragas, V. and Pyragas, K. (2007a). Delayed feedback control of periodic orbits without torsion in nonautonomous chaotic systems: theory and experiment. *Phys. Rev. E*, **76**, pp.026203-1-6.

Tamaševičius, A., Tamaševičiūtė, E., Mykolaitis, G. and Bumelienė, S. (2007b). Stabilization of unstable periodic orbit in chaotic Duffing-Holmes oscillator by second order resonant negative feedback. *Lithuanian J. Phys.*, **47**, pp. 235-239.

Tamaševičiūtė, E., Tamaševičius, A., Mykolaitis, G., Bumelienė, S. and Lindberg, E. (2008a). Analogue electrical circuit for simulation of the Duffing-Holmes equation. *Nonlinear Analysis: Modelling and Control*, **13**(2), pp.241-252.

Tamaševičiūtė, E., Tamaševičius, A., Mykolaitis, G., Pyragienė, T. and Bumelienė, S. (2008b). Extended version of the resonant negative feedback chaos control method. In *Proc. 2nd Int. Conf. on Nonlinear Science and Complexity,* Porto, Portugal, July 28-31, in press.