Conflict Detection and Resolution in Air Traffic Control ⋆

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Abstract: The work deals with application of the optimal control methods to problem of conflict detection and resolution in air traffic control. Motion of two conflicting aircrafts in the horizontal plane is described by the standard systems of ordinary differential equations of the third order (two geometrical coordinates and the heading). Information about parameters of the aircrafts motion is assumed to be known exactly. The conflict situation between aircrafts is detected on the basis of forecast of their motion, and parameters of the conflict are defined. By this information one aircraft is chosen for the resolving manoeuver, the second one does not participate in the resolution. Parameters of the resolving manoeuver and its elements are computed accordingly to the normative and regulation rules for air traffic control. Necessary and sufficient conditions for existence of the resolving manoeuver have been found in the constructive form.

Keywords: Aircraft, conflict situation, collision, control, resolution, algorithms, software.

1. INTRODUCTION AND PROBLEM FORMULATION

Problem of creating practically meaning schemes and manoeuvres for conflict resolution between a pair of aircrafts is extremely important for providing safety in air traffic control.

Consideration of existing formulations of mentioned problems shows that analysis of conflict situation and manoeuvres recommended Anodina et al. (1992), Lipin and Olyanyuk (1999) are rather approximate by both the multi criteria character of the problem and technical reasons. Attempts for strict mathematical formulations of the problem (see, for example, Tomlin et al. (1998), Tomlin et al. (1997) and further works of the authors) can lead to complicated solutions and even to serious changing in the ideology of the air space.

For guaranteed conflict resolution such algorithms and procedures are necessary that would directly take into account demands of the normative documents and regulation rules Belkin et al. (1988), Korolev (2000) and constructively (by their computations) provide the safe distance between conflicting aircrafts.

In the paper, controlled motion of two conflicting aircraft is considered. The motion occurs in the horizontal plane, and initially each aircraft moves along the axis (trace) of its air airway. Dynamics of the motion is described by the standardized (for navigational computations) system of ordinary differential equations (two geometric coordinates and the heading). The values of aircraft velocities are constant and assumed to be known.

The conflict situation between aircrafts is detected on the basis of forecast of their motion, and parameters of the conflict are defined.

The scheme and parameters of resolving manoeuver are computed. The manoeuver by its construction provides guaranteed safe distance (determined by the regulation rules) between aircrafts for all time interval of their approach and by–pass. Moreover, there are crucial additional technological demands:
– conflict resolution must be realized by manoeuver of only one aircraft in the conflicting pair;
– its "S-wise" structure is prescribed in advance; by normative and regulation rules it must consist of the part for active "avoidance" manoeuver providing the safe distance, the part for safe delaying during by–passing the aircrafts (it is usually has he prescribed value and here the motion has the constant direction of velocity), and the part for manoeuver of return onto the trace of the initial airway;
– the manoeuver in the whole must have the minimal lateral deviation of the manoeuvering aircraft from the axis (trace) of its airway;
– the whole time length of the manoeuver must be minimal, and, as a result, avoidance and return parts are to be performed for the minimal time accordingly to the maximal value of admissible control, and as a consequence, the point of beginning of the avoidance part must not be "too far" from the zone of conflict; – the periodic analysis of possible conflict is to begin in advance, i.e., at such initial aircrafts positions, from which the conflict resolution can be performed with reserve, i.e., at sufficient initial distance and under time reserve.

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Problem formulation. *Having the given description of the aircraft dynamics and mentioned formalized technological demands, the problem can be formulated as follows: for the prescribed structure of the manoeuver, it is necessary to choose one manoeuvering aircraft, to construct the aircraft control that for the minimal time provides the minimal distance between aircrafts not smaller than the safe one under minimal deviation of the manoeuvering aircraft from its trace and with the minimal time of return onto the trace.*

Apparently, such a “formulation” is too far from a strict mathematical one. So, using engineering reasonings, the problem is solved by the following sequence of procedures.

1. The forecast procedure is performed, and parameters of the minimal approach are found (for defining the parameters of the closest approach and decision making on the presence of the conflict).

2. Procedure of classification of the type of approach is made for concretization of the conflict type in dependence on parameters of the detected conflict and for choice necessary computational formulas. (Remind that in the case of detection of the conflict between two aircrafts, one can suggest four possible schemes of the manoeuver: either the aircraft number 1 performs the right or the left turns, or the aircraft number 2 performs the right or the left turns.)

3. For each possible scheme, realizability of corresponding avoidance manoeuver is analyzed, and a collection of realizable ones is formed.

4. By this information (and, in practice, by some additional engineering optimality demands or reasonings) the aircraft and corresponding scheme of its whole manoeuver are chosen and suggested to the operator for realization in the air trafiic control system.

2. INITIAL DATA

Controlled motion of each aircraft in the horizontal plane is described by the following standardized system of ordinary differential equations:

$$\begin{aligned} \dot{x} &= V\mathrm{Cos}\psi, \\ \dot{z} &= V\mathrm{Sin}\psi, \\ \dot{\psi} &= \beta/V, \quad |\beta(t)| \le \beta_{\max}. \end{aligned} \tag{1}$$

Here, x is the ordinate of aircraft position along the OX–axis (directed to the North); z is the abscissa of aircraft position along the OZ–axis (directed to the East); ψ is the heading (the angle is counted clock-wise from the OX–axis); β is the aircraft control, the lateral acceleration with geometrical restriction $\beta_{\max}$ (i.e., by modulus); V is the aircraft velocity. For each aircraft the values $\beta_{1,\max}$, $\beta_{2,\max}$, V_1, and V_2 are known, and velocities are assumed to be constant.

Accordingly to normative and regulation rules, the value of the admissible safe distance R_{s} is given.

For simplicity (and unification) of description the following conditional standardized coordinate system is used. It is (conditionally) assumed that the aircraft with *lower* velocity always has the number “1”, moves to the North

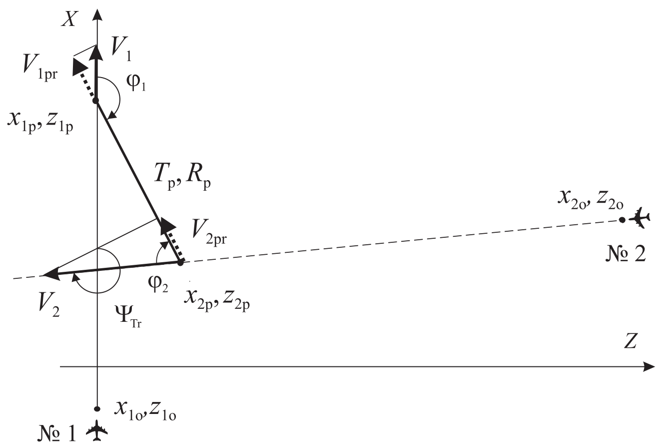

Fig. 1. Approach of aircrafts and possible conflict

and the direction of the axis of its airway always $\Psi_1 = 0$. The aircraft with larger velocity is always numerated as “2”, and the trace of its air way crosses the trace of air way of the first aircraft with the direction $\Psi_2 = \Psi_{\mathrm{Tr}}$ (Fig.1), and this angle assumed to be the “trace angle”. There also, the aircrafts initial positions $x_{1\mathrm{o}}$, $z_{1\mathrm{o}}$ and $x_{2\mathrm{o}}$, $z_{2\mathrm{o}}$ are shown. At the conflict instant T_{p}, the following parameters of the conflict are marked: their positions $x_{1\mathrm{p}}$,$z_{1\mathrm{p}}$ and $x_{2\mathrm{p}}$, $z_{2\mathrm{p}}$, the thick line marks the vector R_{p} of the relative distances, the angles φ_1 and φ_2 of this vector with respect to the aircraft traces. The length of the solid arrows shows the values of the aircraft velocities V_1 and V_2. The vectors $V_{1\mathrm{pr}}$, $V_{2\mathrm{pr}}$ (in dots) show the projections of velocities onto the line “1 – 2”. Remind that at the instant of the conflict $V_{1\mathrm{pr}} - V_{2\mathrm{pr}} = 0$.

3. FORECAST OF CONFLICT

For the accepted model of uniform and linear motion of aircrafts the instant T_{p} of possible conflict coincides with the instant of zero of the derivative $\dot{R} = 0$ of their relative distance R. Its computation is performed by the trivial formula

$$\begin{aligned} T_{\mathrm{p}} = -[(x_{2o} - x_{1o})(V_2\mathrm{Cos}\Psi_{\mathrm{Tr}} - V_1) + z_{2o}V_2\mathrm{Sin}\Psi_{\mathrm{Tr}}]/ \\ [(V_2\mathrm{Cos}\Psi_{\mathrm{Tr}} - V_1)^2 + (V_2\mathrm{Sin}\Psi_{\mathrm{Tr}})^2]. \end{aligned} \tag{2}$$

Parameters of the approach are also computed by trivial formulas:
positions

$$\begin{aligned} x_{1\mathrm{p}} &= x_{1o} + V_1 T_{\mathrm{p}}, \\ z_{1\mathrm{p}} &\equiv 0, \\ x_{2\mathrm{p}} &= x_{2o} + V_2 T_{\mathrm{p}}\mathrm{Cos}\Psi_{\mathrm{Tr}}, \\ z_{2\mathrm{p}} &= z_{2o} + V_2 T_{\mathrm{p}}\mathrm{Sin}\Psi_{\mathrm{Tr}}, \end{aligned} \tag{3}$$

the minimal distance R_{p}

$$R_{\mathrm{p}} = \sqrt{(x_{2\mathrm{p}} - x_{1\mathrm{p}})^2 + (z_{2\mathrm{p}} - z_{1\mathrm{p}})^2}, \tag{4}$$

the auxiliary angles (Fig.1)

$$\varphi_1 = \begin{cases} \pi - \mathrm{Arctg}(z_{2\mathrm{p}}/(x_{1\mathrm{p}} - x_{2\mathrm{p}})), \\ \qquad \text{if } x_{1\mathrm{p}} > x_{2\mathrm{p}}, \quad z_{2\mathrm{p}} > 0, \\ \pi - \mathrm{Arctg}(z_{2\mathrm{p}}/(x_{1\mathrm{p}} - x_{2\mathrm{p}})), \\ \qquad \text{if } x_{1\mathrm{p}} > x_{2\mathrm{p}}, \quad z_{2\mathrm{p}} < 0, \\ \mathrm{Arctg}(z_{2\mathrm{p}}/(x_{2\mathrm{p}} - x_{1\mathrm{p}})), \\ \qquad \text{if } x_{1\mathrm{p}} < x_{2\mathrm{p}}, \quad z_{2\mathrm{p}} > 0, \\ 2\pi + \mathrm{Arctg}(z_{2\mathrm{p}}/(x_{2\mathrm{p}} - x_{1\mathrm{p}})), \\ \qquad \text{if } x_{1\mathrm{p}} < x_{2\mathrm{p}}, \quad z_{2\mathrm{p}} < 0\,, \end{cases} \tag{5}$$

$$\varphi_2 = \begin{cases} \pi - (\Psi_{\mathrm{Tr}} - \varphi_1), & \text{if } 0 < \Psi_{\mathrm{Tr}} < \pi, \\ \pi + \Psi_{\mathrm{Tr}} - \varphi_1, & \text{if } \pi < \Psi_{\mathrm{Tr}} < 2\pi\,, \end{cases} \tag{6}$$

the cross–point of traces

$$\begin{aligned} & x_{\mathrm{Tr}} = x_{2o} - z_{2o}/\mathrm{tg}\Psi_{\mathrm{Tr}}, \\ & z_{\mathrm{Tr}} \equiv 0. \end{aligned} \tag{7}$$

Further, the *main condition* is checked

$$\begin{aligned} & \text{if } R_{\mathrm{p}} \geq R_{\mathrm{s}}, \text{then conflict is absent;} \\ & \text{otherwise, conflict is detected.} \end{aligned} \tag{8}$$

4. CLASSIFICATION OF CONFLICT TYPE

If the conflict has been detected, the computed parameters of the approach allows to classify the conflict type in dependence of the features: relative placing of aircrafts, orientation of the line "1 – 2", which of these aircrafts has the larger velocity, *etc.*

In the whole, there are only 24 essentially differing types. This collection is analyzed in advance and is stored in the computer memory as a standard information together with corresponding mathematical formulas for analysis of realizability of each scheme of possible manoeuver and computation of parameters and results of each manoeuver.

5. EXAMPLE OF CONFLICT AND ITS RESOLUTION

Consider conflict of one simple type and explain on it the essence of the suggested methods and all mentioned procedures for its resolving. The case is presented in Fig.2. The initial positions at t_0 are marked as (x_{1o},z_{1o}) and (x_{2o},z_{2o}). The motion directions are given by velocity vectors (arrows in solid) and for simplicity of description it is given that $V_1 < V_2$. The directions of traces are $\Psi_1 = 0$ and $\Psi_2 = \Psi_{Tr} = 1.5\pi$.

Let after procedure of forecast we detected the conflict, i.e., obtained $R_{\mathrm{p}} < R_{\mathrm{s}}$. This type is uniquely identified by the following conditions: the number 2 with the larger velocity at the conflict instant is behind of the number 1, at the right–hand side with respect to the trace of the aircraft number 1 and crosses its trace from the right to the left. These conditions are formalized in the following elementary formulas:

$$\begin{aligned} & x_{1\mathrm{p}} > x_{2\mathrm{p}}, \\ & z_{2\mathrm{p}} > 0, \\ & \pi < \Psi_{\mathrm{Tr}} < 2\pi. \end{aligned} \tag{9}$$

At the first sight, there are four possible schemes of the manoeuver: the number 1 performs the right and the left turns, and the number 2 performs the right and the left turns.

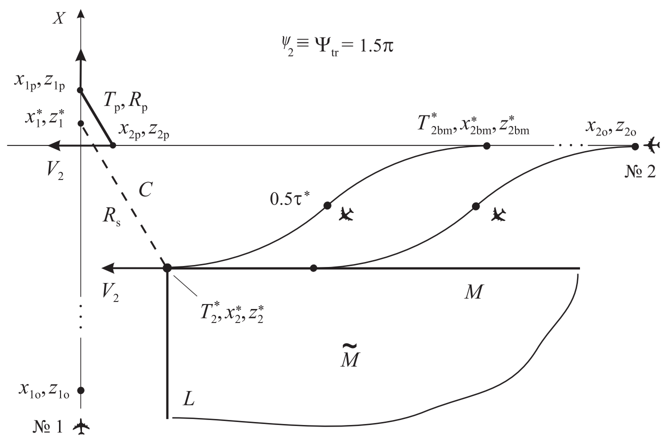

Fig. 2. Conflict and avoidance manoeuver of the minimal length

Consider the left turn manoeuver of aircraft number 2. Under the prescribed structure of the whole manoeuver, the most demands (mentioned in Section 1) are satisfied if the following conditions are satisfied:
– the avoidance manoeuver is implemented for the minimal time on the maximal value of the lateral acceleration with one its switching and equal time length of its halves; – the avoidance manoeuver must end with the previous direction of the velocity V_2; – under the given length of the delay part with motion on the constant direction of velocity, the return manoeuver must inversely repeat the avoidance manoeuver; – it is evident that in the whole, such a manoeuver simultaneously provides the minimal lateral deviation of the manoeuvering aircraft from its initial trace.

If to keep at these ideas, then it is left to choose the point of the beginning the avoidance manoeuver and its time length to provide the safe distance at the instant of the closest approach. So, the initial non–strictly formulated (Section 1) problem is reduced to the following concrete computational problems:

a) is it possible (under the given input data on the aircrafts) to provide the safe distance R_{s}?

b) if it is possible, then how to compute necessary beginning point and the time length of the manoeuver?

Introduce now the following important assumptions and geometrical constructions.

Let us *fix the orientation* of the conflict line "1–2".

Introduce *the auxiliary segment–line* C of length R_{s} with its left end sliding along the trace 1 up– and downward from the point $x_{1\mathrm{p}}$.

The right ends of the line C consist *a special line* L that is parallel to Trace 1.

Now, let begin the standard avoidance maneuvers of the same time length τ from various initial points $x_{2\mathrm{o}}$,$z_{2\mathrm{o}}$ on the Trace 2. Then their ends compose *the special line* M that is parallel to trace 2 and outstanding at the realized lateral deviation ΔB.

Denote by x_2^*,z_2^* the point of intersection of lines M and L, and select a standard manoeuver that comes to this point at the instant T_2^*. If at this instant the aircraft number 1 is at the point x_1^*,z_1^*, then it means that the point x_2^*,z_2^* is one of the closest approach of number 2 to the number 1 after performing the avoidance manoeuver from the beginning point $x_{2\mathrm{bm}}^*$,$z_{2\mathrm{bm}}^*$ at the initial instant $T_{2\mathrm{bm}}^*$, and the safe distance has been provided.

Underline very important properties of the set $\widetilde{M}$ constrained by lines M and L.

1) All trajectories beginning at earlier points (at the right from the point $x_{2\mathrm{bm}}^*$,$z_{2\mathrm{bm}}^*$) and going after the manoeuver along the line M, all get the point x_2^*,z_2^* *at the same instance* T_2^*

2) All trajectories beginning at earlier points (at the right from the point $x_{2\mathrm{bm}}^*$,$z_{2\mathrm{bm}}^*$) and performed under more long ($\tau > \tau^*$) and "deep" manoeuver finish inside the set $\widetilde{M}$ and come at the line L at some instant T_2 that is larger than the instant T_1 at which the number 1 had been at the point x_1,z_1 at the left end of the auxiliary line C. It implies that at the instant T_2 the safe distance will be provided with reserve.

3) All trajectories beginning at the later points (at the left from the point $x_{2\mathrm{bm}}^*$,$z_{2\mathrm{bm}}^*$) and performed under any manoeuver length τ and with any admissible control $\beta(t)$ come onto the continuation of the line L *earlier* the instant T_2^*. This fact implies that such manoeuvres provide the minimal distance smaller than the necessary safe value R_s. It means also that the avoidance manoeuver of the length τ^* beginning from the point $x_{2\mathrm{bm}}^*$,$z_{2\mathrm{bm}}^*$ at the instant $T_{2\mathrm{bm}}^*$ is the left–limit one, which provides the necessary safe distance R_s.

Construct now the main condition for analysis of *realizability* of necessary avoidance manoeuver by the following simple geometric constructions. Suppose the realized lateral deviation ΔB to be a variable parameter. Each its value implies corresponding point $x(\Delta B)$,$z(\Delta B^)$ on the line L, the left end $x_1(\Delta B)$,$z_1(\Delta B^)$ of the auxiliary line C on the trace 1, and the instant $T_1(\Delta B^*)$ of coming the number 1 to this point.

Simultaneously, the value ΔB implies the necessary time length $\tau(\Delta B)$ of the number 2 manoeuver in the back–ward time from the point $x(\Delta B)$,$z(\Delta B^)$ to some beginning point $x_{2\mathrm{bm}}(\Delta B)$,$z_{2\mathrm{bm}}(\Delta B)$. Since by its motion the number 2 can be at this point at the instant $T_{2\mathrm{bm}}(\Delta B)$, the instant of his coming in the direct–time to the point $x(\Delta B)$,$z(\Delta B^)$ will be $T_2(\Delta B) = T_{2\mathrm{bm}}(\Delta B) + \tau(\Delta B)$.

Note (Fig.3) that under increasing the value ΔB from zero the value $T_2(\Delta B)$ monotonically increases being smaller the instant $T_1(\Delta B)$. But in the contrast, the value $T_1 \Delta B)$ monotonically decreases being larger the instant $T_2(\Delta B)$.

Implemented considerations allow one to formulate the main *necessary and sufficient condition* of realizability of the avoidance manoeuver. This is the equality of instances

$$T_2^*(\Delta B^*) = T_1^*(\Delta B^*), \tag{10}$$

of coming the number 2 to the point $x_2^*(\Delta B^*)$, $z_2^*(\Delta B^{ast})$ and, correspondingly, of coming the number 1 to the point $x_1^*(\Delta B^*)$, $z_1^*(\Delta B^*)$ under some minimally admissible value

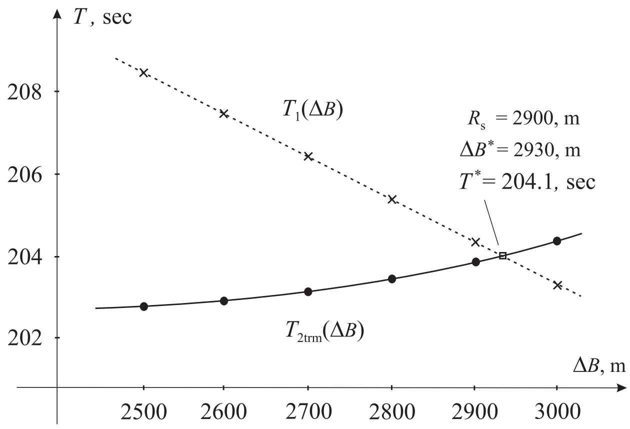

Fig. 3. Numerical example of dependencies $T_1(\Delta B)$ and $T_2(\Delta B)$, and existence of the root ΔB^* for Eq.(8)

ΔB^* of the realized lateral deviation manouevering aurcraft 2.

It means that if Eq.(8) has the root ΔB^*, the necessary avoidance manoeuver exists. Otherwise, such a manoeuver (under the prescribed its structure) can not be built.

Note that condition (8) is *constructive*: during finding its root all necessary parameters of the avoidance manoeuver have been defined.

Eq.(8) is transcendental, but by virtue of monotonicity (Fig.3) of the left– and right–hand functions, it is solved fast numerically. Additionally, there is special algorithm of detecting the case of the root absence.

Similar investigations are carried out for other schemes of the avoidance manoeuver: the right of number 2 and the right and the left turns of the number 1. In general case, some of these manoeuver would be realizable but some are not.

Under the given safe distance R_s, the realizable avoidance manoeuvres differ by their parameters, such as the produced lateral deviation ΔB^*, and the time length τ^*.

By this information (and, in practice, by some additional engineering optimality demands or reasonings) the aircraft and corresponding scheme of its whole manoeuver are chosen and suggested to the operator for realization in the air traffic control system.

Remind that the whole manoeuver (resolving the detected conflict) is composed of further prescribed part safe delaying during by–passing the aircrafts and the part for manoeuver of return onto the initial airway. The last part is the mirror inverse of the avoidance manoeuver. Example of the whole manoeuver resolving the conflict is presented in Fig.4. Here, on the back return manoeuver the characters points and there instants are marked: $T_{2\mathrm{lt}}$, $x_{2\mathrm{lt}}$, $z_{2\mathrm{lt}}$ for the end of the delay by–pass part; $T_{2\mathrm{sw}}$, $x_{2\mathrm{sw}}$, $z_{2\mathrm{sw}}$ for the point of the control switching; $T_{2\mathrm{rt}}$, $x_{2\mathrm{rt}}$,$z_{2\mathrm{rt}}$ for the end point on the trace 2. Figure 5 presents another type of the conflict and its resolution by preferable manoeuver of the first slower aircraft to the right. In practice, there exist possible "degenerated" types of the conflict when it is impossible to calculate reliably parameters of the conflict. It can happen, for example, because of noised information

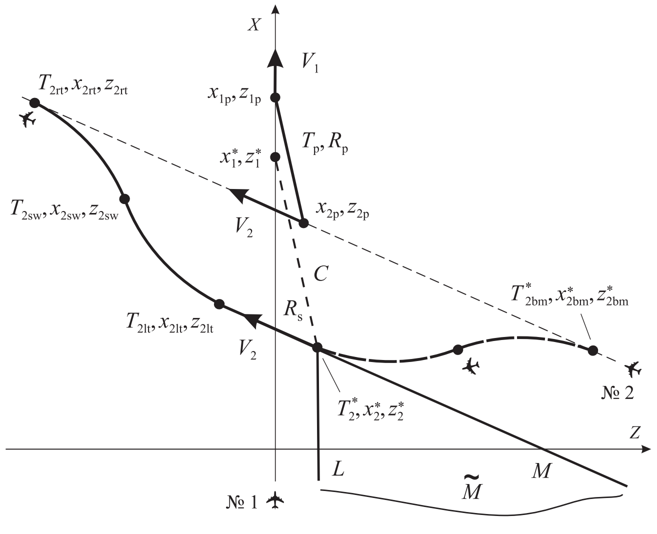

Fig. 4. Example of the whole manoeuver resolving the conflict

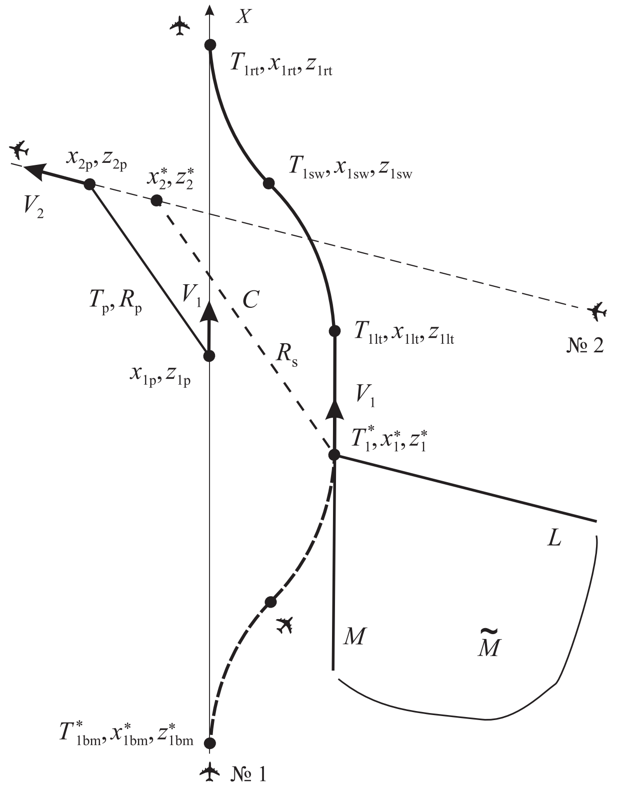

Fig. 5. Example of the conflict type and the resolving manoeuver of the slower aircraft

about the initial positions of aircrafts or by virtue of computational errors. Such a type is presented in Fig.6. Here, the small square marks the set of possible uncertain points of the conflict. Nevertheless, the suggested approach has reliably built the resolving manoeuver.

6. CONCLUSION

Algorithms for conflict detection and resolution have been elaborated. The algorithms take directly into account demands of technological and regulation documents on air traffic control. The algorithms are new, constructive and can be used in perspective air traffic control systems.

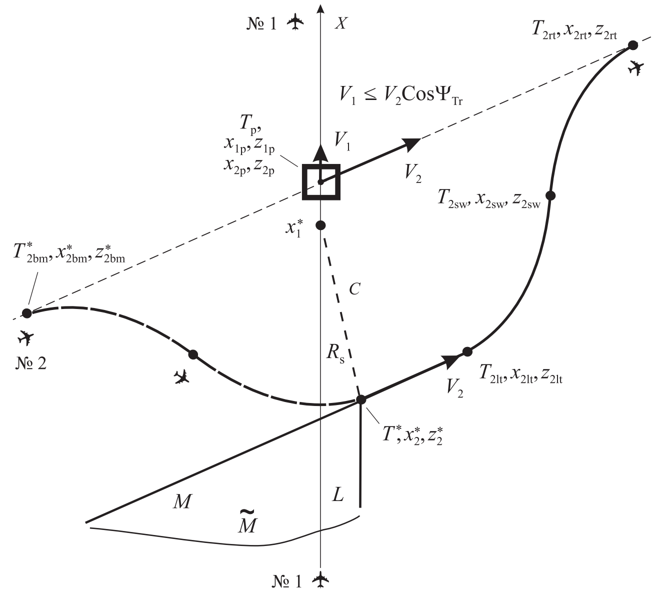

Fig. 6. Degenerated type of the conflict and the resolving manoeuver

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