Dissipative Processes at Phase Synchronization in the Nonlinear Oscillators' Lattice and Heat Emission at the Premelting Stage

L.A. Bityutskaya, E. N. Bormontov, A.N.Shebanov, E.V.Bogatikov

Synchronization processes in 2D lattice of coupled Rossler oscillators are studied. It has been detected that during formation of local synchronized clusters energy redistribution between the potential and kinetic components occurs. The detected effect is seen as a mechanism of experimentally observed heat emission effect at the premelting stage of crystalline substances.

I. INTRODUCTION

Investigation of phase synchronization in chaotic systems is one of the most important achievements in the nonlinear dynamics. At present there is a series of research devoted to study of phase synchronization in model as well as real systems [1-2]. Special methods of phase synchronization mode identification have been worked out [3]; necessary conditions for onset of synchronization effect in the nonlinear dynamic systems have been detected [4]; a phenomenon of synchronous clusters formation in spatially distributed nonlinear systems has been found out. At the same time the energy effects accompanying nonlinear system transition into a synchronized condition are left outside the research area in most studies. On the other hand, being one of the primary self-organization mechanisms, chaotic synchronization can help to find explanation for such experimentally observed energy dissipation effects as heat emission at the premelting stage [5]. The approach examining the synchronization phenomena in condensed matter turned out to be very productive when explaining the collective effects in crystalline lattice dynamics [2]. However, most studies basically consider the synchronization of different branches of phonon oscillations and its influence on crystalline lattice stability, but they do not touch upon energy dissipation issues. Therefore, investigation of energy redistribution processes in the nonlinear dynamic systems in the chaotic synchronization conditions is a task of current importance.

II. METHODS

Using the universal feature of the phase synchronization phenomena as the subject of investigation of dissipation processes at synchronization it is convenient to chose a well-studied nonlinear system with known behavior.

L.A. Bityutskaya, E. N. Bormontov, A.N.Shebanov, E.V.Bogatikov are with Department of Physics, Voronezh State University, University Sqr. 1, 394006 Voronezh, Russia; me144@phys.vsu.ru

Rossler system is attributed to a number of the most extensively studied dynamic systems where phase synchronization was explored. The equation system for coupled Rossler oscillators, which describes oscillations of 2D square lattice, is put in a dimensionless form as follows:

$$\begin{aligned}\frac{dx_{i,j}}{dt} &= -\omega_{i,j}y_{i,j} - z_{i,j} + \gamma(x_{i,j-1} - 2x_{i,j} + x_{i,j+1}),\\ \frac{dy_{i,j}}{dt} &= \omega_{i,j}x_{i,j} + a\,y_{i,j} + \gamma(y_{i,j-1} - 2y_{i,j} + y_{i,j+1}),\\ \frac{dz_{i,j}}{dt} &= 0.4 + z_{i,j}(x_{i,j} - 8.5),\end{aligned} \qquad (1)$$

where x, y, z – oscillator's coordinates characterizing its deviation from equilibrium position, $\omega_{i,j}$ – natural frequency, a – the parameter determining oscillator's chaotic mode; γ – the oscillators' connection parameter; indices i and j identify the oscillator's number in horizontal and vertical direction, respectively.

In order to study the dynamics of synchronization process in Rossler oscillator system, we realize identification of synchronization and desynchronization moments for each oscillator as well as calculation of mean kinetic and potential energy values. As a synchronization criterion we used phase difference constancy among adjacent oscillators. Estimation of the kinetic energy was carried out according to the formula:

$$E_{ij} = \frac{1}{2}\left((dx_{i,j}/dt)^2 + (dy_{i,j}/dt)^2 + (dz_{i,j}/dt)^2\right) \qquad (2)$$

The potential energy of interaction of the adjacent oscillators was calculated according to Hooke's law:

$$U_{i,j} = \gamma\left(\frac{(x_{i,j-1} - 2x_{i,j} + x_{i,j+1})^2}{2} + \frac{(y_{i-1,j} - 2y_{i,j} + y_{i+1,j})^2}{2}\right). \qquad (3)$$

Computational modeling was carried out for the system with dimensions from 5x5 to 500x500 of oscillators with periodic boundary conditions. Euler method with the time step of 10-3 was applied for modeling; frequency spread of the oscillators was $\Delta\omega$=0.1. It was determined that the systems with dimensions 5x5 ÷ 10x10 of oscillators are capable of full synchronization at certain coupling force values. An increase of system dimensions leads to clustering effect: at some threshold values of γ parameter separate synchronized areas with different lifetime form in the system. Starting from the oscillators' dimension of 50x50 there are no qualitative dynamic changes in the studied system.

III. DISSIPATIVE PROCESSES IN ROSSLER SYSTEM

To visualize the synchronization process we used representation in a certain colour spectrum of synchronized state lifetime for each oscillator (fig.1). For the oscillator system of 50x50 cluster lifetime distribution was defined (fig.2). It was established that the clusters with a long lifetime include a minimal number of oscillators not exceeding 2-3; the maximal lifetime of a synchronized oscillator is ~100-150 of vibration periods.

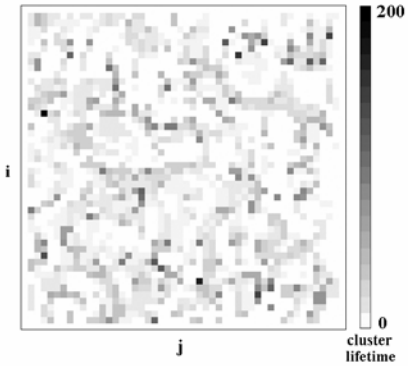

Fig.1. Formation of synchronized clusters in the lattice of Rossler coupled oscillators.

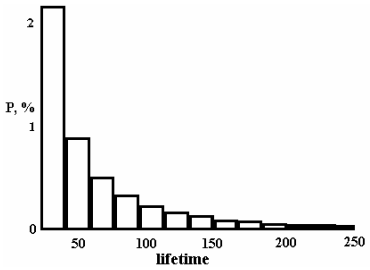

Fig.2. Probability density of lifetime of the synchronized clusters.

Dependence of synchronized cluster fraction on the chaotic mode and the coupling parameter has a threshold character (fig.3). At the same time the part of the synchronized oscillators with a long lifetime even in intense chaos conditions reaches 5-10% (fig. 4b).

The synchronization effect is accompanied by potential energy decreasing of the system which turns to kinetic oscillators' energy (fig. 4).

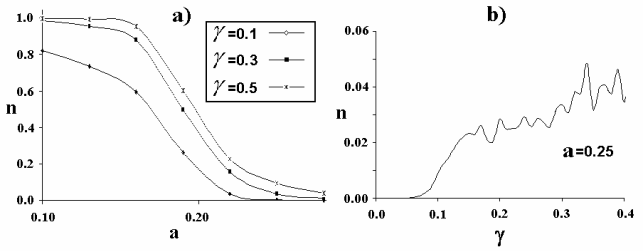

Fig.3. Dependence of synchronized cluster fraction on the Rossler system parameters.

The dependence of mean potential and mean kinetic energy of the system on the coupling parameter γ has three specific region (fig.5):

- in area I (0 <γ<0.03) an increase of the coupling parameter causes an increase of system potential energy that naturally results from the potential energy definition (3); system kinetic energy in this area decreases;

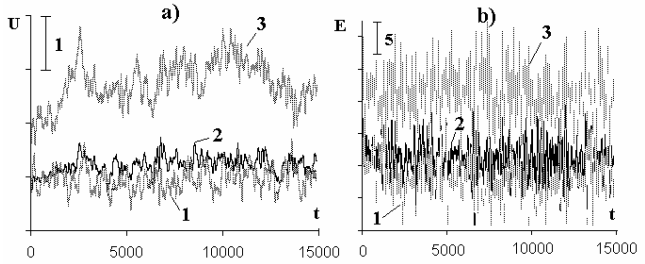

Fig.4. Potential energy redistribution among the synchronized (1) and desynchronized (3) oscillators at synchronization (desynchronization) moments; 2 – mean potential of all the oscillators' energy – (a); kinetic energy redistribution among the synchronized (3) and desynchronized (1) oscillators, 2 – mean kinetic energy – (b).

-in area II (0.03<γ<0.06) there is a simultaneous increase of potential and kinetic energy;

-in area III (γ>0.06) when the coupling force increases, decreasing of system potential energy occurs due to the synchronization process; kinetic energy of the system in this area continues to grow.

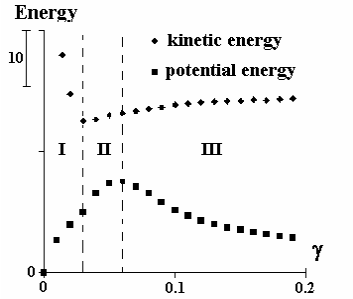

Fig.5. Potential and kinetic energy dependence on the coupling force.

IV. PREMELTING EFFECT MODEL

The transition of a part of nonlinear oscillator interaction potential energy into kinetic energy can be used to explain the experimentally observed heat emission effect when approaching the melting point of crystalline substances [5]. Let consider *x*, *y*, *z* as atom deviation from the crystalline lattice point, *a* – as a parameter controlling the level of vibration anharmonicity, γ – as a parameter providing growth of mean potential energy with increasing temperature of the crystalline lattice at mean vibration amplitude constant which is a uncontrollable parameter within the accepted model. In this case temperature growth of a solid accompanied by an increase of atom amplitude vibration and by intensification of vibration anharmonicity is equivalent to the simultaneous change of two parameters

in Rossler coupled oscillator system – that is chaotic parameter increase a and coupling parameter growth γ. Influence of these parameters on the atom vibration synchronization level is of the opposite character – intensification of the chaotic state hampers vibration synchronization, while intensification of interaction among the adjacent atoms resulting from increase in heat vibration amplitude eases synchronization appearance. To determine the conditions of heat emission appearance caused by synchronization of crystal lattice heat vibration it is necessary to find the dependence U (a, γ) which is defined by the chosen dynamic model, as well as the dependence a (γ) which is a characteristic parameter of a specific crystalline lattice within the accepted model. Figure 6 shows the example of the parametric diagram demonstrating the required variation range of parameter γ for heat emission depending on dγ increment at a=0.3, da=0.03.

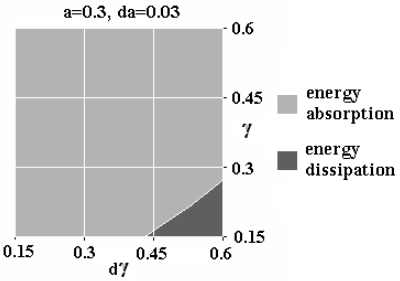

Fig.6. Parametric diagram for determination of Rossler system parameter range which allows observing energy dissipation.

According to the shown diagram, heat emission is possible in the coupling force value range γ<0.2, which according to fig.6 is most appropriate to behaviour of an actual crystalline lattice. At the same time for a wide range of parameters we can observe energy absorption as a result of decrease of the synchronized atoms' part.

Within the accepted model, premelting will go as follows. Lets denote the dependence of synchronized atoms' fraction in the crystal on the temperature by n(T). Let formation of synchronized clusters occur at approaching of threshold temperature Tc of a solid. In this case, first, crystal heat capacity should lower owing to a decrease of the number of degrees of freedom in the crystal:

$$C_V = 3N_A k[1 - 1/d \cdot n(T)], \quad d > 1 .$$

Second, heat release due to transition of a part of atom potential energy into kinetic energy should occur. Considering the fact that at the classical approximation the heat release rate as a result of synchronization equals

$$Q = \frac{1}{C_V} \cdot s \cdot \frac{3}{2} kT \cdot N_A \frac{\partial n}{\partial t},$$

one-dimensional equation of heat conductivity taking into account heat vibration synchronization will be the following:

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \Big/ 1 - \frac{sT}{2[1 - 1/d \cdot n(T)]} \frac{\partial n}{\partial T} \qquad (4)$$

where s – atom interaction potential energy fraction changing to thermal motion energy; a – thermal conductivity coefficient (for simplicity sake its dependence on temperature was not taken into account). The necessary condition for heat emission as a result of heat vibration synchronization is an increasing dependence of n(T), as well as external heating. However, since the equation (4) contains positive feedback, at certain parameter values at the premelting stage the mode of thermal structure metastable localization can be brought about, and at T>Tc there can be emission of heat in the form of macroscopic fluctuations [6]. The local short-lived large energy fluctuations studied by Y.L. Khait can function as macroscopic dissipative structures' nuclei. In studies [7] they demonstrated that at approaching the melting point the probability of formation of crystal local heating areas with dimensions up to ~50 A and lifetime of ~1-2 periods of atom vibrations increases exponentially. The energy of atoms within these areas increases by >>kT value which can lead to formation of synchronized clusters and further self-heating at average crystal temperature <Tc, and also at absence of external heating.

V. CONCLUSIONS

The given qualitative model of premelting process of crystalline substance using phase synchronization of thermal atom vibrations as the main mechanism allows explaining such experimentally observed premelting effect features as a possibility of appearance of both endo- and exothermal effects, macroscopic fluctuation ability, existence of temperature rage of premelting effect onset. A quantitative theory should base on the equations of dynamics describing crystalline lattice vibrations under anharmonicity conditions. However, the universal character of self-organization effects allows expecting that the character of the considered effects will not change when studying a more realistic model.

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