FIXED-TIME CONTROL PROTOCOL FOR EQUIDISTANT DEPLOYMENT OF SECOND-ORDER MOBILE AGENTS ON A LINE SEGMENT

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Abstract

The problem of the equidistant deployment of a group of mobile agents on a line segment is studied. It is assumed that agent dynamics is modeled by double integrators, and each agent receives information on its velocity and distances to some of its neighbors. New nonlinear nonhomogeneous decentralized control algorithms are proposed providing agent convergence to the prescribed positions in a fixed time for any initial conditions. The theoretical results are illustrated via computer simulations.

Key words

Mobile agent, double integrator, equidistant deployment, fixed-time stability, distributed protocol.

1 Introduction

Formation control is actual and intensively developing topic of research, see, e.g., [Cortes and Egerstedt, 2017; Imangazieva, 2025; Konovalov and Matveev, 2025; Proskurnikov and Fradkov, 2016] and references therein. An interesting and important class of formation control problems is the deployment of mobile agents on a curve or line segment. Various approaches to solution of such problems were proposed in [Wagner and Bruckstein, 1997; Kvinto and Parsegov, 2012; Proskurnikov and Parsegov, 2016; Terushkin and Fridman, 2021; Qu, Xu, Song and Fan, 2020; Aleksandrov, Fradkov and Semenov, 2020; Aleksandrov, Fradkov and Semenov, 2023]. However, it should be noted that the results of the mentioned works are based on the use of linear control protocols that guarantee an exponential convergence rate of agents to the prescribed positions. At the same time, in numerous applications, a predefined time of transient processes is required.

In the paper [Parsegov, Polyakov and Shcherbakov, 2012], for a group of mobile agents modeling by first-order integrators, a nonlinear control protocol was designed that provides an equidistant deployment of agents on the segment in a fixed time for any initial conditions. In [Zimenko, Efimov, Polyakov and Ping, 2023], a decentralized homogeneous control protocol was proposed for the more realistic models of agents in the form of double integrators. With the aid of the Implicit Lyapunov Function method, sufficient conditions in terms of Linear Matrix Inequalities for the finite-time convergence of agents to the uniform distribution were derived. In [Aleksandrov, Efimov and Dashkovskiy, 2023], fixed-time nonlinear nonhomogeneous protocol was constructed for agents of the second order.

The objective of the present contribution is a further development of the approach proposed in [Aleksandrov, Efimov and Dashkovskiy, 2023]. We study the problem of equidistant deployment of a group of the second order agents on the line segment. New control algorithms are proposed guaranteeing agent convergence to the prescribed positions in a fixed time.

Let us note that in [Parsegov, Polyakov and Shcherbakov, 2012; Zimenko, Efimov, Polyakov and Ping, 2023; Aleksandrov, Efimov and Dashkovskiy, 2023], it was assumed that, under the design of decentralized controls, each agent exploits the information received from its two nearest indexed neighbors. In this paper, we consider more general scenario where each agent might receive information from several of its left neighbors and several of its right neighbors (not necessarily nearest neighbors are used).

It is worth mentioning that, in [Aleksandrov and Andriyanova, 2022], via a numerical simulation, it was shown that in some cases the increasing number of neighbors reduces the time of transient processes. Therefore, it is useful to have the opportunity to receive information from several neighbors. Moreover, compared with [Aleksandrov, Efimov and Dashkovskiy, 2023], instead of information on relative agent velocities, information on absolute velocities is exploited.

2 Preliminaries

In this paper, standard notation is used. Let $\mathbb{R}$ be the field of real numbers, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the vector spaces of n-tuples of real numbers and of $n \times n$ matrices, respectively. For a vector $\eta \in \mathbb{R}^n$, the inequality $\eta \ll 0$ means that all its components are negative. Let $\Omega = \mathrm{diag}\{\omega_1, \ldots, \omega_n\} \in \mathbb{R}^{n\times n}$ be a diagonal matrix with the numbers $\omega_1, \ldots, \omega_n$ on the main diagonal.

We will use Young's inequality (see [Hardy, Littlewood and Polya, 1988]):

$$ab \le \frac{1}{p}a^p + \frac{1}{q}b^q$$

for $a \ge 0$, $b \ge 0$, $p > 1$, $q = p/(p-1)$.

Consider the differential equation system

$$\dot{x} = \Psi(t, x), \tag{1}$$

were $t \ge 0$, $x(t) \in \mathbb{R}^k$, vector function $\Psi(t, x)$ is continuous for $t \ge 0$ and $x \in \mathbb{R}^k$. Let $\Psi(t, 0) = 0$ for all $t \ge 0$. Thus, the system (1) admits the zero solution.

Denote by $x(t, t_0, x_0)$ a solution of (1) with the initial conditions $t_0 \ge 0$, $x_0 \in \mathbb{R}^k$.

Definition 1. [Polyakov, 2012] *The zero solution of (1) is finite-time stable if it is globally asymptotically stable, and any solution reaches the origin at some finite time instant, i.e., for every $x_0 \in \mathbb{R}^k$, there exists $T(x_0) \ge 0$ such that $x(t, t_0, x_0) = 0$ for $t_0 \ge 0$, $t \ge t_0 + T(x_0)$. Here $T(x_0) : \mathbb{R}^k \to [0, +\infty)$ is a settling-time function.*

Definition 2. [Polyakov, 2012] *The zero solution of (1) is globally fixed-time stable if it is globally finite-time stable, and the settling-time function $T(x_0)$ is bounded, i.e., there exists $T_{\max} > 0$ such that $T(x_0) \le T_{\max}$ for any $x_0 \in \mathbb{R}^k$.*

3 Statement of the problem

Let a group of m mobile agents on a line be given. The agents are interpreted as numbered points whose coordinates are denoted by $z_1(t), \ldots, z_m(t) \in \mathbb{R}$. The double integrators

$$\ddot{z}_s(t) = u_s, \quad s = 1, \ldots, m, \tag{2}$$

will be used as agent models. Here $u_1, \ldots, u_m$ are control protocols. Denote by $z(t) = (z_1(t), \ldots, z_m(t))^\top$ the state vector of (1).

Our objective is to design a decentralized control protocol guaranteeing the fixed-time equidistant agent deployment on a given line segment with endpoints b and e ($b < e$). Hence, all solutions of the corresponding closed-loop system should reach the equilibrium position $\bar{z} = (\bar{z}_1, \ldots, \bar{z}_m)^\top$ in a fixed time for any initial conditions, where $\bar{z}_s = b + s(e - b)/(m + 1)$, $s = 1, \ldots, m$. Such a protocol should be defined on the basis of local interactions between agents.

4 Assumptions

The endpoints of the segment will be interpreted as auxiliary static agents, i.e., $z_0(t) = b$, $z_{m+1}(t) = e$ for $t \in [0, +\infty)$.

Assumption 1. *Each agent receives information about the distances between itself and some of its left neighbors and some of its right neighbors. In addition, each agent knows the differences between its index and indices of agents from which the signals are received.*

Remark 1. Neighbors are understood in terms of agent indices.

Remark 2. Compared with [Aleksandrov, Efimov and Dashkovskiy, 2023] where only information from nearest neighbors was used, here we consider the case where agents can exploit information from multiple left and right neighbors (not necessarily nearest ones).

Assumption 2. *The communication graph of the considered multi-agent system is undirected.*

Assumption 3. *Each agent can measure its velocity.*

Remark 3. Unlike [Aleksandrov, Efimov and Dashkovskiy, 2023], in this paper, to design control protocols, instead of relative agent velocities with respect to their nearest neighbors, absolute velocities will be used.

Denote by Q_{sl} and Q_{sr} the sets of indices of left and right neighbors, respectively, from which the sth agent receives information, $s = 1, \ldots, m$.

Assumption 4. *Let $Q_{sl} \ne \emptyset$, $Q_{sr} \ne \emptyset$ for all $s = 1, \ldots, m$.*

Applying the approach proposed in [Aleksandrov and Andriyanova, 2022], define values p_{sj} for $s = 1, \ldots, m$, $j = 0, \ldots, m + 1$ by the formulae

$$p_{sj} = \begin{cases} \dfrac{\beta_s}{(s-j)M_{sl}} & \text{for } j \in Q_{sl}, \\[2ex] \dfrac{\beta_s}{(j-s)M_{sr}} & \text{for } j \in Q_{sr}, \\[2ex] 0 \quad \text{for } j \notin Q_s, \end{cases}$$

where $Q_s = Q_{sl} \cup Q_{sr}$, M_{sl} and M_{sr} are the numbers of elements of the sets Q_{sl} and Q_{sr}, respectively, β_s are positive tuning parameters.

Next, consider the matrix $P = \{p_{sj}\}_{s,j=1}^m$.

Assumption 5. *There exist values of parameters $\beta_1,\dots,\beta_m$ for which the matrix P is symmetric.*

Remark 4. In particular, Assumption 5 is fulfilled if each agent receives information only from one left neighbor and one right neighbor. In such a scenario, $M_{sl} = M_{sr} = 1$ and one can take $\beta_s = 1$, $s = 1,\dots,m$.

5 Main result

Let Assumptions 1–5 be satisfied and values of the parameters $\beta_1,\dots,\beta_m$ be chosen according to Assumption 5.

Consider the matrix $D = \mathrm{diag}\{d_1,\dots,d_m\}$, where

$$d_s = \sum_{j=0}^{m+1} p_{sj}, \quad s = 1,\dots,m.$$

From the results of the paper [Aleksandrov and Andriyanova, 2022] it follows that if $A = P - D$, $\omega = (\omega_1,\dots,\omega_m)^\top$ and

$$\omega_s = 2 - 2^{1-s}, \quad s = 1,\dots,m,$$

then

$$A\omega \ll 0. \tag{3}$$

Construct a control protocol as follows

$$u_s(t) = y_s^\rho(t) + y_s^\sigma(t) - \omega_s^{1-\mu}\dot{z}_s^\mu(t) - \omega_s^{1-\nu}\dot{z}_s^\nu(t), \tag{4}$$

where ρ,σ,μ,ν are positive rational numbers with odd numerators and denominators,

$$y_s(t) = \sum_{j=0}^{m+1} p_{sj}(z_j(t) - z_s(t)), \quad s = 1,\dots,m. \tag{5}$$

It is easy to verify that the system (2) with the control (4) admits the equilibrium position $\bar{z}$.

Theorem 1. *If*

$$0 < \rho < \mu < 1, \quad \sigma > \nu > 1, \tag{6}$$

then the protocol (4) provides global fixed-time deployment of agents (2) on the segment $[b,e]$.

Proof. With the aid of the substitution (5), we arrive at the system

$$B\ddot{y}(t) + \Omega_1 F_1(\dot{y}(t)) + \Omega_2 F_2(\dot{y}(t)) + \frac{\partial G(y(t))}{\partial y} = 0. \tag{7}$$

Here $B = -A^{-1}$, $\Omega_1 = \mathrm{diag}\{\omega_1^{1-\mu},\dots,\omega_m^{1-\mu}\}$, $\Omega_2 = \mathrm{diag}\{\omega_1^{1-\nu},\dots,\omega_m^{1-\nu}\}$, $F_1(\dot{y}) = (x_1^\mu,\dots,x_m^\mu)^\top$, $F_2(\dot{y}) = (x_1^\nu,\dots,x_n^\nu)^\top$, where x_s are components of the vector $x = B\dot{y}$,

$$G(y) = \sum_{i=1}^m \left(\frac{1}{\rho+1} y_i^{\rho+1} + \frac{1}{\sigma+1} y_i^{\sigma+1} \right).$$

It is worth noticing that the matrix B is symmetric and positive definite.

The equilibrium position $\bar{z}$ of the closed-loop system corresponds to the zero solution of (7). From the results of the paper [Aleksandrov, Efimov and Dashkovskiy, 2023] it follows that if inequalities (6) are valid and the functions

$$W_i(\dot{y}) = -\dot{y}^\top \Omega_i F_i(\dot{y}), \quad i = 1,2, \tag{8}$$

are negative definite, then the zero solution of (7) is globally fixed-time stable.

To prove the negative definiteness of the functions (8), we will use the approach proposed in [Aleksandrov, 2024]. One obtains

$$W_1(\dot{y}) = x^\top A \Omega_1 F_1(-Ax) = \sum_{s=1}^m x_s \sum_{j=1}^m a_{sj}\omega_j^{1-\mu} x_j^\mu,$$

where a_{sj} are entries of A.

Let $\xi_s = x_s/\omega_s$, $s = 1,\dots,m$. Then

$$W_1(\dot{y}) = \sum_{s=1}^m \omega_s \xi_s \sum_{j=1}^m a_{sj}\omega_j \xi_j^\mu.$$

$$\le \sum_{s=1}^m \omega_s^2 a_{ss} |\xi_s|^{\mu+1} + \sum_{s,j=1,\ s\neq j}^m \omega_s|\xi_s| \sum_{j=1}^m a_{sj}\omega_j |\xi_j|^\mu.$$

Using Young's inequality, we obtain

$$W_1(\dot{y}) \le \frac{1}{\mu+1}\sum_{s=1}^m \omega_s \sum_{j=1}^m a_{sj}\omega_j \left(\xi_s^{\mu+1} + \mu \xi_j^{\mu+1} \right)$$

$$= \frac{1}{\mu+1}\sum_{s=1}^m \omega_s \xi_s^{\mu+1} \sum_{j=1}^m a_{sj}\omega_j$$

$$+ \frac{\mu}{\mu+1}\sum_{j=1}^m \omega_j \xi_j^{\mu+1} \sum_{s=1}^m a_{sj}\omega_s.$$

From (3) and the symmetry of the matrix A it follows the existence of positive constants γ_1 and γ_2 such that

$$W_1(\dot{y}) \le -\gamma_1 \sum_{s=1}^m \xi_s^{\mu+1} \le -\gamma_2 \sum_{s=1}^m y_s^{\mu+1}.$$

To prove the negative definiteness of the function $W_1(\dot{y})$, similar arguments can be applied. The proof is completed. □

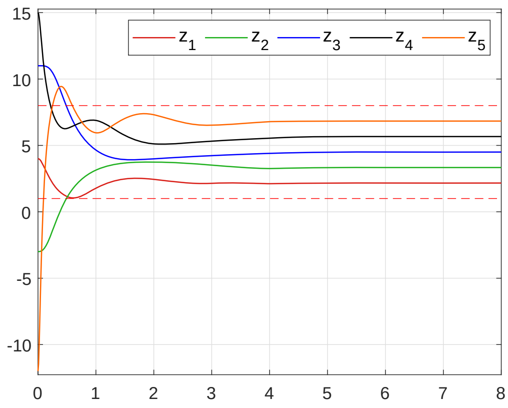

Figure 1. Agent time history.

Table 1. Network topology and agent initial positions

agent number	initial position	neighbors
1	4	0, 2, 3
2	-3	1, 5
3	11	1, 4
4	15	3, 5
5	-12	2, 4, 6

Remark 5. In the proof of Theorem 1, it is shown that the constraints imposed in [Aleksandrov, Efimov and Dashkovskiy, 2023] are fulfilled for the system (7). Therefore, to derive estimates of the upper bound on the convergence rate of solutions, the results of the above work can be applied.

In the case where each agent receives information only from two its nearest neighbors, instead of the protocol (4), the following one

$$u_s(t) = y_s^{\rho}(t) + y_s^{\sigma}(t) - \dot{z}_s^{\mu}(t) - \dot{z}_s^{\nu}(t), \quad s = 1, \ldots, m, \tag{9}$$

can be used.

Corollary 1. *Let each agent receive information only from two of its nearest neighbors and the inequalities (6) be valid. Then the protocol (9) provides global fixed-time deployment of agents (2) on the segment* $[b, e]$.

The proof of this corollary is similar to that of Theorem 1.

6 Results of a computer simulation

To demonstrate the efficiency of the developed approach, consider a group consisting of five agents, i.e., $m = 5$. Let $[b, e] = [1, 8]$, $t_0 = 0$, $\rho = 1/3$, $\mu = 3/5$, $\sigma = 5$, $\nu = 3$.

For simulation, we choose network topology and agent initial positions presented in Table 1. In this case we can take $\beta_1 = 2$, $\beta_2 = \beta_3 = \beta_4 = 1$, $\beta_5 = 2$.

In Fig. 1, the dependencies of the agent coordinates on time are given. The simulation results demonstrate fixed-time deployment of agents on the segment $[1, 8]$.

7 Conclusion

In this contribution, new decentralized control protocols are designed providing equidistant deployment of second-order mobile agents on a line segment. Application of such controls guarantees fixed-time agent convergence to uniform distribution for any initial conditions. Unlike the results of other works, the proposed protocols may be nonhomogeneous, and to construct them, each agent uses information on its velocity and distances to some of its neighbors (not necessarily nearest neighbors).

At the same time, the practical applicability of the result remains limited, since control constraints, collision avoidance, measurement noise, communication delays, and switching topologies are not taken into account. An extension of the developed approach to the case of influence of these factors will be an interesting direction for future research.

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