# MOVING HORIZON SIMULTANEOUS ESTIMATION OF PROCESS GAIN AND DISTURBANCES FOR AN OXYGEN CONVERTER GAS RECOVERY PROCESS MODEL

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Abstract: This paper proposes a moving horizon simultaneous estimation of process gain and disturbances for discrete-time linear systems with unknown process gain so as to minimize a moving-horizon performance index. The proposed method uses discrete-time Euler-Lagrange equations in order to derive the proposed adaptive disturbance estimator. A numerical simulation for an oxygen converter gas recovery process shows the efficiency of the proposed method.

Keywords: adaptive observer, disturbance estimation, moving-horizon estimation, an oxygen converter gas recovery process

## 1. INTRODUCTION

Simultaneous estimation of disturbance and a process gain is an important issue in the field of industrial process control because disturbance and change of a process gain over a lapse of time are inevitable in real processes. Hence, several researches have been studied on the topics of disturbance and parameters estimation.

An adaptive observer (Kreisselmeier, 1977) is a standard approach for estimating both state variables and parameters of controlled processes. However, since it is not designed for estimating disturbances in itself, some modifications are required to realize simultaneous estimation of disturbance and a process gain. Using an extended kalman filter was presented in Keller and Darouach (1999) for simultaneous estimation of disturbance and process parameters by regarding unknown parameters as constant state variables the initial values of which are unknown. However, it just concerns random disturbances mean values of which are constant. Hence, it is not adequate for simultaneous estimation of disturbance with large variation and a process gain.

Therefore, this paper proposes an adaptive disturbance estimator for discrete-time, single-input, single-output, linear time-invariant processes to estimate both unknown process gain and distrubances with large variation. The strategy for the proposed adaptive disturbance estimator is to minimize a moving-horizon performance index which consist of quadratic forms of estimated output error and disturbances. Ohtsuka (1999) has already proposed an adaptive disturbance estimator which minimizes a moving-horizon performance index. However, while it deals with a continuous-time system, this paper deals with a discrete-time system. Hence, the proposed method is an extension of the earlier work to a discrete-time version. That can be done by using discrete-time Euler-Lagrange equations. Furthermore, this paper gives a calculation algorithm in the form of the Ricatti type of recursion formula.

The proposed method is applied to the approximated process model of an oxygen converter gas recovery process (Yoshida *et al.*, 1988). Through a numerical simulation the efficiency of the proposed method will be shown.

## 2. MOVING HORIZON ADAPTIVE DISTURBANCE ESTIMATOR

### *2.1 Case of a Nonlinear Process*

Consider a discrete-time, nonlinear process as shown here.

$$\boldsymbol{x}_{k+1} = \boldsymbol{f}(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{d}_k) \tag{1}$$

$$\boldsymbol{y}_k = \boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{v}_k) \tag{2}$$

where $\boldsymbol{x}_k$, $\boldsymbol{u}_k$, $\boldsymbol{d}_k$ and $\boldsymbol{v}_k$ are the state vectors, the control input signals, the disturbance signals, and measured noise signals at the time $k$. For the process Eqn. (1) and Eqn. (2), we consider estimating the state vector $\boldsymbol{x}_k$ and disturbance $\boldsymbol{d}_k$.

This paper formulates it as a receding horizon optimization problem where the following performance index should be optimized.

$$\begin{aligned} J_k = {} & \eta[\hat{\boldsymbol{x}}_{0,k}, \boldsymbol{u}_{0,k}, \boldsymbol{y}_{0,k}] \\ & + \varphi[\hat{\boldsymbol{x}}_{-N,k}, \boldsymbol{u}_{-N,k}, \boldsymbol{y}_{-N,k}] \\ & + \sum_{\tau=-N}^{-1} L[\hat{\boldsymbol{x}}_{\tau,k}, \boldsymbol{u}_{\tau,k}, \boldsymbol{y}_{\tau,k}, \hat{\boldsymbol{d}}_{\tau,k}] \qquad (3) \\ s.t. \quad & \hat{\boldsymbol{x}}_{\tau+1,k} = \boldsymbol{f}(\hat{\boldsymbol{x}}_{\tau,k}, \boldsymbol{u}_{\tau,k}, \hat{\boldsymbol{d}}_{\tau,k}), \\ & \qquad \tau = -N, -N+1, \cdots, -1 \qquad (4) \end{aligned}$$

The performance index evaluates the integrated value based on the input-output data from the present time $k$ to the past $N$ step past time. When $L[\,\cdot\,]$ evaluates the norm of the error signal between the estimated output $\hat{\boldsymbol{y}}_k = \boldsymbol{h}(\hat{\boldsymbol{x}}_k, 0)$ and measured output $\boldsymbol{y}_k$, the optimal values of

$$\begin{aligned} & \hat{\boldsymbol{x}}_{\tau,k}, \quad \tau = -N, -N+1, \cdots, 0, \\ & \hat{\boldsymbol{d}}_{\tau,k}, \quad \tau = -N, -N+1, \cdots, -1 \end{aligned}$$

become the optimal state vector and the estimated disturbance based on the evaluation of input-output signals from the present time $k$ to the $N$ steps past time. The performance index $J_k$ moves the evaluation range as the present time $k$ goes by. Therefore, the estimated value of state variable at the present time $k$ becomes $\hat{\boldsymbol{x}}_{0,k}$ and the estimated disturbance becomes $\hat{\boldsymbol{d}}_{0,k}$.

The next theorem gives the necessary condition of $\hat{\boldsymbol{x}}_{\tau,k}$ and $\hat{\boldsymbol{d}}_{0,k}$ where the performance index $J_k$ should be optimized, which is the discrete-time Euler-Lagrange equation. Subsequently, we denote $L[\,\cdot\,]$, $\boldsymbol{f}[\,\cdot\,]$, $\eta[\,\cdot\,]$ and $\varphi[\,\cdot\,]$ in Eqn. (1) and Eqn. (3) as $L_{\tau,k}$, $\boldsymbol{f}_{\tau,k}$, $\eta_{0,k}$ and $\varphi_{-N,k}$.

*Theorem 1.* Let the Hamiltonian be defined as

$$H_{\tau,k} = L_{\tau,k} + \boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}} \boldsymbol{f}_{\tau,k} \tag{5}$$

Then, the necessary condition of $\hat{\boldsymbol{x}}_{\tau,k}$ and $\hat{\boldsymbol{d}}_{\tau,k}$ which optimize the performance index Eqn. (3) and Eqn. (4) is given by the following discrete-time Euler-Lagrange equations.

$$\frac{\partial}{\partial \hat{\boldsymbol{x}}_{\tau,k}} H_{\tau,k} = \boldsymbol{\lambda}_{\tau,k}^{\mathrm{T}}, \quad \tau = -1, -2, \cdots, -N+1 \tag{6}$$

$$\hat{\boldsymbol{x}}_{\tau+1,k} = \boldsymbol{f}_{\tau,k}, \quad \tau = -N, -N+1, \cdots, -1 \tag{7}$$

$$\frac{\partial}{\partial \hat{\boldsymbol{d}}_{\tau,k}} H_{\tau,k} = \boldsymbol{0}, \quad \tau = -N, -N+1, \cdots, -1 \tag{8}$$

$$\frac{\partial}{\partial \hat{\boldsymbol{x}}_{0,k}} \eta_{0,k} = \boldsymbol{\lambda}_{0,k}^{\mathrm{T}} \tag{9}$$

$$\frac{\partial}{\partial \hat{\boldsymbol{x}}_{-N,k}} \varphi_{-N,k} = -\frac{\partial}{\partial \hat{\boldsymbol{x}}_{-N,k}} H_{-N,k} \tag{10}$$

where, $\boldsymbol{\lambda}_{\tau,k}$ is a costate vector the order of which is the same as one of the state vector $\hat{\boldsymbol{x}}_{\tau,k}$.

**(Proof)** Incorporating the constraints of difference equations Eqn. (4) in terms of $\hat{\boldsymbol{x}}_{\tau,k}$ into the performance index $J_k$ in Eqn. (3) using costate $\boldsymbol{\lambda}_{\tau,k}$ we get the following equation.

$$\begin{aligned} J_k^* = {} & \eta_{0,k} + \varphi_{-N,k} \\ & + \sum_{\tau=-N}^{-1} \left\{ L_{\tau,k} + \boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}} \left( \boldsymbol{f}_{\tau,k} - \hat{\boldsymbol{x}}_{\tau,k} - \Delta\hat{\boldsymbol{x}}_{\tau,k} \right) \right\} \end{aligned} \tag{11}$$

where $\Delta\hat{\boldsymbol{x}}_{\tau,k}$ is increments of $\hat{\boldsymbol{x}}_{\tau,k}$, which is defined as

$$\Delta\hat{\boldsymbol{x}}_{\tau,k} = \hat{\boldsymbol{x}}_{\tau+1,k} - \hat{\boldsymbol{x}}_{\tau,k} \tag{12}$$

Hence, when $\hat{\boldsymbol{x}}_{\tau,k}$ is satisfied with the difference equation Eqn. (4), $J_k^*$ corresponds to $J_k$.

Now, consider the first variation $\delta J_k^*$ of $J_k^*$. Since the independent variables of $J_k^*$ are $\hat{\boldsymbol{x}}_{\tau,k}$, $\Delta\hat{\boldsymbol{x}}_{\tau,k}$, $\hat{\boldsymbol{d}}_{\tau,k}$ and $\boldsymbol{\lambda}_{\tau+1,k}$, the first variation $\delta J_k^*$ becomes

$$\begin{aligned}\delta J_k^* = &\frac{\partial}{\partial \hat{\boldsymbol{x}}_{0,k}}\eta_{0,k}\delta\hat{\boldsymbol{x}}_{0,k} + \frac{\partial}{\partial \hat{\boldsymbol{x}}_{-N,k}}\varphi_{-N,k}\delta\hat{\boldsymbol{x}}_{-N,k}\\ &+\sum_{\tau=-N}^{-1}\left\{\left[\frac{\partial}{\partial \hat{\boldsymbol{x}}_{\tau,k}}H_{\tau,k} - \boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\right]\delta\hat{\boldsymbol{x}}_{\tau,k}\right.\\ &+\frac{\partial}{\partial \hat{\boldsymbol{d}}_{\tau,k}}H_{\tau,k}\delta\hat{\boldsymbol{d}}_{\tau,k}\\ &\left.-\boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\delta\Delta\hat{\boldsymbol{x}}_{\tau,k} + \left(\boldsymbol{f}_{\tau,k} - \hat{\boldsymbol{x}}_{\tau+1,k}\right)^{\mathrm{T}}\delta\boldsymbol{\lambda}_{\tau+1,k}\right\}\end{aligned} \tag{13}$$

Noting that the following equation is derived from Eqn. (12),

$$\begin{aligned}\delta\Delta\hat{\boldsymbol{x}}_{\tau,k} &= \delta\left(\hat{\boldsymbol{x}}_{\tau+1,k} - \hat{\boldsymbol{x}}_{\tau,k}\right)\\ &= \delta\hat{\boldsymbol{x}}_{\tau+1,k} - \delta\hat{\boldsymbol{x}}_{\tau,k}\end{aligned} \tag{14}$$

we can get

$$\begin{aligned}&-\sum_{\tau=-N}^{-1}\boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\delta\Delta\hat{\boldsymbol{x}}_{\tau,k}\\ &= -\sum_{\tau=-N}^{-1}\boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\left(\delta\hat{\boldsymbol{x}}_{\tau+1,k} - \delta\hat{\boldsymbol{x}}_{\tau,k}\right)\\ &= -\sum_{\tau=-N}^{-1}\boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\delta\hat{\boldsymbol{x}}_{\tau+1,k} + \sum_{\tau=-N}^{-1}\boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\delta\hat{\boldsymbol{x}}_{\tau,k}\end{aligned} \tag{15}$$

Replacing $\tau$ with $\tau' = \tau + 1$ in the first term of the righthand side of the above equation, it follows that

$$-\sum_{\tau=-N}^{-1}\boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\delta\hat{\boldsymbol{x}}_{\tau+1,k} = -\sum_{\tau'=-N+1}^{0}\boldsymbol{\lambda}_{\tau',k}^{\mathrm{T}}\delta\hat{\boldsymbol{x}}_{\tau',k} \tag{16}$$

Hence, we can get

$$\begin{aligned}&-\sum_{\tau=-N}^{-1}\boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\delta\Delta\hat{\boldsymbol{x}}_{\tau,k}\\ &= \sum_{\tau=-N+1}^{-1}\left(\boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}} - \boldsymbol{\lambda}_{\tau,k}^{\mathrm{T}}\right)\delta\hat{\boldsymbol{x}}_{\tau,k}\\ &\quad -\boldsymbol{\lambda}_{0,k}^{\mathrm{T}}\delta\hat{\boldsymbol{x}}_{0,k} + \boldsymbol{\lambda}_{-N+1,k}^{\mathrm{T}}\delta\hat{\boldsymbol{x}}_{-N,k}\end{aligned} \tag{17}$$

In addition, noting that the third term of the righthand side of Eqn. (13) can be rewritten into the following equation

$$\begin{aligned}&\sum_{\tau=-N}^{-1}\left\{\left[\frac{\partial}{\partial \hat{\boldsymbol{x}}_{\tau,k}}H_{\tau,k} - \boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\right]\delta\hat{\boldsymbol{x}}_{\tau,k}\right\}\\ &= \sum_{\tau=-N+1}^{-1}\left\{\left(\frac{\partial}{\partial \hat{\boldsymbol{x}}_{\tau,k}}H_{\tau,k} - \boldsymbol{\lambda}_{\tau+1,k}^{\mathrm{T}}\right)\delta\hat{\boldsymbol{x}}_{\tau,k}\right\}\\ &\quad +\left(\frac{\partial}{\partial \hat{\boldsymbol{x}}_{-N,k}}H_{-N,k} - \boldsymbol{\lambda}_{-N+1,k}^{\mathrm{T}}\right)\delta\hat{\boldsymbol{x}}_{-N,k}\end{aligned} \tag{18}$$

$\delta J_k^*$ in Eqn. (13) becomes

$$\begin{aligned}\delta J_k^* = &\left(\frac{\partial}{\partial \hat{\boldsymbol{x}}_{0,k}}\eta_{0,k} - \boldsymbol{\lambda}_{0,k}^{\mathrm{T}}\right)\delta\hat{\boldsymbol{x}}_{0,k}\\ &+\frac{\partial}{\partial \hat{\boldsymbol{x}}_{-N,k}}\left(H_{-N,k} + \varphi_{-N,k}\right)\delta\hat{\boldsymbol{x}}_{-N,k}\\ &+\sum_{\tau=-N+1}^{-1}\left\{\left(\frac{\partial}{\partial \hat{\boldsymbol{x}}_{\tau,k}}H_{\tau,k} - \boldsymbol{\lambda}_{\tau,k}^{\mathrm{T}}\right)\delta\hat{\boldsymbol{x}}_{\tau,k}\right\}\\ &+\sum_{\tau=-N}^{-1}\left\{\frac{\partial}{\partial \hat{\boldsymbol{d}}_{\tau,k}}H_{\tau,k}\delta\hat{\boldsymbol{d}}_{\tau,k}\right\}\\ &+\sum_{\tau=-N}^{-1}\left\{\left(\boldsymbol{f}_{\tau,k} - \hat{\boldsymbol{x}}_{\tau+1,k}\right)\delta\boldsymbol{\lambda}_{\tau+1,k}\right\}\end{aligned} \tag{19}$$

Since the first variation should be zero from the necessary condtion of optimality, the conclusions of the theorem can be introduced. □

From the theorem 1, we can see that the optimal condition of estimated disturbances and states for the performance index Eqn. (3) is given by two-point boundary value problems of difference equation

### *2.2 Case of a Linear System with Unknown Process Gain*

Consider a single-input, single-output, discrete-time, linear system with unknown process gain given in the next state-space equations.

$$\boldsymbol{x}[k+1] = \boldsymbol{A}\boldsymbol{x}[k] + \theta\boldsymbol{B}u[k] + \boldsymbol{E}d[k] \tag{20}$$

$$y[k] = \boldsymbol{C}\boldsymbol{x}[k] \tag{21}$$

where $\boldsymbol{x}[\cdot] \in \boldsymbol{R}^n$, $u[\cdot] \in \boldsymbol{R}^1$ $y[\cdot] \in \boldsymbol{R}^1$ and $d[\cdot] \in \boldsymbol{R}^1$ are state vectors, input signals, output signals and disturbances, respectivey. $\theta \in \boldsymbol{R}^1$ is an unknown process gain. The design method is given for the time-invariant systems $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C})$ for simplicity of discussion. However, it can be straightforwardly extended to the time-variant case $(\boldsymbol{A}[k], \boldsymbol{B}[k], \boldsymbol{C}[k])$.

We regard unkown process gain $\hat{\theta}[k]$ as a state variable the initial value of which is unknown, and define the extended state vector in the following way.

$$\hat{\boldsymbol{x}}_e[k] = \begin{bmatrix}\hat{\boldsymbol{x}}[k]\\ \hat{\theta}[k]\end{bmatrix} \tag{22}$$

In addtion, let the estimated disturbances be denoted as $\hat{d}[k]$. Then, we can give the state estimation model in the following way.

$$\hat{\boldsymbol{x}}_e[k+1] = \boldsymbol{A}_e[k]\hat{\boldsymbol{x}}_e[k] + \boldsymbol{B}_e\hat{d}[k] \tag{23}$$
$$\hat{y}[k] = \boldsymbol{C}_e\hat{\boldsymbol{x}}_e[k] \tag{24}$$

where

$$\boldsymbol{A}_e[k] \triangleq \begin{bmatrix} \boldsymbol{A}[k] & \boldsymbol{B}u[k] \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}, \quad \boldsymbol{B}_e \triangleq \begin{bmatrix} \boldsymbol{E} \\ 0 \end{bmatrix},$$
$$\boldsymbol{C}_e \triangleq \begin{bmatrix} \boldsymbol{C} & 0 \end{bmatrix}$$

Let the estimated error $\boldsymbol{\varepsilon}[\cdot]$ be defined as,

$$\boldsymbol{\varepsilon}[\cdot] = \hat{y}[\cdot] - y[\cdot] \tag{25}$$

and the performance index be defined as

$$\begin{aligned} J_k = {} & \boldsymbol{\varepsilon}[k-N]^{\mathrm{T}}\boldsymbol{Q}_s\boldsymbol{\varepsilon}[k-N] \\ & + \boldsymbol{\varepsilon}[k]^{\mathrm{T}}\boldsymbol{Q}_f\boldsymbol{\varepsilon}[k] \\ & + \sum_{\tau=k-N}^{k-1}\left\{\boldsymbol{\varepsilon}[\tau]^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\varepsilon}[\tau] + \hat{d}[\tau]^{\mathrm{T}}\boldsymbol{R}\hat{d}[\tau]\right\} \end{aligned} \tag{26}$$

where $\boldsymbol{Q}_s$,$\boldsymbol{Q}_f$,$\boldsymbol{Q}$ is positive semi-definite matrix, and $\boldsymbol{R}$ is a positive parameter. The performance index consists of the sum of the quadratic form of estimation errors and estimated disturbance values from the present time $k$ to the $N$-steps past time $k-N$.

Then the discrete Euler-Lagrange equation which give a necessary condition of optimal estimation of disturbances and state vector can be derived in the following two-point boundary-value (TPBV) recursion formulas.

$$\begin{aligned} \boldsymbol{\lambda}[\tau] &= \boldsymbol{A}_e[\tau]^{\mathrm{T}}\boldsymbol{\lambda}[\tau+1] + 2\boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}_e\hat{\boldsymbol{x}}_e[\tau] \\ &\quad -2\boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}y[\tau], \qquad (27) \\ &\quad \tau = k-1, k-2, \cdots, k-N+1 \\ \hat{\boldsymbol{x}}_e[\tau+1] &= \boldsymbol{A}_e[\tau]\hat{\boldsymbol{x}}_e[\tau] + \boldsymbol{B}_e\hat{d}[\tau], \qquad (28) \\ &\quad \tau = k-N, k-N+1, \cdots, k-1 \\ \hat{d}[\tau] &= -\frac{1}{2}\boldsymbol{R}^{-1}\boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{\lambda}[\tau+1], \qquad (29) \\ &\quad \tau = k-N, k-N+1, \cdots, k-1 \\ \boldsymbol{\lambda}[k] &= 2\boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}_f\boldsymbol{C}_e\hat{\boldsymbol{x}}_e[k] - 2\boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}_f y[k] \qquad (30) \\ \boldsymbol{0} &= \boldsymbol{C}_e^{\mathrm{T}}(\boldsymbol{Q}_s+\boldsymbol{Q})y[k-N] \\ &\quad -\boldsymbol{C}_e^{\mathrm{T}}(\boldsymbol{Q}_s+\boldsymbol{Q})\boldsymbol{C}_e\hat{\boldsymbol{x}}_e[k-N] \\ &\quad -\frac{1}{2}\boldsymbol{A}_e[k-N]^{\mathrm{T}}\boldsymbol{\lambda}[k-N+1] \qquad (31) \end{aligned}$$

The next proposition gives a simultaneous disturbance and state vector estimation algorithms Ricatti type recursion formulas using the optimal necessary conditions Eqn. (27) $\sim$ Eqn. (31).

*Proposition 2.* Let the costate $\boldsymbol{\lambda}[\tau]$ be defined as

$$\boldsymbol{\lambda}[\tau] = 2\boldsymbol{P}[\tau]\hat{\boldsymbol{x}}_e[\tau] + 2\alpha[\tau] \tag{32}$$

Then, a simultaneous disturbance and state vector estimation algorithms is given in the following steps.

(1) Let the terminal condition be defined as

$$\boldsymbol{P}[k] = \boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}_f\boldsymbol{C}_e \tag{33}$$
$$\alpha[k] = -\boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}_f y[k] \tag{34}$$

Then, solve the following difference equation backward in the manner of $\tau = k-1, k-2, \cdots k-N$ to get $\boldsymbol{P}[k-1], \boldsymbol{P}[k-2], \cdots, \boldsymbol{P}[k-N]$, $\alpha[k-1], \alpha[k-2], \cdots, \alpha[k-N]$,

$$\begin{aligned} \boldsymbol{P}[\tau] &= \boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}_e + \boldsymbol{A}_e[\tau]^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{A}_e[\tau] \\ &\quad -\boldsymbol{A}_e[\tau]^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{B}_e \\ &\quad \times\left(\boldsymbol{R} + \boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{B}_e\right)^{-1} \\ &\quad \times\boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{A}_e[\tau] \qquad (35) \\ \alpha[\tau] &= \boldsymbol{A}_e[\tau]^{\mathrm{T}}\alpha[\tau+1] \\ &\quad -\boldsymbol{A}_e^{\mathrm{T}}[\tau]\boldsymbol{P}[\tau+1]\boldsymbol{B}_e \\ &\quad \times\left(\boldsymbol{R} + \boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{B}_e\right)^{-1} \\ &\quad \times\boldsymbol{B}_e^{\mathrm{T}}\alpha[\tau+1] - \boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}y[\tau] \qquad (36) \end{aligned}$$

(2) Using $\boldsymbol{P}[k-N]$,$\alpha[k-N]$, Solve the following linear equation to get $\hat{\boldsymbol{x}}_e[k-N]$.

$$\begin{aligned} &\left(\boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}_s\boldsymbol{C}_e + \boldsymbol{P}[k-N]\right)\hat{\boldsymbol{x}}_e[k-N] \\ &\quad = \boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}_s y[k-N] - \alpha[k-N] \end{aligned} \tag{37}$$

(3) Calculate the estimated disturbance and state vectors using the following equation and state equation Eqn. (28)

$$\begin{aligned} \hat{d}[\tau] &= -(\boldsymbol{R} + \boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{B}_e)^{-1} \\ &\quad \times\boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{A}_e[\tau]\hat{\boldsymbol{x}}_e[\tau] \\ &\quad -(\boldsymbol{R} + \boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{B}_e)^{-1}\boldsymbol{B}_e^{\mathrm{T}}\alpha[\tau+1] \end{aligned} \tag{38}$$

to get

$$\begin{aligned} &\hat{\boldsymbol{x}}_e[\tau], \quad \tau = k-N, k-N+1, \cdots, k, \\ &\hat{\boldsymbol{d}}[\tau], \quad \tau = k-N, k-N+1, \cdots, k-1 \end{aligned}$$

**(Proof)** The conditions Eqn. (30) can be straightforwardly derived from Eqn. (32) when $\boldsymbol{P}[k]$ and $\alpha[k]$ are Eqn. (33) and Eqn. (34), respectively.

From Eqn. (28), Eqn. (29) and Eqn. (32), it follows that

$$\begin{aligned} \hat{d}[\tau] &= -\boldsymbol{R}^{-1}\boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\hat{\boldsymbol{x}}_e[\tau+1] - \boldsymbol{R}^{-1}\boldsymbol{B}_e^{\mathrm{T}}\alpha[\tau+1] \\ &= -\boldsymbol{R}^{-1}\boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{A}_e[\tau]\hat{\boldsymbol{x}}_e[\tau] \\ &\quad -\boldsymbol{R}^{-1}\boldsymbol{B}_e^{\mathrm{T}}\boldsymbol{P}[\tau+1]\boldsymbol{B}_e\hat{d}[\tau] \\ &\quad -\boldsymbol{R}^{-1}\boldsymbol{B}_e^{\mathrm{T}}\alpha[\tau+1] \end{aligned} \tag{39}$$

which leads to estimated disturbances Eqn. (38).

Applying Eqn. (32) to Eqn. (27), we can get

$$\begin{aligned} &2\boldsymbol{P}[\tau]\hat{\boldsymbol{x}}_e[\tau] + 2\alpha[\tau] \\ &= \boldsymbol{A}_e[\tau]^{\mathrm{T}} \left(2\boldsymbol{P}[\tau+1]\hat{\boldsymbol{x}}_e[\tau+1] + 2\alpha[\tau+1]\right) \\ &\quad +2\boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{C}_e\hat{\boldsymbol{x}}_e[\tau] - 2\boldsymbol{C}_e^{\mathrm{T}}\boldsymbol{Q}y[\tau] \end{aligned} \tag{40}$$

Then, we can see that the optimal condition Eqn. (27) is satisfied from Eqn. (35) and Eqn. (36) by using state equations Eqn. (28) and estimated disturbances Eqn. (38), and rearranging in terms of $\boldsymbol{P}[\tau]$ and $\alpha[\tau]$.

Finally, we can get Eqn. (37) using Eqn. (35) and Eqn. (36) at $\tau = k - N$ after applying Eqn. (31) to Eqn. (32) at $\tau = k - N + 1$ and Eqn. (28) and Eqn. (38) at $\tau = k - N$ and rearranging in terms of $\hat{\boldsymbol{x}}_e[k-N]$. □

## 3. AN APPLICATION TO OXYGEN CONVERTER GAS RECOVERY PROCESS MODEL

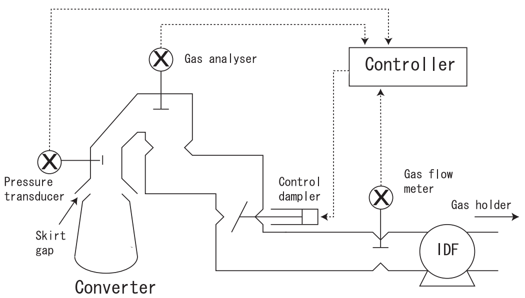


Fig. 1. Oxygen Converter Gas Recovery Process

In the Oxygen Converter Gas Recovery (OG) Processes depicted in Fig. 1 the pressure control in the converter is required to be a prescribed value using a control damper as manipulated variables in order to improve the efficiency of gas recovery process. However, the generated gas volume, which is regarded as disturbances, is unknown, and the process gain from the control damper open to the pressure in the converter greately varies. Hence, the process gain should be unknown parameter to be estimated.

### *3.1 Oxygen Converter Gas Recovery Process Model*

An approximated model of OG Process model is given by the following equations (Yoshida *et al.*, 1988).

$$P_0(s) = \frac{1}{1+T_P s}\left(\frac{K_P}{1+T_D s}e^{-L_D s}U(s) + K_P K_G f_0(s)\right) \tag{41}$$

where $T_P$ is the pressure transducer time constant, $T_D$ is the control damper time constant, $L_D$ is the time lag of the control damper, $K_P$ is the process gain, and $K_G$ is the gain from the generated gas flow to the pressure in the converter. The manipulated variable $u(\cdot)$ is the damper open, and the controlled variable $P_0(\cdot)$ is the pressure in the converter. $f_0(\cdot)$ is the generated gas flow which works as disturbance which causes deviation from the regulated set value of the pressure in the converter. This paper assumes that the parameters $T_P, T_D, L_D$ are known a priori, but $K_P, K_G$ are unknown parameters, and the disturbance $f_0(s)$ is unknown. This section considers the problem to estimate the process gain $K_P$ and the disturbance $K_G f_0(s)$ simultaneously in the on-line manner from the time series data of pressure in the converter and the control damper open which are the input-output signals.

Let $x_1(t)$ and $x_2(t)$ be defined as

$$\mathcal{L}[x_1(t)] = \frac{K_P}{1+T_D s}e^{-L_D s}U(s) \tag{42}$$

$$\mathcal{L}[x_2(t)] = \frac{1}{1+T_P s}\left(x_1(s) + K_P K_G f_0(s)\right) \tag{43}$$

then, the state realization of the process Eqn. (42) is given by

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} &= \boldsymbol{A}_c \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + K_p \boldsymbol{B}_c u(t - L_D) \\ &\quad + \boldsymbol{E}_c K_P K_G f_0(t) \end{aligned} \tag{44}$$

$$y(t) = \boldsymbol{C}_c \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{45}$$

where

$$\boldsymbol{A}_c \triangleq \begin{bmatrix} -\dfrac{1}{T_D} & 0 \\ \dfrac{1}{T_P} & -\dfrac{1}{T_P} \end{bmatrix}, \quad \boldsymbol{B}_c \triangleq \begin{bmatrix} \dfrac{1}{T_D} \\ 0 \end{bmatrix},$$

$$\boldsymbol{E}_c \triangleq \begin{bmatrix} 0 \\ \dfrac{1}{T_P} \end{bmatrix}, \quad \boldsymbol{C}_c \triangleq \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The next, let the equivalent discrete-time model be derived using zero order hold, where sampling time is $T_s$.

$$\begin{aligned} \begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} &= \boldsymbol{A}_d \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + K_p \boldsymbol{B}_d u[k - l_d] \\ &\quad + \boldsymbol{E}_d d[k] \end{aligned} \tag{46}$$

$$y[k] = \boldsymbol{C}_d \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} \tag{47}$$

where

$$\boldsymbol{A}_d \triangleq e^{\boldsymbol{A}_c T_s}, \quad \boldsymbol{B}_d \triangleq \int_0^{T_s} e^{\boldsymbol{A}_c \tau}\boldsymbol{B}_c d\tau,$$

$$\boldsymbol{E}_d \triangleq \int_0^{T_s} e^{\boldsymbol{A}_c \tau}\boldsymbol{E}_c d\tau, \quad \boldsymbol{C}_d \triangleq \boldsymbol{C}_c$$

$$L_D \triangleq l_d \cdot T_s, \quad d[k] \triangleq K_P K_G f_0[k]$$

Then, replace the unknown process gain $K_P$ by unknown parameter $\theta$, and define the extended state vector in which the estimated parameter $\hat{\theta}$ is incorporated as Eqn. (22), and define the extended system matrices as

$$\boldsymbol{A}_e[k] \triangleq \begin{bmatrix} \boldsymbol{A}_d & \boldsymbol{B}_d u[k-l_d] \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}, \quad \boldsymbol{B}_e \triangleq \begin{bmatrix} \boldsymbol{E}_d \\ 0 \end{bmatrix},$$
$$\boldsymbol{C}_e \triangleq \begin{bmatrix} \boldsymbol{C}_d & 0 \end{bmatrix}$$

we can formulate the Eqn. (24) from the OG process model Eqn. (46) and Eqn. (47). Hence, we can apply the proposition 2 to the OG process model, and estimate both disturbance and process gain simultaneously.

### 3.2 A Simulation Result

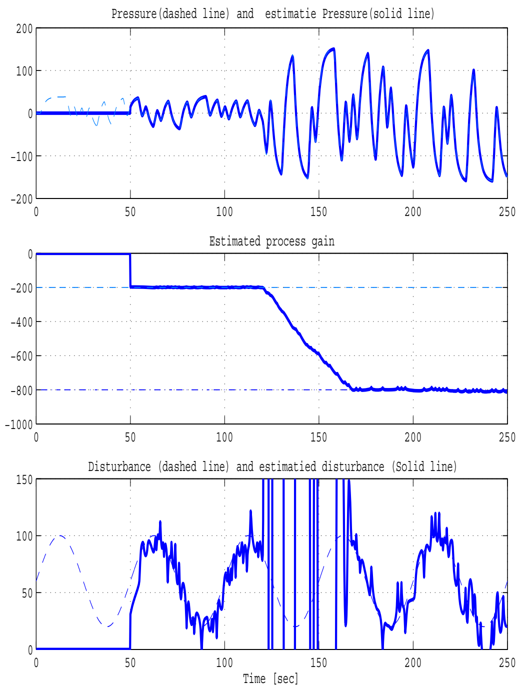


Fig. 2. Simulation result of moving horizon optimal estimation of pressure, process gain and disturbance

In a numerical simulation, we set the parameters $T_D = 0.5$, $T_P = 2.0$, $L_D = 2.0$ $K_g = 1.0 \times 10^{-4}$ in Eqn. (46) and Eqn. (47), and the sampling time $T_s = 0.2[sec]$.

It is assumed that the process gain keeps $K_P = -200$ until the time $120[sec]$, then changes into $K_P = -800$. The disturbance $d(t)$ is assumed to be the following equation.

$$d(t) = 40K_pK_G \sin(0.04\pi t) + 60K_pK_G \tag{48}$$

The manipulated variable, which is the control damper open, is a PRBS signal with range $[-0.2, 0.2]$. The parameters of the performance index are set to be the evaluation range $N = 250$[step] (50[sec]), and the weighting matrices $\boldsymbol{Q} = \boldsymbol{Q}_s = \boldsymbol{Q}_f = 100$, $\boldsymbol{R} = 1$.

Fig. 2 shows that simulation results of moving horizon optimal estimation of pressure, process gain and disturbance for an approximated second order OG process model. From the results, it follows that a process gain can be estimated very well even when it changes from $K_P = -200$ to $K_P = -800$ at the time $120[sec]$, and disturbance can also be estimated almost well although large oscillation of the estimated disturbance signal occurs from the time at which the process gain changes to the time at which the estimation of the process gain succeed to follow the new value $K_P = -800$. From a numerical simulation, we can see that the proposed method works well for the approximated second order OG process model with the large disturbance.

## 4. CONCLUDING REMARKS

This paper newly proposed a disrete-time moving-horizon adaptive disturbance estimator to estimate both the process gain and distrubances simultaneously so as to minimize a moving horizon performance index. In addition, a numerical simulation for the approximated second order OG process model of an oxygen converter gas recovery process was done, and we made assure that the proposed method worked well for the given model through a numerical simulation.

Application of the proposed method to the input-output data from a real process, and the control system design using estimated disturbance and the process gain remain for future works.

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