AUTO-TUNING METHOD OF EXPANDED PID CONTROL FOR MIMO SYSTEMS

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Abstract: This paper is concerned with auto-tuning of the expanded PID control for MIMO linear system (r-input, m-output, nth-order). The purpose is to give an adaptive law of time-varying PID parameter matrices so as to asymptotically stabilize the closed-loop system using expanded PID control for the regulation problem. The proposed method is derived from satisfying the Lyapunov's stability theorem. For its execution it is necessary for a certain hypothetical system to satisfy almost strictly positive real (ASPR) property. So we also propose the method letting its hypothetical system be ASPR. In our method, since it is not necessary for the controlled MIMO system to be stable and/or minimum phase, it is useful for large class systems. The effectiveness of the proposed method is confirmed with unstable MIMO system by a numerical simulations.

Keywords: expanded PID control, auto-tuning, MIMO linear system, almost strictly positive real (ASPR)

1. INTRODUCTION

PID control is usually well known as a classical dynamic control for SISO system. So there exist lots of researches on tuning methods of PID control for SISO system. But it is often difficult to apply them to MIMO system. Although there are several researches of PID control for MIMO system, they are restricted to stable and/or minimum phase system in many cases. Thus there is enough room to study in case of general MIMO system with non minimum phase and/or unstable.

As the tuning method of PID control for MIMO system, there are several researches (C. Lin and Lee, 2004; Shimizu and Tamura, 2005; Tamura and Shimizu, 2006; T. Yamamoto and Kitamori, 1992). In (C. Lin and Lee, 2004), PID parameter matrices are determined by solving LMI after formulating PID control as a static output feedback of the extended system. Also as a method based on approach from static output feedback, the eigenvalue assignment method by PID control is proposed in (Tamura and Shimizu, 2006). (Shimizu and Tamura, 2005) proposed the expanded PID control and the adjustment method applying the high gain output feedback. (T. Yamamoto and Kitamori, 1992) proposed the adaptive tuning method of two-degree-of-freedom PID control by using the model matching method.

In this paper, we propose the new auto-tuning method of the expanded PID control(Shimizu and Tamura, 2005) for regulator problem of the general MIMO system. Proposed auto-tuning method is derived from satisfying the Lyapunov's stability theorem. For its execution it is necessary for a certain hypothetical system to satisfy almost strictly positive real (ASPR) property (H. Kaufman and Sobel, 1994; Bar-Kana and Kaufman, 1985). Therefore we also propose the method letting its hypothetical system be ASPR.

In our method, since it is not necessary for the controlled MIMO system to be stable and/or minimum phase, it is useful for large class systems. Moreover for practicability, an approximate differential of the output is used instead of the strictly output differential of one.

Finally, the effectiveness of the proposed method is confirmed with unstable MIMO system.

2. FORMULATION OF PROBLEM

Consider the following MIMO system

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t), \quad (1)$$

$$\boldsymbol{y}(t) = C\boldsymbol{x}(t) \quad (2)$$

where $\boldsymbol{x}(\cdot) \in \mathbb{R}^n, \boldsymbol{u}(\cdot) \in \mathbb{R}^r$ and $\boldsymbol{y}(\cdot) \in \mathbb{R}^m$ are the state vector, the control input vector and the output vector, respectively. Assume that $\{A, B, C\}$ is controllable and observable.

The expanded PID control (Shimizu and Tamura, 2005) for the regulator problem of (1), (2) is given as

$$\dot{\boldsymbol{u}}(t) = -K_I\boldsymbol{y}(t) - K_P\dot{\boldsymbol{y}}(t) - K_D\ddot{\boldsymbol{y}}(t) - K_U\boldsymbol{u}(t) \quad (3)$$

where $K_P, K_I, K_D \in \mathbb{R}^{r\times m}$ are PID parameter matrices and $K_U \in \mathbb{R}^{r\times r}$ denotes the expanded parameter matrix.

Here, we consider using the approximate differential of the output $\boldsymbol{w}(t), \dot{\boldsymbol{w}}(t)$ given as

$$\dot{\boldsymbol{w}}(t) = -E\boldsymbol{w}(t) + E\dot{\boldsymbol{y}}(t), \quad \boldsymbol{w}(0) = \boldsymbol{0}, \quad (4)$$

$$\text{where} \quad E = \text{diag}\{\ 1/\tau_1,\ 1/\tau_2,\ \cdots,\ 1/\tau_m\}$$

for practicability instead of the strictly output differential $\dot{\boldsymbol{y}}(t), \ddot{\boldsymbol{y}}(t)$ in (3). Note that $\tau_i > 0$ are usually given at the span between 0.1 to 0.4. Accordingly the expanded PID control with the approximate differential is formulated as

$$\dot{\boldsymbol{u}}(t) = -K_I\boldsymbol{y}(t) - K_P\boldsymbol{w}(t) - K_D\dot{\boldsymbol{w}}(t) - K_U\boldsymbol{u}(t). \quad (5)$$

To investigate the auto-tuning method of PID parameter matrices of (5), we define the PID parameter matrices as the time-varying $K_P(t), K_I(t), K_D(t)$, $K_U(t)$. Therefore, the following time-varying expanded PID control

$$\dot{\boldsymbol{u}}(t) = -K_I(t)\boldsymbol{y}(t) - K_P(t)\boldsymbol{w}(t) - K_D(t)\dot{\boldsymbol{w}}(t) - K_U(t)\boldsymbol{u}(t) \quad (6)$$

is obtained.

In this paper we propose the new auto-tuning method of $K_I(t), K_P(t), K_D(t), K_U(t)$ in order to asymptotically stabilize the closed loop system (1), (2), (4) and (6).

3. NEW AUTO-TUNING METHOD OF EXPANDED PID CONTROL

We propose the new auto-tuning method of expanded PID control (6) by the following theorem.
Theorem: When the following hypothetical system $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$:

$$\widetilde{A} = \begin{bmatrix} A & O & B \\ ECA & -E & ECB \\ O & O & O \end{bmatrix}, \quad \widetilde{B} = \begin{bmatrix} O \\ O \\ I_r \end{bmatrix}, \quad (7)$$

$$\widetilde{C} = \begin{bmatrix} \widetilde{C}_1 & \widetilde{C}_2 & \widetilde{C}_3 \end{bmatrix} = [H_IC + H_DECA, \; H_P - H_DE, H_DECB + H_U] \quad (8)$$

has ASPR property [3,4]($\widetilde{C}\widetilde{B} > 0$ and minimum phase), giving the time-varying PID parameter matrices in (6) such that

$$K_I(t) = L_I(t)H_I, \;\; K_P(t) = L_P(t)H_P, \quad (9a)$$

$$K_D(t) = L_D(t)H_D, \;\; K_U(t) = L_U(t)H_U, \quad (9b)$$

$$\dot{L}_I(t) = \boldsymbol{\zeta}(t)(H_I\boldsymbol{y}(t))^TT_I, \quad (10a)$$

$$\dot{L}_P(t) = \boldsymbol{\zeta}(t)(H_P\boldsymbol{w}(t))^TT_P, \quad (10b)$$

$$\dot{L}_D(t) = \boldsymbol{\zeta}(t)(H_D\dot{\boldsymbol{w}}(t))^TT_D, \quad (10c)$$

$$\dot{L}_U(t) = \boldsymbol{\zeta}(t)(H_U\boldsymbol{u}(t))^TT_U, \quad (10d)$$

$$\boldsymbol{\zeta}(t) = H_I\boldsymbol{y}(t) + H_P\boldsymbol{w}(t) + H_D\dot{\boldsymbol{w}}(t) + H_U\boldsymbol{u}(t) \quad (11)$$

let the closed loop system (1), (2), (4) and (6) be asymptotically stable. Here, $H_P, H_I, H_D, H_U \in \mathbb{R}^{r\times m}$ denote the intermediate parameter matrices, $L_P(t), L_I(t), L_D(t), L_U(t) \in \mathbb{R}^{r\times r}$ denote adaptive parameter matrices and T_I, T_P, T_D, T_U are arbitrary positive definite martices.
(proof) We choose the Lyapunov function candidate as follows:

$$V(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \\ \boldsymbol{u}(t) \end{bmatrix}^T \widetilde{P} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \\ \boldsymbol{u}(t) \end{bmatrix} + \text{Trace}\left\{(L_I(t) - \widetilde{F})T_I^{-1}(L_I(t) - \widetilde{F})^T\right\} + \text{Trace}\left\{(L_P(t) - \widetilde{F})T_P^{-1}(L_P(t) - \widetilde{F})^T\right\} + \text{Trace}\left\{(L_D(t) - \widetilde{F})T_D^{-1}(L_D(t) - \widetilde{F})^T\right\} + \text{Trace}\left\{(L_U(t) - \widetilde{F})T_U^{-1}(L_U(t) - \widetilde{F})^T\right\} \quad (12)$$

where $\widetilde{P} \in \mathbb{R}^{(n+m+r)\times(n+m+r)}$ is a positve definite matirix, $\widetilde{F} \in \mathbb{R}^{r\times r}$ is a hypothetical gain matrix for only proof, and $T_I, T_P, T_D, T_U \in \mathbb{R}^{r\times r}$ are arbitrary positive definite matrices.

The time derivative of (12) is calculated as

$$
\begin{aligned}
\dot{V}(t) &= 2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T \widetilde{P}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{w}}(t)\\ \dot{\boldsymbol{u}}(t)\end{bmatrix}\\
&+2\mathrm{Trace}\left\{(L_I(t)-\widetilde{F})T_I^{-1}\dot{L}_I(t)^T\right\}\\
&+2\mathrm{Trace}\left\{(L_P(t)-\widetilde{F})T_P^{-1}\dot{L}_P(t)^T\right\}\\
&+2\mathrm{Trace}\left\{(L_D(t)-\widetilde{F})T_D^{-1}\dot{L}_D(t)^T\right\}\\
&+2\mathrm{Trace}\left\{(L_U(t)-\widetilde{F})T_U^{-1}\dot{L}_U(t)^T\right\}\\
&= 2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T \widetilde{P}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{w}}(t)\\ \dot{\boldsymbol{u}}(t)\end{bmatrix} - 2\boldsymbol{\zeta}(t)^T\widetilde{F}\boldsymbol{\zeta}(t)\\
&+2\boldsymbol{\zeta}(t)^T\big(L_I(t)H_I\boldsymbol{y}(t)+L_P(t)H_P\boldsymbol{w}(t)\\
&+L_D(t)H_D\dot{\boldsymbol{w}}(t)+L_U(t)H_U\boldsymbol{u}(t)\big) \qquad (13)
\end{aligned}
$$

with (10), (11).

Here the following relations

$$
\begin{aligned}
\boldsymbol{y}(t) &= C\boldsymbol{x}(t), &(14a)\\
\dot{\boldsymbol{y}}(t) &= CA\boldsymbol{x}(t)+CB\boldsymbol{u}(t), &(14b)\\
\dot{\boldsymbol{w}}(t) &= -E\boldsymbol{w}(t)+EC(A\boldsymbol{x}(t)+B\boldsymbol{u}(t)) &(14c)
\end{aligned}
$$

obtained from (2) are substituted into (6), (11) in order to get

$$
\begin{aligned}
\dot{\boldsymbol{u}}(t) &= -(K_I(t)C+K_D(t)ECA)\boldsymbol{x}(t)\\
&\quad -(K_P(t)-K_D(t)E)\boldsymbol{w}(t)\\
&\quad -(K_D(t)ECB+K_U(t))\boldsymbol{u}(t), &(15)\\
\boldsymbol{\zeta}(t) &= (H_IC+H_DECA)\boldsymbol{x}(t)\\
&\quad +(H_P-H_DE)\boldsymbol{w}(t)\\
&\quad +(H_DECB+H_U)\boldsymbol{u}(t). &(16)
\end{aligned}
$$

Thus substituting (1), (15) and (16) into (13) gives

$$
\begin{aligned}
\dot{V}(t) &= 2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T \widetilde{P}\begin{bmatrix}A & O & B\\ ECA & -E & ECB\\ O & O & O\end{bmatrix}\\
&\times\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix} - 2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T\widetilde{P}\begin{bmatrix}O\\ O\\ I_r\end{bmatrix}\\
&\times\Big[K_I(t)C+K_D(t)ECA,\ K_P(t)-K_D(t)E,\\
&K_D(t)ECB+K_U(t)\Big]\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}\\
&-2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T\Big[H_IC+H_DECA,\\
&H_P-H_DE,\ H_DECB+H_U\Big]^T\widetilde{F}\\
&\times\Big[H_IC+H_DECA,\ H_P-H_DE,\\
&H_DECB+H_U\Big]\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}\\
&+2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T\Big[H_IC+H_DECA,\\
&H_P-H_DE,\ H_DECB+H_U\Big]^T\\
&\times\Big[L_I(t)H_IC+L_D(t)H_DECA,\\
&L_P(t)H_P-L_D(t)H_DE,\\
&L_D(t)H_DECB+L_U(t)H_U\Big]\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}. \qquad (17)
\end{aligned}
$$

Using (7), (8) , we can express (17) as

$$
\begin{aligned}
\dot{V}(t) &= 2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T \widetilde{P}\widetilde{A}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}\\
&-2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T\widetilde{P}\widetilde{B}\Big[K_I(t)C+K_D(t)ECA,\\
&K_P(t)-K_D(t)E,\ K_D(t)ECB+K_U(t)\Big]\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}\\
&-2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T\widetilde{C}^T\widetilde{F}\widetilde{C}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}\\
&+2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T\widetilde{C}^T\Big[L_I(t)H_IC+L_D(t)H_DECA,\\
&L_P(t)H_P-L_D(t)H_DE,\\
&L_D(t)H_DECB+L_U(t)H_U\Big]\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}. \qquad (18)
\end{aligned}
$$

From (9a), (9b), (18) becomes

$$
\begin{aligned}
\dot{V}(t) &= 2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T \widetilde{P}\widetilde{A}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}\\
&-2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T\widetilde{C}^T\widetilde{F}\widetilde{C}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}\\
&+2\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}^T(\widetilde{C}^T-\widetilde{P}\widetilde{B})\\
&\times\Big[L_I(t)H_IC+L_D(t)H_DECA,\\
&L_P(t)H_P-L_D(t)H_DE,\\
&L_D(t)H_DECB+L_U(t)H_U\Big]\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{w}(t)\\ \boldsymbol{u}(t)\end{bmatrix}. \qquad (19)
\end{aligned}
$$

Here if the hypothetical system $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ defined by (7), (8) is ASPR ($\widetilde{C}\widetilde{B} > 0$ and minimum phase), there exists the positive definite matrix $\widetilde{P}$ and the gain matrix $\widetilde{F}$ which satisfy the following relations (Bar-Kana and Kaufman, 1985; H. Kaufman and Sobel, 1994):

$$\widetilde{P}(\widetilde{A} - \widetilde{B}\widetilde{F}\widetilde{C}) + (\widetilde{A} - \widetilde{B}\widetilde{F}\widetilde{C})^T \widetilde{P} < 0, \tag{20}$$
$$\widetilde{P}\widetilde{B} = \widetilde{C}^T. \tag{21}$$

Then (19) becomes

$$\dot{V}(t) = 2 \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \\ \boldsymbol{u}(t) \end{bmatrix}^T \Big(\widetilde{P}(\widetilde{A} - \widetilde{B}\widetilde{F}\widetilde{C}) + (\widetilde{A} - \widetilde{B}\widetilde{F}\widetilde{C})^T P\Big) \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{w}(t) \\ \boldsymbol{u}(t) \end{bmatrix} < 0. \tag{22}$$

Accordingly, it can be concluded that if the hypothetical system $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ has ASPR property ($\widetilde{C}\widetilde{B} > 0$ and minimum phase), the resultant closed loop sytem (1), (2), (4) and (6) is asymptotically stable by tuning PID parameter matrices as (9), (10), and (11), and all element of PID parameter matrices are bounded. □

4. HYPOTHETICAL SYSTEM

To execute Theorem, we have to check if the hypothetical system $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ defined by (7), (8) satisfies ASPR ($\widetilde{C}\widetilde{B} > 0$ and minimum phase). First check $\widetilde{C}\widetilde{B} > 0$, so the $\widetilde{C}\widetilde{B}$ is calculated as

$$\widetilde{C}\widetilde{B} = \widetilde{C}_3 = H_D ECB + H_U. \tag{23}$$

In order to let (23) be positive definite matrix, we set the intermediate parameter matrix H_U as follows:

$$H_U = -H_D ECB + D, \quad D > 0 \tag{24}$$

where $D \in \mathbb{R}^{r\times r}$ is arbitrary positive definite matrix. Accordingly, $\widetilde{C}\widetilde{B}$ of (23) becomes

$$\widetilde{C}\widetilde{B} = \widetilde{C}_3 = D > 0, \tag{25}$$

so $\widetilde{C}\widetilde{B} > 0$ is satisfied.

Furthermore, check the minimum phase property (zero dynamics asymptotically stable) of the hypothetical system $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$. Here since $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ has relative degree $\{1, 1, \cdots, 1\}$ from (25), we can transform $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ into normal form, and the zero dynamics can be obtained (Isidori, 1995). Hence let us consider the following state transformation for $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$.

$$\begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{\eta} \end{bmatrix} = \begin{bmatrix} \widetilde{C}_1 \ \widetilde{C}_2 \ \widetilde{C}_3 \\ \widetilde{T} \end{bmatrix} \widetilde{\boldsymbol{x}}, \tag{26}$$

where $\widetilde{T} = \begin{bmatrix} I_n & O & O \\ O & I_m & O \end{bmatrix}$, $\widetilde{T}\widetilde{B} = O$,

and $\boldsymbol{\xi} \in \mathbb{R}^r$ from $\boldsymbol{\xi} = \widetilde{\boldsymbol{y}}$ so $\boldsymbol{\eta} \in \mathbb{R}^{n+m}$. As a result, the normal form of the hypothetical system $\{\widetilde{A}, \widetilde{B}, \widetilde{C}\}$ defined by (7), (8) is concretely obtained as follows:

$$\dot{\boldsymbol{\xi}} = Q_{11}\boldsymbol{\xi} + Q_{12}\boldsymbol{\eta} + \widetilde{C}_3\boldsymbol{v}, \tag{27a}$$
$$\boldsymbol{\eta} = Q_{21}\boldsymbol{\xi} + Q_{22}\boldsymbol{\eta}, \tag{27b}$$
$$\widetilde{\boldsymbol{y}} = \boldsymbol{\xi} \tag{28}$$

where

$$Q_{11} = \begin{bmatrix} \widetilde{C}_1 \ \widetilde{C}_2 \end{bmatrix} \begin{bmatrix} B \\ ECB \end{bmatrix} \widetilde{C}_3^{-1},$$
$$Q_{12} = \begin{bmatrix} \widetilde{C}_1 \ \widetilde{C}_2 \end{bmatrix} \left(\begin{bmatrix} A & O \\ ECA & -E \end{bmatrix} - \begin{bmatrix} B \\ ECB \end{bmatrix} \begin{bmatrix} \widetilde{C}_1 \ \widetilde{C}_2 \end{bmatrix} \right),$$
$$Q_{21} = \begin{bmatrix} B \\ ECB \end{bmatrix} \widetilde{C}_3^{-1},$$
$$Q_{22} = \begin{bmatrix} A & O \\ ECA & -E \end{bmatrix} - \begin{bmatrix} B \\ ECB \end{bmatrix} \widetilde{C}_3^{-1} \begin{bmatrix} \widetilde{C}_1 \ \widetilde{C}_2 \end{bmatrix}.$$

From (27b), the zero dynamics is obtained as follows

$$\dot{\boldsymbol{\eta}}(t) = \left(\widehat{A} - \widehat{B}\widetilde{C}_3^{-1} \begin{bmatrix} \widetilde{C}_1 \ \widetilde{C}_2 \end{bmatrix} \right) \boldsymbol{\eta}(t), \tag{29}$$

where $\widehat{A} \triangleq \begin{bmatrix} A & O \\ ECA & -E \end{bmatrix}$, $\widehat{B} \triangleq \begin{bmatrix} B \\ ECB \end{bmatrix}$.

In order to satisfy the minimum phase property of the hypothetical system, the zero dynamics (29) has to be asymptotically stable. Since H_I, H_P, H_D which we can set arbitrarily are contained in (29), it is possible to determine the H_I, H_P, H_D satisfying its assumption. Then if such H_I, H_P, H_D are determined, we can execute Theorem and give the auto-tuning method of $K_P(t), K_I(t), K_D(t), K_U(t)$ so as to asymptotically stabilize the closed loop system (1), (2), (4) and (6).

5. DETERMINATION METHOD OF H_I, H_P, H_D, H_U

The most important task in our method is to determine the intermediate parameter matrices H_I, H_P, H_D, H_U stabilizing the zero dynamics (29). In this section, we propose a simple method determining H_I, H_P, H_D, H_U.

So we transform the partial matrix $\widetilde{C}_3^{-1} \begin{bmatrix} \widetilde{C}_1 \ \widetilde{C}_2 \end{bmatrix}$ of the zero dynamics matrix (29) into

$$\widetilde{C}_3^{-1}\left[\widetilde{C}_1\ \widetilde{C}_2\right] = D^{-1} \times \left[H_I C + H_D ECA\ \ H_P - H_D E\right] = D^{-1}\left[H_I\ H_P\ H_D\right]\begin{bmatrix} C & O \\ O & I_m \\ ECA & -E \end{bmatrix} \tag{30}$$

from (8), (25). Putting

$$\widehat{H}_I = D^{-1}H_I, \widehat{H}_P = D^{-1}H_P, \widehat{H}_D = D^{-1}H_D, \tag{31}$$

$$\widehat{H} = \left[\widehat{H}_I\ \widehat{H}_P\ \widehat{H}_D\right], \widehat{C} = \begin{bmatrix} C & O \\ O & I_m \\ ECA & -E \end{bmatrix} \tag{32}$$

makes (30) be

$$\widetilde{C}_3^{-1}\left[\widetilde{C}_1\ \widetilde{C}_2\right] = \widehat{H}\widehat{C}. \tag{33}$$

Hence the zero dynamics (29) can be regarded as the hypothetical output feedback system:

$$\dot{\boldsymbol{\eta}}(t) = (\widehat{A} - \widehat{B}\widehat{H}\widehat{C})\boldsymbol{\eta}(t). \tag{34}$$

Thus determining the output feedback gain $\widehat{H}$ stabilizing (34), we can determine H_I, H_P, H_D with the relations (31), (32).

Now, to obtain such an output feedback gain $\widehat{H}$, we apply the eigenvalue assignment method (Tamura and Shimizu, 2006) to the subsystem $\{\widehat{A}, \widehat{B}, \widehat{C}\}$. For its application it is required that $\{\widehat{A}, \widehat{B}, \widehat{C}\}$ is controllable and observable and $2m + r > n$, so let us suppose that and get the such output feedback gain $\widehat{H}$. Accordinly, from (31), (32) the H_I, H_P, H_D are calculated as

$$H_I = D\widehat{H}_I,\ H_P = D\widehat{H}_P,\ H_D = D\widehat{H}_D \tag{35}$$

where $D > 0$ can be arbitrarily set. Then using above H_D and D, we can obtain H_U from (24) as

$$H_U = -H_D ECB + D. \tag{36}$$

Remark: When we apply the eigenvalue assignment method to subsystem $\{\widehat{A}, \widehat{B}, \widehat{C}\}$, it is important how to choose the desired eigenvalues. So as such the desired eigenvalue, we can consider choosing the optimal eigenvalues calculated from the optimal closed loop $\widehat{A} - \widehat{B}\widehat{F}$ with $\widehat{F} = \widehat{R}^{-1}\widehat{B}^T\widehat{P}$ obtained by solving the Riccati equation $\widehat{P}\widehat{A} + \widehat{A}^T\widehat{P} + \widehat{Q} - \widehat{P}\widehat{B}\widehat{R}^{-1}\widehat{B}^T\widehat{P} = O,\ \widehat{Q} > 0, \widehat{R} > 0$.

Design procedure of H_I, H_P, H_D and H_U is now given as follows:

[Design Procedure]

Step 1: Set the desired eigenvalues for (34) (e.g. using the method of Remark).

Step 2: Apply the eigenvalues assignment method (Tamura and Shimizu, 2006) to $\{\widehat{A}, \widehat{B}, \widehat{C}\}$, and determine the output feedback gain $\widehat{H}$ assigning th desired eigenvalue given in Step1 .

Step 3: Give the adequate $D > 0$, and determine the H_I, H_P, H_D, H_U from (35), (36).

6. SIMULATIONS

Consider the following MIMO linear system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 3 & 0 & -3 & 1 \\ -1 & 1 & 4 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\boldsymbol{u},$$

$$\boldsymbol{y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\boldsymbol{x}$$

with unstable and non-minimum phase.

First set the approximate differential parameter $E = \mathrm{diag}\{1/\tau_1, 1/\tau_2\}$ of (4) as $\tau_1 = 0.3,\ \tau_2 = 0.3$. Following Step 1 of Design Procedure, we set the desired eigenvalues for (34). So using method of Remark under setting $\widehat{P} = I_6, \widehat{R} = I_2$ in Riccati equaion, we set such the desired eigenvalues as $-4.234, -4.394 \pm 1.241i, -0.4428, -1.198 \pm 0.2077i$.

From Step2, we get $\widehat{H}$ which assigns the desired eigenvalues as $\widehat{H} = \begin{bmatrix} -7.287 & -2.818 & -3.250 & 4.399 & -1.016 & -0.0564 \\ 12.87 & 7.177 & 10.05 & -5.009 & 2.983 & -1.531 \end{bmatrix}$.

By Step3, we calculate H_I, H_P, H_D and H_U as

$$H_I = \begin{bmatrix} -14.58 & -5.638 \\ 25.74 & 14.35 \end{bmatrix}, H_P = \begin{bmatrix} -6.501 & 8.799 \\ 20.10 & -10.02 \end{bmatrix},$$

$$H_D = \begin{bmatrix} -2.032 & 2.507 \\ 5.966 & -3.063 \end{bmatrix}, H_U = \begin{bmatrix} 8.772 & -8.356 \\ -19.89 & 12.21 \end{bmatrix}$$

after setting $D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$. We set the initial values as $\boldsymbol{x}(0) = [5, 0, 0, 0]^T, \boldsymbol{u}(0) = \boldsymbol{0}, L_I(0) = L_P(0) = L_D(0) = L_U(0) = O$, and put $T_I = T_P = T_D = T_U = I_2$ in (10a) $\sim$ (10d). The simulation results are shown in Fig. 1 $\sim$ Fig. 5. It is observed that time-varying part of the PID parameter matrices $L_I(t), L_P(t), L_D(t), L_U(t)$ are auto-tuned for asymptotic stability.

7. CONCLUSIONS

We have proposed the new auto-tuning method of the expanded PID control for MIMO system. For its execution it is necessary for a certain hypothetical system to satisfy ASPR property. Therefore we also have proposed the simple method letting its hypothetical system be ASPR. In our method, since it is not necessary for the controlled MIMO system to be stable and/or minimum phase, it is useful for large class systems.

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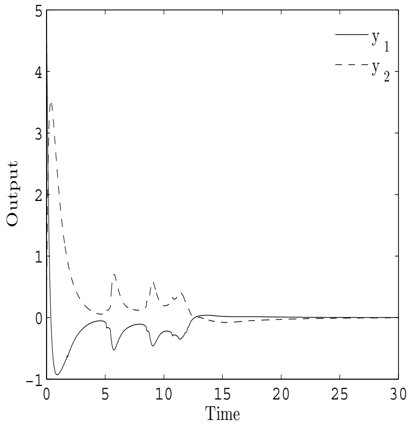

Fig.1. Trajectories of outputs

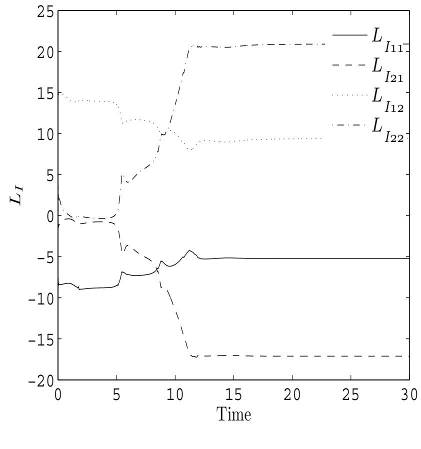

Fig.2. Trajectories of $L_I(t)$

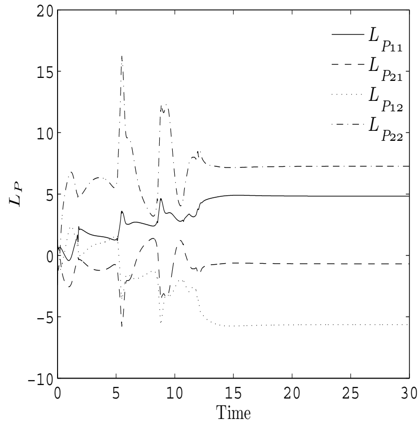

Fig.3. Trajectories of $L_P(t)$

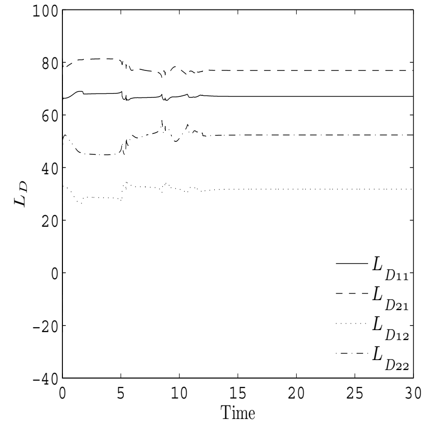

Fig.4. Trajectories of $L_D(t)$

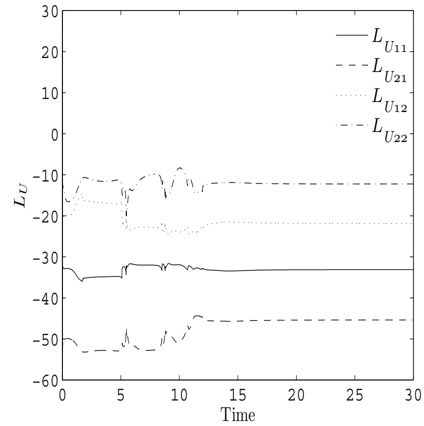

Fig.5. Trajectories of $L_U(t)$