A CYBERNETIC VIEW ON INTERNATIONAL STABILITY UNDER MULTIPLE SANCTIONS AND COUNTER-SANCTIONS

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Abstract

In the paper, a simple discrete dynamical model for the dynamics of two antagonistic parties (opponents) under iterated sanctions and counter-sanctions is proposed. The model is inspired by the Osipov-Lanchester model for combats. Simple stability criteria are derived both for the full information case and for the stochastic uncertainty case. A cybernetic (controlled) version of the model described by bilinear difference equations is proposed. The results provide some important qualitative conclusions that are interpreted in terms of international stability preservation. The risks of global instability caused by a further increase of sanctions intensity are formulated and a possible way of controlling the situation by small actions based on the mutual trust of parties is proposed. The author hopes that the proposed mathematical models can bring humanity closer to understanding and updating the mechanisms of achieving sustainable and safe development even in the face of tough sanctions confrontation.

Key words

International stability, modelling sanctions and counter-sanctions.

1 Introduction

The year 2022 has already become very hard for international relations, including international research collaborations. Relationships that have existed for decades and centuries are being destroyed. The question of maintaining international stability and the survival of mankind in the face of the nuclear threat has again become critically important. The building of the modern world order is on fire, and the question of how to put out the fire has become of utmost importance.

What is the mission of science and scientists in this difficult time? It seems to be twofold. First, to maintain personal and creative connections between scientists from different countries, including countries in conflict in order to better understand the situation and its dynamics and keep the integrity and connectivity of the international research network. Second, to try to predict the dynamics of the conflict by scientific methods and warn politicians and the population of the planet about the most dangerous threats and risks.

As for the scientific forecast of the dynamics of the conflict, the specificity of the current situation lies in the repeated application of sanctions by all parties involved in the conflict. It seems however that no mathematical models that allow the prediction of international stability and instability under conditions of multiple sanctions and counter-sanctions have appeared in the literature, although there have been attempts to use game theoretic models or control theoretic models to describe conflict situations [Tsebelis, 1990; Erbe and Kopacek, 2008].

A very simple model for the dynamics of a bilateral conflict under multiple sanctions was proposed recently [Fradkov, 2022]. It is based on an analogy with the classical combat models of the Osipov-Lanchester type [Helmbold, 1993; Washburn and Kress, 2009]. In this article, the international stability challenges under sanctions are discussed based on the model [Fradkov, 2022]. A controlled version of the model is described and perspectives to achieve peace and international stability in the model systems are discussed.

2 Models of military operations

The role of sanctions is similar to the role of military operations. However, mathematical models of sanctions differ from the models of military operations and have been little studied before. The peculiarity of the current period of international relations is in that the sanctions are applied by different parties iteratively, in packages and stages. They have a dramatic effect on international relations. However, there are still no simple mathematical models describing the main effect of sanctions and allowing one to derive stability conditions.

Below we propose a simple model of repeated sanctions and counter-sanctions mutual dynamics, motivated by and having similarities with the Osipov-Lanchester model. Stability conditions for the overall systems are derived both for deterministic case and in presence of stochastic uncertainty.

In this section, the simplest Osipov-Lanchester model is recalled for completeness, following [Helmbold, 1993; Washburn and Kress, 2009][1] The model is described by a system of two linear differential equations

$$\dot{R}(t) = -\alpha G, \qquad (1)$$
$$\dot{G} = -\beta R,$$

where R, G are numbers of the units representing power of each opponent, α, β are firing intensities (efficiencies). Using Euler's method a discrete time version of (1) can be written as follows

$$R_{n+1} = R_n - \alpha \Delta t G_n, \qquad (2)$$
$$G_{n+1} = G_n - \beta \Delta t R_n.$$

where Δt is sampling interval. The system (2) can be considered as a model for warfare in their own right, with each iteration corresponding to a separate battle.

3 Mathematical model of multiple sanctions and its analysis

According to Galtung's definition [Galtung, 1967] sanctions are "actions initiated by one or more international actors (the 'senders') against one or more others (the 'receivers') with either or both of two purposes: to punish the receivers by depriving them of some value and/or to make the receivers comply with certain norms the senders deem important." The mathematical model of [Fradkov, 2022] is based on the assumption that efficiency of an economical or political sanction or a sanction package is mainly determined by its negative influence on the opponent with respect to the opponent's counter-sanction package at the previous stage. Given the diplomatic reciprocity ("mirror response") principle [Keohane, 1986], the sanction intensity at the next stage should be almost similar or at least not weaker than the intensity of counter-sanction at the previous stage. Note that the meaning of the term "not weaker" depends on the public opinion stronger than on the real economic efficacy. This claim is illustrated by the whole international sanction history of the last decade and especially in 2022 [Attia and Grauvogel, 2022]. It means that the model should not have long memory: the strength of the new sanction usually depends on the strength of the sanction during the previous stage (sampling interval).

Assuming linearity of all dependencies for simplicity introduce the following equations [Fradkov, 2022]:

$$x_{n+1} = x_n + a(y_n - y_{n-1}), \qquad (3)$$
$$y_{n+1} = y_n + b(x_n - x_{n-1}),$$
$$n = 1, 2, \dots$$

where x_n, y_n are values of sanction and counter-sanction pressure at n^{th} sampling instant, a, b are positive cross-gain parameters. Thus, equations (3) constitute a simple linear model of sanction dynamics. Cross-gains a, b in general are unknown and different for different parties of the conflict. They depend on economic situation in the opponent countries, on public traditions and attitude on media pressure, etc.

In order to transform (3) into a more convenient form for stability analysis introduce difference variables $v_n = x_n - x_{n-1}, w_n = y_n - y_{n-1}$. Then (3) take the form

$$v_{n+1} = aw_n, w_{n+1} = bv_n, \ n = 1, 2, \dots \qquad (4)$$

It is easy to find solutions to (4). Shifting n to $n-1$ in the second equation of (4), substituting it into the first one and performing similar procedure for the first equation of (4) we come up with two scalar equations:

$$v_{n+1} = qv_{n-1}, \qquad (5)$$
$$w_{n+1} = qw_{n-1},$$
$$n = 2, 3, \dots$$

where $q = ab > 0$ is total gain of the system.

Analysis of the linear systems (4) and (5) provides some important conclusions concerning asymptotic behavior of the process. The only parameter influencing stability of (6) is the total gain q. The system is asymptotically stable, i.e. $v_n \to 0, w_n \to 0$ as $n \to \infty$ for any initial conditions v_0, w_0 if and only if $q < 1$. In this case also total amounts of sanctions (solutions of (2) are bounded, i.e. system is reasonably stable. If $q > 1$ then the solutions of (5) and solutions of (3) tend to infinity for all nonzero initial conditions, i.e. instability occurs. The boundary case $q = 1$ is exceptional and can be neglected in practice.

The first important observation is that the strategy parameters α, β of the opponents enter expression for q symmetrically. It means that both opponents are equally responsible for stability of the process.

The second important observation is that each opponent is able to ensure stability of the process by means of proper choice of its strategy (decreasing its own gain α or β). In other words it is profitable for each party to decrease q since increase of q leads to instability which

[1] The model below was proposed independently during World War I by Mikhail Osipov in 1915 in Russia and by Frederick Lanchester in 1916 in Great Britain, see [Helmbold, 1993].

is disadvantageous both for that party and for the whole system.

For completeness the solutions to (3) are shown below in the closed form. Assume that x_0, x_1 and y_0, y_1 are given. Then solutions to (3) are represented explicitly as power functions

$$x_{2n} = (q^n - 1)/(q-1)(y_1 - y_0)/\beta,$$
$$x_{2n+1} = (q^n - 1)/(q-1)(x_1 - x_0)/\beta, \quad (6)$$
$$n = 1, 2, \ldots$$

Relations (7) and similar expressions for y_n represent long term cumulative effect of sanctions. It seems however that sanctions of the past weakly influence economic wealth and public opinion in long term since each opponent takes all measures to suppress the effect of the sanctions as soon as possible.

The case of random uncertainty in gains a, b (also considered in [Fradkov, 2022]) will be studied in the next section. It will be shown that random disturbances of a, b have zero mean, stability conditions are more strict than in the case of known a, b. However symmetry with respect to parties still holds under uncertainty.

4 Stochastic model of the repeated sanctions dynamics

Simplistic model (3) has an apparent drawback that it may be supersensitive to the parameter values α and β. In reality the values of α and β depend in a complex way on many factors, including economical, political ones and on public opinion. Moreover the values of α and β may change from stage to stage. The simplest model for uncertainty is randomness. Therefore a model with random parameters is proposed and analyzed in this section.

Suppose that α and β in (3) are replaced with $\alpha + \xi_n$ and $\beta + \eta_n$, respectively. The stochastic version of (3) is as follows:

$$v_{n+1} = (\alpha + \xi_n) w_n, \quad (7)$$
$$w_{n+1} = (\beta + \eta_n) v_n,$$
$$n = 1, 2, \ldots$$

The random errors are assumed to have zero means and bounded variances:

$$E\xi_n = E\eta_n = 0,$$

$$E\xi_n^2 = \sigma_x^2, E\eta_n^2 = \sigma_y^2,$$

$$n = 1, 2, \ldots$$

where E stands for mathematical expectation. Assume additionally that the errors of different opponents are uncorrelated: $E\xi_n\eta_m = 0$ for all $m, n = 0, 1, 2, \ldots$ and analyze mean square stability of (7).

Under imposed assumptions the mean squares of the variables in (7) satisfy the relations $Ev_{n+1}^2 = E[(\alpha + \xi_n)(\beta + \eta_n)]^2 Ev_{n-1}^2$. It means that the stability of the system depends on the averaged gain $\bar{q} = E[(\alpha + \xi_n)(\beta + \eta_n)]^2$. Evaluation of the averaged gain yields:

$$\bar{q} = E(\alpha^2 + 2\alpha\xi_n + \xi_n^2)(\beta^2 + 2\beta\xi_n + \xi_n^2)$$

Taking into account zero correlations we obtain

$$\bar{q} = (\alpha^2 + \sigma_x^2)(\beta_y^2 + \sigma_y^2). \quad (8)$$

It is seen that the same conclusions hold for stochastic case if the gain q is replaced with the averaged gain $\bar{q}$. It is seen also that stability conditions for stochastic case are more strict since $\bar{q} > q$. Note that the stability may be achieved in principle only in the case when condition $(\sigma_x^2 + \sigma_y^2) < 1$ holds.

5 Controlled model and stability control

Suppose that control is available as the change of the coefficients α and β. Although true values of α and β are unknown, each party obviously has an ability to change their values due to the strengthening and weakening of influence of the measured effect of sanctions at the next stage. Introducing controlled factors u_n, z_n rewrite the model equation (3) in the following form.

$$v_{n+1} = \alpha_n w_n, w_{n+1} = \beta_n v_n, n = 1, 2, \ldots \quad (9)$$

where $\alpha_n = \alpha u_n, \beta_n = \beta z_n$, and u_n, z_n are controlling factors, $0 < u_n < 1, 0 < z_n < 1$. Since the control goal is achievement of the stability, we do not consider the value of controlling factors greater than 1. Thus the controlled model is described by bilinear difference equations (9). Note that the model total gain $q_n = \alpha\beta u_n z_n$ depends symmetrically on the value of the control variables of both parties.

The bilinear nature of the model suggests the mechanism of control aimed at providing stability. In order to decrease the total gain q_n each party should choose control variables strictly less than one. The following result may be easily proven.

Proposition 1. For any initial conditions v_1, w_1, any parameter values $\alpha\beta$ and any value of the threshold $0 < \varepsilon < 1$ the choice of control factors satisfying $0 < u_n < 1 - \varepsilon, 0 < z_n < 1 - \varepsilon$ ensures after some number of steps n_* stability condition $q_n \leq q < 1$ for all $n > n_*$.

The above result provides a control rule which is extremely simple: each party at each stage should change its controlling variable in such way that the total gain q_k strictly decreases. It is amazing that in order to achieve stability an arbitrarily small decrease of u_n, z_n is sufficient independently of the value of unknown parameters. Such a control looks like just a demonstration of good will: let your opponent know that you reduce sanctions if the opponent will do the same. Apparently we have shown mathematically that such a procedure ensures international stability.

6 Conclusions

In the paper a simple discrete dynamical model for dynamics of two antagonistic parties (opponents) under iterated sanctions and counter-sanctions is proposed. The model is inspired with Osipov-Lanchester model for combats. Simple stability criteria are derived both for the full information case and for stochastic uncertainty case. A cybernetic (controlled) version of the model described by bilinear difference equations is proposed. The results provide some important qualitative conclusions that are interpreted in terms of international stability preservation.

An important observation is that the model parameters a, b of the opponents enter expression for q symmetrically. It means that both opponents (parties) are equally responsible for stability of the process i.e. for peaceful sustainable development. Apparently, an alternative to the sustainable, peaceful development of conflicts is an unstable, uncontrolled transition to armed clashes.

Avoiding the escalation of military conflicts and preventing the wars is the main priority of mankind, as was proclaimed by Albert Einstein and a number of prominent scientists in 1946 and later. Unfortunately the scientists' warnings are not taken seriously by politicians in too many cases. More than 65 years of the Russel–Einstein Manifesto history did not show significant success in convincing governments to avoid wars.

On the other hand there is at least one positive example of changing opinion of politicians based on scientific results and mathematical modeling: the history of the *nuclear winter* phenomenon [Turco et al., 1984; Aleksandrov and Stenchikov, 1984]. In the case of nuclear winter the research results were obtained independently by US and Russian scientists and properly promoted to their governments. It followed from them that with the explosion of even a small part of the warheads that existed at that time, in a few months the Earth would be buried under a layer of ash, all living things would freeze and life on the planet would cease. This led to the understanding that there will be no winners in a nuclear war. Alas, today's politicians do not perceive this scientific fact as a dire warning, using nuclear threats as a conventional political argument. The risks of a catastrophe are now extremely high and continue to grow "by leaps and bounds" of short-sighted public statements.

Summarizing, the proposed step-by-step algorithm is by no means a panacea for smoothing out contradictions between the politicians of countries with competing interests. However, the author believes that introducing more mathematics and cybernetics into the area of peace research may help researchers to come up with further solid proposals to their governments. Reflection on the mathematical models can bring humanity closer to the understanding and update of the mechanisms for achieving sustainable and safe development even in the face of tough sanctions confrontation.

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