ON THE CONTROL OF CHAOS IN RAYLEIGH- BÉNARD CONVECTION IN MAXWELLIAN FLUIDS USING BACKSTEPPING DESIGN

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Abstract
The problem of controlling chaos in Rayleigh-Bénard convection in weakly elastic fluids is examined here. The nonlinear model presented in the article is based on the model proposed by Khayat. Chaotic behavior in this nonlinear system is controlled using backstepping design. By applying proposed controller, the dynamic of the system is stabilized on the desired trajectory. Simulation results show the high performance of the method for chaos elimination in this system.

Key words
Chaos, Chaos Control, Khayat System, Rayleigh-Bénard Convection

1 Introduction

Chaotic phenomena have been observed in numerous fields of science such as physics, chemistry, biology, and ecology. An interesting subject in chaos theory is to eliminate the chaotic behavior by means of control systems. In the recent years, chaos control has been used in many theoretical and engineering problems and many papers have been published over the past two decades about controlling chaos in systems ranging from economics to biology with various methods of control. The first document in control of chaotic systems, named OGY, returns to 1990s [Ott, Grebogi and York, 1990; T. Shinbort, Ott, Grebogi and Yorke,1990]. In this scheme, an analytical method using low energy control signals was introduced for stabilizing Unstable Periodic Orbits (UPO) in a chaotic attractor. The main disadvantage of this method is its dependency to time of filling the chaotic attractor by trajectories. Moreover, in this period of time, the Pyragas method, based on delayed feedback control, was presented [K. Pyragas,1992]. This approach also uses the recurrence of chaotic trajectories. Pyragas method has been extensively employed for stabilizing periodic solutions of nonlinear systems [Chen and Yu, 1999;Arecchi, Boccaletti, Ciofini, and Meucci, 1998]. Recently, different nonlinear control techniques are used for chaos control. Some of them are: controlling the chaos in Lorenz system, using feedback linearization [Hwang, Fung, Hsieh, and Li, 1999; Alasty and Salarieh, 2005], controlling the chaos by variable structure systems [Chen and Yu, 1999; Yu, 1997, Konishi, Hirai, Kokame, 1998; Tsai, Fuh, and Chang, 2002], and feedback linearization in discrete time systems, with assumption that the normal form are attainable [Fuh and Tsai, 2002; Liaw and Tung, 1996]. Furthermore, fuzzy methods and neural networks have been applied for chaos control [Calvo and Cartwright, 1998; Guan and Chen, 2003; Ramesh and Narayanan, 2001; Alasty, Salarieh, 2004; Alasty and Salarieh, 2005; Poznyak, Yu and Sanchez, 1999]. One of these nonlinear methods is backstepping design, which is also employed to chaos suppression in articles.[Lu and Zhang, 2001; Yassen,2006]

It is known that, in some processes the objective is to suppress (laminarize) chaotic or turbulent motions and maintain a steady, time-dependent flow. This will help to make right predictions of flow and reduce undesirable temperature oscillations and fluctuations which may increase safety in operational condition and reduce drag.[Singer and Bau, 1991; Wagner, Bertozzi, Howle, 2003] Thus, the general problem of how one controls body force driven convection is an area of practical as well as scientific interest.

In the field of heat transfer and fluids dynamics, one common example of a dynamical system that exhibits chaotic behavior under specific circumstances is the Rayleigh-Bénard convection. [Bénard, 1900;

Rayleigh,1916] The phenomenon of thermal instability in a horizontal layer of viscous fluid heated from below was first observed by Bénard in 1900.[Bénard, 1900] The problem was analyzed theoretically by Lord Rayleigh in 1916. [Rayleigh, 1916] The Rayleigh-Bénard convection problem consists of a horizontal fluid layer subjected to heating uniformly underside and cooling uniformly on the upper side. The imposed temperature difference must exceed a finite critical value before the first signs of motion and convective heat transfer are detected and the fluid leaves the state of steady conduction. The onset of convection is expressed by the critical Rayleigh number (RaH). Immediately above RaH, the flow consists of counterrotating two-dimensional rolls, the cross sections of which are almost square. This flow pattern is commonly recognized as Bénard Cells, or Bénard convection. The cellular flow becomes significantly more complicated as RaH exceeds by one or more orders of magnitude the critical value. The two-dimensional rolls break up to three-dimensional hexagonal cells. As the temperature difference increases, the cells become narrower and, eventually, the flow becomes oscillatory and turbulent [Bejan, 1984]. Here, the key parameter is the Rayleigh number, which is proportional to the temperature difference across the fluid layer. Small values of Ra correspond to simple often time-independent flows; intermediate values of Ra correspond to complex chaotic flows; and very large values of Ra correspond to strongly driven turbulent flow. The importance of this problem lies in the fact that this system can exhibit not only aperiodic time dynamics (chaos), but also spatiotemporal irregular behavior (turbulence), where the dynamics are aperiodic in time and space.

The possibility of bifurcation and chaotic dynamics in this problem is greatly in interest. [Cross and Hohenberg, 1993; Khayat, 1995; Khayat 1994] This problem has been extensively studied for Newtonian fluids as the attempting to find the (two-dimensional) solution to the full conservation equations for a Newtonian fluid. Saltzman [Saltzman, 1962] and Lorenz [Lorenz, 1963] derived a set of ordinary differential equations by expanding the stream function and temperature in double Fourier series via Galerkin technique in space, with the coefficients being a function of time alone. From the infinite set of differential equations, keeping only one term for the stream function, and two terms for the temperature, Lorenz obtained a three-dimensional nonlinear system. In [Khayat 1994], the Galerkin method is used to derive a four-dimensional dynamical system which constitutes an extension of the Lorenz model to include viscoelastic fluids. A similar truncation procedure to the Fourier representation of the flow and temperature fields is adopted with the constitutive equation of the Oldroyd-B type. In [Khayat, 1995], a four-dimensional dynamical system was derived for an upper-convected Maxwellian fluid. The Upper Convected Maxwell model is a generalization of the Maxwell fluid using the Upper-convected time derivative, which is widely used in polymer rheology for the description of behavior of a viscoelastic fluid under large deformations. For the case of small deformation, the nonlinearities introduced by the Upper Convected Derivative disappear and the model becomes an ordinary model of Maxwell material. Maxwell's equation is obtained from the Oldroyd-B equation by setting fluid retardation to zero. These studies can be viewed as studies to check the influence of fluid elasticity on the transition to turbulence during thermal convection of a viscoelastic fluid.

Active control of convection processes has attracted a great deal of research interest in recent years. [Singer and Bau, 1991; Wagner, Bertozzi, Howle, 2003]. The very first work on actively controlling the Rayleigh-Bénard system, was performed by Tang & Bau.[Tang and Bau, 1993.] They considered control through perturbation of the lower boundary temperature in proportion to the temperature at the mid-height of the fluid layer. They also considered control actuation by generating a velocity profile at the lower boundary. On the other hand, imposing a given flux at a boundary is considered in theoretical and experimental to actively control the problem. [Wagner, Bertozzi, Howle, 2003, Tang, and Bau, 1995; Howle, 1997; Howle, 1997]. In the experiments, shadowgraphic visualization is used to measure the wave pattern [Howle, 1997; Howle, 1997]. A controller, then, used this wave pattern information as an input to the control law.

In this paper, we consider the problem of controlling convection in the Rayleigh-Bénard system. A backstepping based controller is designed to stabilize the system on a desired trajectory. Simulation results show that the proposed techniques can be successfully implemented for chaos suppression in the system, even when the system is subjected into a disturbance.

2 Problem Statement

The dynamical system model of upper-convected Maxwellian fluid in problem of Rayleigh-Bénard convection can be obtained based on the model suggested in [Khayat, 1995].The governing equations for the fluid motion of Rayleigh-Bénard convection are the well known Boussinesq equations, a set of nonlinear partial differential equations which via applying Galerkin truncation yield the fluid velocity, pressure, and temperature as a time-based system of ordinary differential equations. This system can be written in scaled form as [Khayat, 1995]:

$$\begin{aligned} \dot{X} &= \Pr\left(Y - P\right) \\ \dot{Y} &= rX - Y - XZ \\ \dot{Z} &= XY - bZ \\ \dot{P} &= \delta\left(X - P\right) \end{aligned} \tag{1}$$

Under certain conditions for the values of $\delta, \Pr, r$ and b, the above described system exhibits chaotic behavior. It is to be noted that the system for $r > 1$

has got three equilibrium points at $(0,0,0,0)^T$ and $(\mu,\mu,r-1,\mu)^T$ where $\mu=\pm\sqrt{b\,(r-1)}$.

The main goal is to stabilize the system in Eq. (1) on any desired trajectory when chaotic motion emerges. It is assumed that every variable in Eq. (1) is measurable [Howle, 1997].

3 Chaos Control

The implementation of nonlinear control techniques for suppressing chaotic dynamics in the system will be achieved by adding a feedback control forcing signal in the second equation in the system (1). The new equations of motion including the form of control force, *u* be written as:

$$\begin{aligned} \dot{X} &= \Pr(Y-P) \\ \dot{Y} &= rX - Y - XZ + u \\ \dot{Z} &= XY - bZ \\ \dot{P} &= \delta(X-P) \end{aligned} \tag{2}$$

3.1 System Standardization

In order to find the proper controller, we need to reformulate the system's control state space equation into a suitable form. To this end, let define the state variables as follow:

$$x_1 = P, x_2 = X, x_3 = Y, x_4 = Z \tag{3}$$

Thus, the control system described in Eq. (2) can be rewritten as,

$$\begin{aligned} \dot{x}_1 &= \delta(x_2 - x_1) \\ \dot{x}_2 &= \Pr(x_3 - x_1) \\ \dot{x}_3 &= rx_2 - x_3 - x_2x_4 + u \\ \dot{x}_4 &= x_2x_3 - bx_4 \end{aligned} \tag{4}$$

2.2 BackStepping Design

To stabilize the dynamic of the system on the desired trajectory $x_{1d}(t)$ using backstepping control, we start from the first equation in Eq. (4). Suppose a stabilizing function of $x_2=\phi_2$ has to be designed as the state x_2 assumed to be the control input. Therefore, one may define the Lyapunov function as,

$$V_1 = \frac{1}{2}(x_1 - x_{1d})^2 \tag{5}$$

Its time derivative along the control system,

$$\dot{V}_1 = (x_1 - x_{1d})(\dot{x}_1 - \dot{x}_{1d}) = (x_1 - x_{1d})\left(\delta(\phi_2 - x_1) - \dot{x}_{1d}\right) \tag{6}$$

will become negative by choosing the control function ϕ_2 as,

$$\phi_2 = \frac{\dot{x}_{1d} + x_1(\delta-1) + x_{1d}}{\delta} \tag{7}$$

Assuming the new control signal for the second equation, $x_3=\phi_3$ and defining new Lyapunov function as,

$$V_2 = V_1 + \frac{1}{2}(\phi_2 - x_2)^2 = V_1 + \frac{1}{2}\left(\frac{\dot{x}_{1d} + x_1(\delta-1) + x_{1d}}{\delta} - x_2\right)^2 \tag{8}$$

one may readily find the first derivative as follows.

$$\begin{aligned} \dot{V}_2 &= -\delta(x_1 - x_{1d})(x_2 - x_1) + (\phi_2 - x_2)(\dot{\phi}_2 - \dot{x}_2) \\ &= -\delta(x_1 - x_{1d})(x_2 - x_1) + (\phi_2 - x_2)\left(\frac{\ddot{x}_{1d} + \dot{x}_1(\delta-1) + \dot{x}_{1d}}{\delta} - \ddot{x}_2\right) \\ &= -\delta(x_1 - x_{1d})(x_2 - x_1) + (\phi_2 - x_2)\left(\frac{\ddot{x}_{1d} + \delta(x_2 - x_1)(\delta-1) + \dot{x}_{1d}}{\delta} - \Pr(\phi_3 - x_1)\right) \end{aligned} \tag{9}$$

By choosing ϕ_3 as,

$$\phi_3 = \frac{x_1\left(\Pr - 2\delta + 2 - \frac{1}{\delta}\right) + x_2(-2+\delta) + \frac{2\dot{x}_{1d} + x_{1d} + \ddot{x}_{1d}}{\delta} + \delta x_{1d}}{\Pr} \tag{10}$$

the Eq. (9) becomes N.D.

Now let define,

$$V_3 = V_2 + \frac{1}{2}(\phi_3 - x_3)^2 \tag{11}$$

then,

$$\begin{aligned} \dot{V}_3 &= -(x_1 - x_{1d})^2 - (\phi_2 - x_2)^2 + (\phi_3 - x_3)(\dot{\phi}_3 - \dot{x}_3) \\ &= \dot{V}_2 + (\phi_3 - x_3)\Bigg[\frac{\dot{x}_1(\Pr - \delta + 1) + \dot{x}_2(-2+\delta) + \dot{\phi}_2 + \frac{\dddot{x}_{1d} + \ddot{x}_{1d}}{\delta}}{\Pr} \\ &\quad - (rx_2 - x_3 - x_2x_4 + u)\Bigg] \end{aligned} \tag{12}$$

become N.D. by choosing the control law, u , as,

$$\begin{aligned} u &= x_2x_4 + x_1\left(\frac{\delta^2 + 3\Pr - 2\delta\Pr - 3\delta + 3}{\Pr} - \frac{1}{\Pr\delta} + \Pr\frac{(\delta-1)}{\delta}\right) \\ &\quad + x_2\left(\frac{\Pr\delta - \delta^2 + 3\delta - 3}{\Pr} - r - \Pr\right) \\ &\quad + x_3(-2+\delta) + \frac{\dddot{x}_{1d} + 3\ddot{x}_{1d} + 3\dot{x}_{1d} + x_{1d}}{\delta\Pr} + \Pr\frac{\dot{x}_{1d} + x_{1d}}{\delta} \end{aligned} \tag{13}$$

Remark 1: The forth equation in Eq. (4) has been never used in controller design procedure. It is obvious as we can interpret it as the internal dynamics of the controlled system.

Let define the controller input as,

$$u = u_1 + u_2 = x_2x_4 + u_2 \tag{14}$$

Thus the dynamic equation of control system in its standard form would be,

$$\begin{aligned} \dot{x}_1 &= \delta(x_2 - x_1) \\ \dot{x}_2 &= \Pr(x_3 - x_1) \\ \dot{x}_3 &= rx_2 - x_3 + u_2 \end{aligned} \tag{15}$$

In the presented standard form, the dynamics of x_1, x_2 and x_3 became decoupled from x_4. This will make the equation,

$$\dot{x}_4 = x_2x_3 - bx_4 \tag{16}$$

representing the internal dynamics of the control system. The convergence of this dynamic is guaranteed as the parameter *b* is positive. Thus it will converge and stabilize as the x_1 and x_3 converge to their desired trajectory.

Remark 2: As x_1 is converging to x_{1d} one may conclude form Eq. (11) that x_2 and x_3 is converging to ϕ_2 and ϕ_3 respectively. Thus,

$$\begin{aligned} x_2 &\to \frac{\dot{x}_{1d}}{\delta} + x_{1d} = x_{2r} \\ x_3 &\to \frac{\ddot{x}_{1d} + 3\dot{x}_{1d}}{\Pr\delta} + x_{1d} = x_{3r} \end{aligned} \tag{17}$$

Also according to Eq. (16) one may obtain,

$$x_4 = x_4(0)e^{-bt} + \int_0^t e^{-b(t-\tau)} x_{2r}(\tau) x_{3r}(\tau) d\tau \quad (18)$$

So as b is positive,

$$x_4 \to \chi(t) \;;\; \chi(t) = \int_0^t e^{-b(t-\tau)} x_{2r}(\tau) x_{3r}(\tau) d\tau \quad (19)$$

Remark 3: Suppose that x_{1d} is C^{∞} and bounded such as,

$$|x_{1d}| \le \alpha_1, |\dot{x}_{1d}| \le \beta_1, |\ddot{x}_{1d}| \le \gamma_1 \quad (20)$$

Thus from Eq. (17) and using the triangular inequality one may obtain,

$$\begin{aligned} |x_{2r}| &\le \frac{|\dot{x}_{1d}|}{\delta} + |x_{1d}| \le \frac{\beta_1}{\delta} + \alpha_1 = \alpha_2 \\ |x_{3r}| &\le \frac{|\ddot{x}_{1d}| + 3|\dot{x}_{1d}|}{\Pr\delta} + |x_{1d}| \le \frac{\gamma + 3\beta_1}{\Pr\delta} + \alpha_1 = \alpha_3 \end{aligned} \quad (21)$$

Also one may conclude from Eqs. (19) and (21) that,

$$|\chi(t)| = \left|\int_0^t e^{-b(t-\tau)} x_{2r}(\tau) x_{3r}(\tau) d\tau\right| \le \alpha_2\alpha_3 \int_0^t e^{-b(t-\tau)} d\tau = \alpha_2\alpha_3 \frac{1-e^{-bt}}{b} \quad (22)$$

Thus the bound is decreasing to $\dfrac{\alpha_2\alpha_3}{b}$ as times goes on.

Remark 4: Suppose that x_{1d} is constant, i.e.

$$x_{1d} = \psi \quad (23)$$

Thus from Eqs. (17) and (19), one may conclude that,

$$\begin{aligned} & x_2 \to x_{1d} = \psi \\ & x_3 \to x_{1d} = \psi \\ & x_4 \to \int_0^t e^{-b(t-\tau)} \psi^2 d\tau = \frac{\psi^2}{b} \end{aligned} \quad (24)$$

where all are constants.

2.3 BackStepping Design in the presence of Uncertainty

In the previous sub-section we derive the backstepping controller which stabilize the control system on desired trajectory for the state variable x_1. In this section the controlled system in Eq. (4) is assumed with bounded disturbance. The new equations of motion including the form of control force, *u*, and with external disturbance can be written as:

$$\begin{aligned} \dot{x}_1 &= \delta(x_2 - x_1) \\ \dot{x}_2 &= \Pr(x_3 - x_1) \\ \dot{x}_3 &= rx_2 - x_3 - x_2x_4 + d + u \\ \dot{x}_4 &= x_2x_3 - bx_4 \end{aligned} \quad (25)$$

where *d* denotes the bounded disturbance, i.e.

$$|d| \le \sigma \quad (26)$$

It is to be noted that, σ is a positive known constant. The procedure of previous section can be applied here. One may find the virtual control actions ϕ_2 and ϕ_3 as described in Eqs. (7) and (10). Lets again consider Eq. (11).

$$V_3 = V_2 + \frac{1}{2}(\phi_3 - x_3)^2 \quad (27)$$

Its time derivative along the control system in Eq. (25) can be derived as,

$$\begin{aligned} \dot{V}_3 &= -(x_1 - x_{1d})^2 - (\phi_2 - x_2)^2 + (\phi_3 - x_3)(\dot{\phi}_3 - \dot{x}_3) \\ &= \dot{V}_2 + (\phi_3 - x_3)[\frac{\dot{x}_1(\Pr - \delta + 1) + \dot{x}_2(-2+\delta) + \dot{\phi}_2 + \frac{\dddot{x}_{1d} + \ddot{x}_{1d}}{\delta}}{\Pr} \\ &\quad -(rx_2 - x_3 - x_2x_4 + d + u)] \end{aligned} \quad (28)$$

Lets consider the control action for un-disturbance system using Eq. (13) as,

$$\begin{aligned} u_{ud} &= x_2x_4 + x_1\left(\frac{\delta^2 + 3\Pr - 2\delta\Pr - 3\delta + 3}{\Pr} - \frac{1}{\Pr\delta} + \Pr\frac{(\delta-1)}{\delta}\right) \\ &\quad + x_2\left(\frac{\Pr\delta - \delta^2 + 3\delta - 3}{\Pr} - r - \Pr\right) \\ &\quad + x_3(-2+\delta) + \frac{\dddot{x}_{1d} + 3\ddot{x}_{1d} + 3\dot{x}_{1d} + x_{1d}}{\delta\Pr} + \Pr\frac{\dot{x}_{1d} + x_{1d}}{\delta} \end{aligned} \quad (29)$$

Define the control action for System in Eq. (25) as,

$$u = u_{ud} + u_d \quad (30)$$

Substituting Eq. (30) in Eq. (28) one may obtain,

$$\dot{V}_3 = -(x_1 - x_{1d})^2 - (\phi_2 - x_2)^2 - (\phi_3 - x_3)^2 + (\phi_3 - x_3)(u_d + d) \quad (31)$$

Defining,

$$u_d = -(\sigma + \theta)\operatorname{sgn}(\phi_3 - x_3) \quad (32)$$

which θ is a positive constant.
Substituting in Eq. (31) results in:

$$\begin{aligned} \dot{V}_3 &= -(x_1 - x_{1d})^2 - (\phi_2 - x_2)^2 - (\phi_3 - x_3)^2 + (\phi_3 - x_3)(u_d + d) \\ &\le -(x_1 - x_{1d})^2 - (\phi_2 - x_2)^2 - (\phi_3 - x_3)^2 - \theta|\phi_3 - x_3| \end{aligned} \quad (33)$$

Thus considering Eqs. (29) and (32) the control law can be summarized as:

$$\begin{aligned} u &= x_2x_4 + x_1\left(\frac{\delta^2 + 3\Pr - 2\delta\Pr - 3\delta + 3}{\Pr} - \frac{1}{\Pr\delta} + \Pr\frac{(\delta-1)}{\delta}\right) \\ &\quad + x_2\left(\frac{\Pr\delta - \delta^2 + 3\delta - 3}{\Pr} - r - \Pr\right) + x_3(-2+\delta) \\ &\quad + \frac{\dddot{x}_{1d} + 3\ddot{x}_{1d} + 3\dot{x}_{1d} + x_{1d}}{\delta\Pr} + \Pr\frac{\dot{x}_{1d} + x_{1d}}{\delta} \\ &\quad -(\sigma+\theta)\operatorname{sgn}\left(\frac{x_1(\Pr - \delta + 2) + x_2(-2+\delta) + \frac{-x_1 + x_{1d} + \ddot{x}_{1d} + 2\dot{x}_{1d}}{\delta}}{\Pr} - x_3\right) \end{aligned} \quad (34)$$

Remark 5: Although the stability of the proposed control technique has been proved theoretically, there are some technical problems such as strong chattering in implementation of control law due to use of sign function in Eq.(32). To overcome this problem one can use the saturation function instead of sign function:

$$sat\left(S/\phi\right) = \begin{cases} S/\phi & \left|S/\phi\right| < 1 \\ sign\left(S/\phi\right) & otherwise \end{cases} \quad (35)$$

where φ is a positive small number. In this case, some steady state error is generated which implies that there exists some error between the stabilized trajectory and the actual one. By decreasing φ to zero, the mentioned error will converge to zero.

4 Simulation Results

The above described control scheme is now used to control the states of a chaotic system. The aforementioned system in Eq. (1) exhibits chaotic behavior with the following parameters:

$$r = 4.6, \Pr = 10, \delta = 3.0703, b = 8/3 \quad (36)$$

These parameters represent a viscoelastic fluid with $De = 0.022$.[Cross and Hohenberg, 1993] For these parameters, the phase plane diagrams of the system chaotic attractor are shown in Fig. (1).

The simulation consists of two parts. First in absence of disturbance, the described controller in section 3.2 is implemented. Then, by considering a bounded disturbance in the model, the controller proposed in section 3.3 is applied to the system. In both methods the controller signal is equal to zero for the first *T=20* time units. For time greater than *T*, the controller turned.

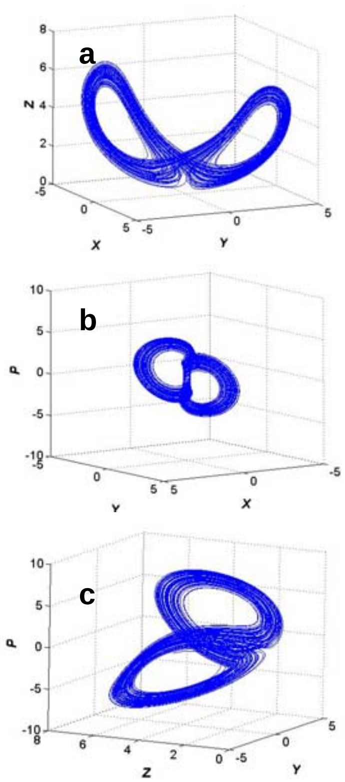

Figure 1. Chaotic attractor of the system in the phase space projected on (a)XYZ space, (b) XYP space, (b) YZP space

The controller in backstepping design of the model in the absence of disturbance follows the pattern defined in Eq. (13). Figure 2 shows the results for this case regulating the $x_{1d} = P_d = \sqrt{b\left(r-1\right)}$. As this point is the *P-component* of the equilibrium of the system in Eq. (1), one may conclude from Eq. (24) that the *X,Y* and *Z* components of the system are converging to their corresponding equilibrium components.

Thus the state variable of the system is converging to its equilibrium point. It is to be noted as the system stabilized on its equilibrium point; the control action will converge to zero in a finite time. On the other hand as the system has got the strange attractor, controlling a state would result the system stabilization on equilibrium manifold.

In Fig. 3 we represent the result of the *P-component* of the system tracks the desired trajectory, $x_{1d} = P_d = 2\sin\left(t\right)$.

In presence of disturbance in the system, the second control algorithm which is introduced in section 3.3 is used to control the chaotic motion of the system. Regarding Eq. (26), the following bound is considered in the simulation:

$$\sigma = 0.1 \quad (37)$$

Also, in Eq. (32) θ is set to 0.1 .

Figure (4) represents the results of the proposed controller in presence of bounded disturbance. This bounded uncertainties assumed to be $d = 0.1\sin\left(10t\right)$.

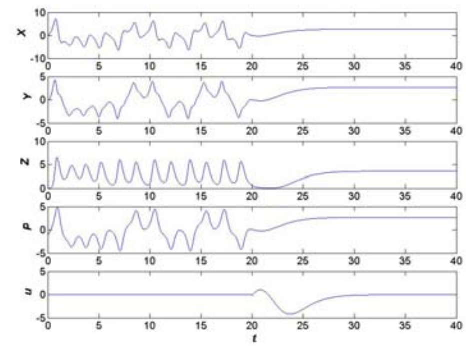

Figure 2. Regulation time response of the states and control input for the controlled system, the controller is turned at t=20

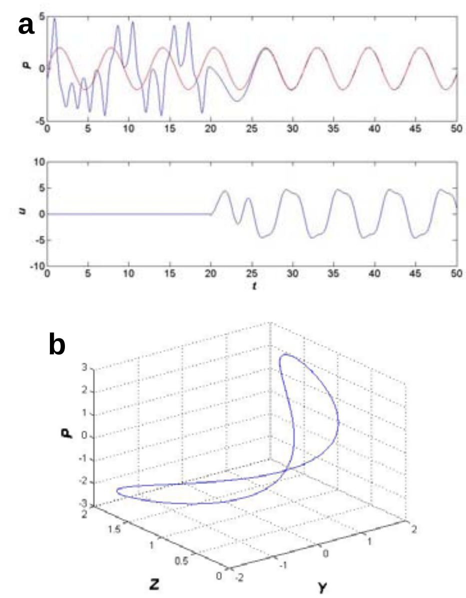

Figure 3. (a) Time response of the P state and control input and (b) phase space projected on YZP for the controlled system which tracks $P_d\left(t\right) = 2\sin\left(t\right)$, the controller is turned at t=20.

As can be seen from the simulation results, the stability of desired trajectory orbits is completely achieved in less than 50 time units.

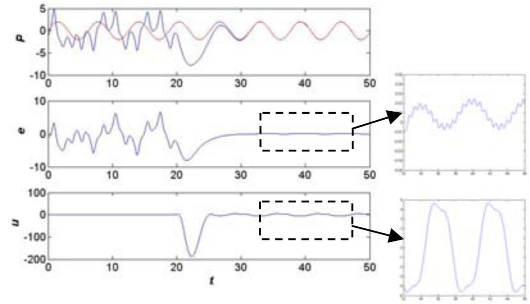

Figure 4. Time response of the P state and control input for the controlled system with disturbance which tracks $P_d(t) = 2\sin(t)$, the controller is turned at t=20.

5 Conclusion

In this paper, the problem of controlling chaos in Rayleigh- Bénard Convection in Maxwellian Fluids is considered. A backstepping design is developed to eliminate chaos based on stabilizing the P-component of the system on a desired trajectory. The modified version of the proposed controller is applied to the system by considering a bounded disturbance in the system. Both techniques are successfully implemented to stabilize the desired trajectory in the system in a reasonable small time and control action. Simulation results confirm the performance of the designed controllers.

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