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INTEGRATED INVERSION OF NUMERICAL GEOPHYSICAL MODELS USING ARTIFICIAL NEURAL NETWORKS

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Abstract

A unified modelling procedure is proposed to jointly interpret the variations observed in geophysical data and to properly take into account the relationship between the intrusive processes and the geophysical variations expected at the ground surface. We focus on the joint inversion of geophysical data by a procedure based on Artificial Neural Network (ANN) for the estimation of the volcanic source parameters. As forward model, we developed a 3D numerical model based on Finite Element Method (FEM) for computing ground deformation, magnetic and gravity changes caused by magmatic overpressure sources, with the aim to consider a more realistic description of Etna volcano, including the effects of topography and medium heterogeneities.

Key words

Identification, Modeling, Numerical methods.

1 Introduction

Geodetic and potential fields investigations have been playing an increasingly important role in Mt. Etna eruptive processes ([Bonaccorso *et al*, 1999]; [Bonforte *et al*, 2008]; [Del Negro *et al*, 2004]; [Napoli *et al*, 2008]; [Carbone *et al*, 2007]; [Carbone *et al*, 2008]). The amount of available data collected represents a valuable database, but limited efforts have been made for an effective integration of different data. Even if complicated models have been proposed, ground deformation, magnetic and gravity data are usually interpreted separately from each other and the joint modelling has remained elusive, despite the obvious benefit in constraining the solution. When the cause of their variations can be ascribed to the same volcanic source, a joint inversion would be advisable in order to identify the source parameters with a greater degree of confidence [Nunnari *et al*, 2001]. For an integrated inversion modelling, complex inverse methods are required to combine forward models with appropriate optimization algorithms and automatically find the best set of parameters that well matches the available observations. Hence, the rationale of the inversion modelling approach requires: (i) solution of forward models, (ii) numerical inversion procedure. The forward problem consists in deriving a relationship between sources and observations. Over the last decades, straightforward analytical solutions for simplified geometric sources have been devised under the assumption of homogeneous elastic half-space medium ([Mogi, 1958]; [Sasai, 1991]; [Hagiwara, 1977]). To overcome this intrinsic limitation and provide more realistic models, which consider various geometries as well as complicated distribution of medium properties, numerical solutions can be investigated. With the aim to consider a more realistic description of Mt Etna, we developed a numerical procedure based on Finite Element Method (FEM) to evaluate geophysical changes caused by overpressure source in a 3D formulation. The FEM-based numerical model is able to include not only complicated distribution of both rock magnetization and elastic rigidity, but also the real topography of the studied area, that are responsible for significant effects [Currenti *et al*, 2007]. We investigate the ability of an inversion procedure based on Artificial Neural Networks (ANNs) for the estimation of the volcanic source parameters from magnetic, gravity and ground deformation data. Artificial Neural networks have been used to invert geophysical models based on analytical models, because once the network is trained, the inversion process requires a very short time, while in traditional optimization algorithm the whole search procedure has to be reiterated. Numerical models are not often used to train neural network for inversion because of the high computational time to obtain the solution of forward model. We performed a hybrid inversion in which neural networks are trained with numerical patterns, obtained using Finite Element Method. Synthetic modelling is performed in order to provide a data set large enough to represent the training and testing sets of the possible models within the model space. Multilayer perceptrons (MPLs), once correctly trained, can solve the inversion problem very fast and with an appreciable degree of accuracy.

2 Forward Numerical Model

As forward models, we developed a numerical procedure based on Finite Element Method (FEM) to evaluate geophysical changes caused by spherical overpressure sources that are quite appropriate for modelling inflation/deflation of magma reservoirs. We used the software Pylith to solve the elastostatic problem for the elastic deformation field [Mogi, 1958]. A computational domain of a 100x100x50 km is considered for the deformation calculation. As for boundary conditions, horizontal and vertical displacements are fixed to zero at the lateral and bottom boundaries respectively, representing the vanish displacement at the infinity. The upper boundary is stress free and represents the ground surface. The accuracy of the numerical solution strongly depends on the mesh resolution and the computational domain size. To properly set up the model, benchmark tests are carried out comparing the numerical solutions of deformation field in a homogeneous half-space with the analytical ones. The analytical solutions of the elastostatic problems are obtained using the simple and common Mogi model embedded in a homogeneous Poisson's medium [Mogi, 1958]. After benchmark tests were carried out, we used numerical model including medium heterogeneities and irregular topography to model a more realistic description of Mt. Etna. As magnetic effect generated by a pressure source, we analyzed the thermomagnetic effect. Thermomagnetic field changes are due to thermal demagnetization or remagnetization due to temperature changes of rocks. When temperature exceeds Curie point, rocks lose their magnetization and then modify the intensity of superficial magnetic field. The magnetic change is expressed as follows:

$$\begin{aligned}\Delta T &= \frac{\Delta M_m}{4\pi r^2}\left\{1-3\left(\frac{x}{r}\cos I-\frac{z}{r}\sin I\right)^2\right\}\\ \Delta M_m &= \frac{4}{3}\pi r^3 m\end{aligned}\qquad(1)$$

Where ΔM_m is the magnetic moment, R is the radius of the sphere, m is the magnetization, I is the magnetic inclination. Gravity field changes are due to additional mass input at some depth. Migration of magmatic mass generates a density variation that can be observed at the surface through gravity field measures. The gravity change due to input of new mass in a spherical source is expressed as follows:

$$\begin{aligned}\Delta g &= G\Delta M_g \frac{z}{(x^2+y^2+z^2)^{\frac{3}{2}}}\\ \Delta M_g &= \frac{4}{3}\pi r^3 \Delta\rho\end{aligned}\qquad(2)$$

where ΔM_g is the mass change, ρ is the density contrast, G is the gravitational constant. Using the forward model described above, we generated a synthetic set of deformation, magnetic and gravity data for training the neural network and find out if multilayer perceptrons (MPLs) could approximate without ambiguity the complex relation between geophysical observations and source parameters. Inversion algorithms based on classic analytical models, that neglect the complexities associated with morphology and medium properties of volcano edifice, could provide an inaccurate estimate of source parameters. Then we implemented numerical inversion training the network with numeric patterns, that can include the real effects of topography and medium heterogeneity. The procedure of pattern generation is divided in several step and is executed automatically. First the computational domain of Mt. Etna is meshed into 598948 isoparametric and arbitrarily distorted tetrahedral elements connected by 103424 nodes. The domain mesh, showed in Fig. 1, generated with the software LaGrit, has a spatial resolution that reaches 300m in the area where the topographic relief is sharper and in the area where the sources are located.

Then the parameters of the sources are generated with random distribution in the ranges reported in Table 1. The volume where the pressure sources are located contains the position of all the pressure sources active during the last decades [Bonforte *et al*, 2008].

Once all the patterns are defined, the meshed sources are iteratively introduced in the domain mesh to finally obtain 1050 complete meshes. Every mesh is characterized by the source in a different random position that is contained in the range reported in Table 1. The numerical solutions are calculated applying a pressure of 100MPa to the source wall and then rescaling them with the random values of ΔV, assuming valid the hypothesis of point source. For each simulation, computed using PyLith, the accuracy was warranted checking the convergency of the FEM solution. The iteration of GMRES solver is stopped when a threshold of

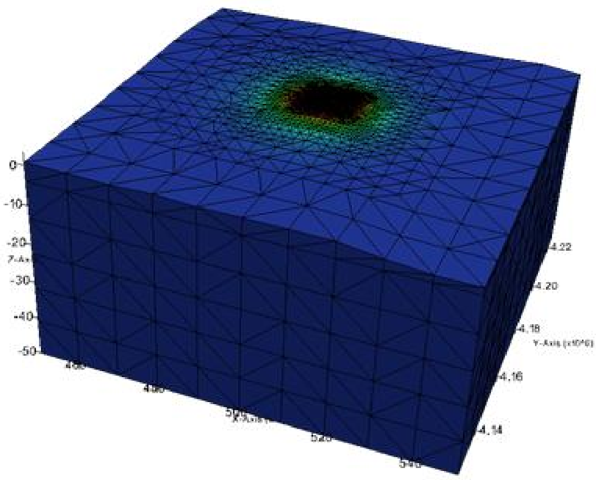

Figure 1. Mesh of the computational domain.

10^{-9} is reached or the number of iterations is higher than 200. By a linear speedup on a cluster of 20 nodes the computing time reduces from 10 days to 12 hours. At the end solutions have been interpolated at the coordinates of the stations of deformation, magnetic and gravity network on Mt. Etna, which map is reported in Fig. 2. The rectangle corresponds to the projection on surface of the volume where the sources are located.

3 ANN Based Inverse Model

Inverse modelling with MLP Artificial Neural Networks (ANNs) can be formalised as follows. Let M be a model whose parameters are indicated by x and the associated output data by d. The inverse modeling can be formally indicated as $x = f^{-1}(d)$, in the hypothesis that f^{-1} exists. In practical situations, it is possible to approach the inverse problem numerically: the forward model $f()$ is used to generate synthetic data di corresponding to model parameters xi, (randomly chosen in the space of parameters). At the same time the pairs (d_i, x_i) are used to approximate the inverse model by approximating $f^{-1}(.)$ by using an approximating function of the form:

Source Parameters	Minimum	Maximum
$X_C[km]$	496	502
$Y_C[km]$	4175	4183
$Z_C[km]$	-9	-1
$\Delta V[m^3]$	5 10^6	10 10^6
$\Delta M_g[kg]$	3 10^9	150 10^9
ΔM_m $[Am]$	1 10^9	4 10^9

Table 1. Ranges of the random generated parameters of the source.

$$f_a(x) = \sum_{j=1}^{NH} c_j \phi(w_j^T x + t_j) + c_0 \quad (3)$$

where $\phi(.)$ represents the sigmoid function (i.e. $\phi(z) = 1/(1 + e^{-z})$), x is the MLP input vector, w_j is a vector of coefficients (weight of the connections), and c_j, tj are additional adjustable coefficients. The main advantage of inverting with MLP neural network consists in the availability of an approximation of f^{-1} thus allowing speeding up the computation of the source parameters that best fits the observed data. The availability of this function allows avoiding a search for the minimum, as happens with a traditional optimisation algorithm such as GAs. As traditional optimisation algorithms cannot learn, i.e. they cannot benefit from solutions obtained previously for similar problems and each new inversion requires the whole search procedure to be re-iterated. Once the neural network has been trained, it is able to approximate the f^{-1} function and to identify, for a set of geophysical observations, the source parameters that better reproduce these variations. The neural network used for the inversion is a three layered network, with 29 inputs (the three component of deformation field, thermomagnetic field and gravity filed at the network stations), 20 hidden neurons, 6 outputs (position of the source, volume change, mass change, magnetic momentum). The deformation inputs are rescaled performing the quantities ux/uz, uy/uz uz instead of ux, uy, uz, so that the analytic function that links the first two quantities with the source parameters becomes easier to be interpo-

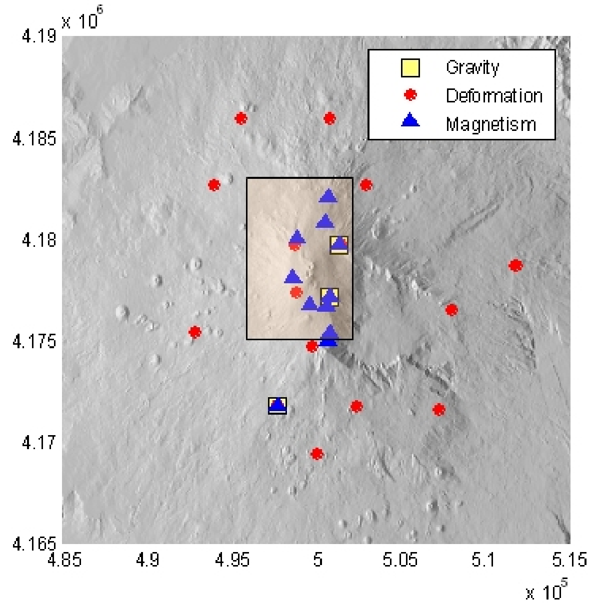

Figure 2. Deformation, gravity and magnetic monitoring network on Mt. Etna.

	Deformation (Analytic)	Deformation (Numeric)	Integrated (Numeric)
$X_C[m]$	239.44	648.78	224.24
$Y_C[m]$	301.05	807.42	620.62
$Z_C[m]$	329.99	672.24	335.80
$\Delta V[m^3]$	0.1 10^6	0.18 10^6	0.25 10^6
$\Delta M_g[kg]$	-	-	0.31 10^9
ΔM_m $[Am]$	-	-	0.07 10^9

Table 2. Performance index RMSE for the inversion of analytic and numeric deformation model and for the integrated numeric model.

lated. The outputs are normalized linearly to finally obtain values in the range [-1;1]. The ANN is trained with numerical patterns, obtained using Finite Element Method. Then the inverse function is tested with a data set that was not used previously for training. The performance index used to test the results is the root mean square error, which expression is:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(P_i - O_i)^2} \qquad (4)$$

where P_i is the calculated value, O_i is the observed value, N is the dimension of data set. We performed the inversion of deformation data for the analytical and numerical solution, to compare the accuracy of the two results. The we inverted deformation, magnetic and gravity data together to investigate if the integrated approach allows for a more accurate solution. The performance index of the three cases are reported in Table 2.

Numerical and analytical inversion of deformation data provide similar results, demonstrating that also the numerical problem can be inverted with good accuracy, even if the inverse function to interpolate is more complicated because of complex distribution of heterogeneity and irregular topography. The mean square errors related to the integrated inversion are smaller than those obtained inverting only deformation data, evidencing the advantages of the integrated approach. This approach, involving geophysical data of different kinds, allows for a more accurate solution than when ground deformation data alone is considered.

4 ANN Based Inverse Model

The main problem in numerical inversion is the high computational time necessary to obtain the solution of forward model. In any optimization algorithm, every iteration the domain has to be meshed again (assuming variable the geometry or position of the source) and the FEM problem has to be solved, requiring a significant computational effort. If we wish to compute the geophysical changes for a large number of models for inversion, analytical models are quite attractive because they are inexpensive to compute. Ideally, we would like a method that retains the computational convenience of analytical solutions but is also capable of including, at least approximately, the effects of topography and heterogeneity. The Artificial Neural Networks can be trained to map the forward numerical model, with the aim to obtain an approximate immediate solution of forward model to be used in any inversion algorithm. Therefore this method permits to overcome the intrinsic limitation of neural approach for inversion that is the dependence on stations configurations: being the structure of the neural network fixed, if a station doesnt work or if a new station is added the neural network has to be trained again, while mapping the forward model independently for any station, the inverse process becomes much flexible even for variations of the configuration of the stations. The neural network was trained to interpolate the function , where ui is the deformation at i-th station, Xc, Yc, Zc give the position of the source, xi, yi are the coordinates of the station. Once the network is trained, we obtain the solution of the forward model and the inversion can be made with any technique at a second time. We computed the mapping of numerical models for two station, one in the summit of volcano and the other at a lower altitude. As performance index we also computed a normalized misfit, to have a normalized measure of the error, which expression is:

$$E_{abs\%} = \frac{\sum |u^{obs} - u^{calc}|}{\sum |u^{obs}|} 100\% \qquad (5)$$

The results are reported in Table 3.

		Station 8 X=498.810 km Y=4177.400 km	Station 10 X=492.820 km Y=4175.400 km
RMSE [m]	u_x	6.51 10^{-6}	3.70 10^{-6}
	u_y	7.72 10^{-6}	2.50 10^{-6}
	u_z	9.12 10^{-6}	2.30 10^{-6}
Normalized misfit [%]	u_x	11.9	6.03
	u_y	10.9	7.75
	u_z	6.90	5.03

Table 3. Performance indexes RMSE and normalized misfit for two stations, located near and far from the summit of volcano.

The results show that the neural network can map with good precision the forward numerical model of deformation field. It is worth noting that results at station 10, that is located out from the summit, are characterized by a smaller error than that related to a summit station, where numerical solution is strongly influenced by topography and heterogeneity (Fig.2). This result evidences the significant effect that these parameters could have on the solution.

5 Conclusions

A hybrid approach is proposed using neural networks and numerical FEM models for geophysical modelling. This method permits to interpret geophysical data avoiding the intrinsic limitation of analytical solutions and providing a more realistic description of volcanic processes. Firstly, ANN is used in inverse scheme to identify the source parameters from geophysical observations. The results show that, notwithstanding the high nonlinearity of the considered inverse problems, it can be unambiguously solved with acceptable accuracy. The work highlights the usefulness of integrated version of geophysical data, that permits to constrain the solution better than when single data type are considered. Secondly, neural approach is also used to map the forward numerical model, reducing the high computational time usually necessary to run the forward model in inversion problems. This approach permits to have a straightforward solution of numerical model, that can substitute the analytical solution in any inversion technique, providing a more realistic description of geophysical changes in volcanic area by taking into account the real topography and medium heterogeneities.

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