Synthesis of Limited Control in Magnetic Suspension at Devices of the Navigation and Attitude Control Systems

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Abstract: Magnetic suspension is applied at devices of the inertial navigation and attitude control systems in order to raise accuracy exclude the wear-out of moving elements and avoid excitement springy slow-damping oscillations of construction. Qualitative achievement of these advantages is possible if optimal control in magnetic suspension is used. The optimization problem is not easy for limited control. It is proposed the decision of this problem in a case of square-law criterion of transient processes quality.

Keywords: optimal limited control, magnetic suspension, navigation and attitude control systems

1. INTRODUCTION

At exception of mechanical contact suspending bodies with respect to a base, a big loading capacity allows a magnetic suspension to have application into different areas of technology. In particular, magnetic suspension is applied for non-contact suspension of sensitive elements at instruments and power devices of the navigation and attitude control systems: Morrison, M.M. (1988), Somov Ye.I. et all (1991), Schweitzer G et all, (1994), Voronkov, V.S. (1997), Somov Ye. I. (2006). Magnetic suspension allows raising accuracy, excluding the wear-out of moving elements and avoiding the excitement springy slow-damping oscillations of construction. The achievement enumerated aims is provided not only absence of mechanical contact suspending bodies with base but also choice of control law in magnetic suspension. Synthesis of control law becomes complicated if account real existing limits of control law. It is proposed the decision of this problem in a case of square-law criterion of transient processes quality.

2. MATHEMATICAL MODEL OF CONTROL OBJECT

2.1. The scheme of magnetic suspension

The scheme of the simplest magnetic suspension is shown on Fig. 1. The stabilization of a gap between ferromagnetic body 1 and a core of electromagnet 2 is realized by means of the auto control system. This system is controlling voltage on electromagnet depending on signals sensors 3,7 of a gap and an electromagnet current. Notes on this figure are following: 1 is suspended ferromagnetic body, 2 is an electromagnet with ferromagnetic core, 3 is a sensor of a gap δ with sensitivity α, 4 is a speed correction section of sensor signal of a gap with constant time τ, 5 is adding section, 6 is a power amplifier with gain factor β, 7 is a sensor of electromagnet current I with sensitivity r.

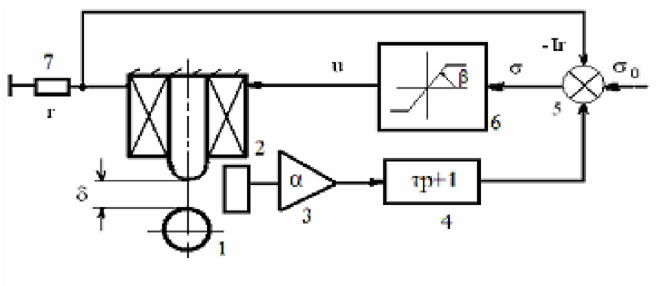

Fig 1 The scheme of the simplest magnetic suspension.

2.2 Mathematical model of magnetic suspension dynamics

Magnetic suspension is an electro mechanic system which dynamics is subordinated to the laws of mechanics and electricity. Mechanical moving the body on magnetic suspension axis if it is located vertically is subordinated the following second Newton's law

$$m\ddot{\delta} = mg - F(\delta, I) \tag{1}$$

according to which acceleration of body on axis suspension is defined its mass m and difference between a force of weight mg and a lifting force of electromagnet $F(\delta, I)$ as certain nonlinear function of a gap δ and a current I in electromagnet. Change the current I in electromagnet is subordinated the following second Kirhgoff's law

$$RI = u + E(\dot{\delta}, \dot{I}), \tag{2}$$

according to which fall of voltage on resistance R of electromagnet is defined by means of the amount controlling voltages u and Faraday's emp of induction $E(\dot{\delta}, \dot{I})$ as certain nonlinear function of a gap and a current of electromagnet change velocity.

Condition of system equilibrium is defined such values of clearance δ_0, current I_0 and voltages u_0 of electromagnet, under which its lifting force balances the weight force of body when emp of induction is zero

$$mg = F(\delta_0, I_0), \quad RI_0 = u_0 \quad . \tag{3}$$

Linearization of nonlinear functions of electromagnet lifting force and emp of induction in vicinities of equilibrium condition (3) leads to linear mathematical model of the simplest magnetic suspension

$$m\ddot{\delta} = a(\delta - \delta_0) - b(I - I_0) \tag{4}$$

$$-b\dot{\delta} + L\dot{I} + R(I - I_0) = u - u_0$$

where *a, b, L* are factors of linearization having the following physical sense: *a, b* are steep nesses of force and magnetic flow characteristics with respect to changing a gap and a current, L is an inductance of electromagnet under the nominal gap. The voltage from power amplifier given to electromagnet is depending on feedback signal $(\sigma - \sigma_0)$ which is formed from bias signal σ_0 and signals from a gap and a current of electromagnet sensors as shown in fig. 1. Output voltage of power amplifier has nonlinear dependency from input. Under symmetrical limitations output voltage $\pm\bar{u}$ this dependency is nearly too piecewise-linear function

$$u(\sigma - \sigma_0) = \begin{cases} +\bar{u}, \beta(\sigma - \sigma_0) \geq \bar{u}, \\ \beta(\sigma - \sigma_0), |\beta(\sigma - \sigma_0)| \leq \bar{u}, \\ -\bar{u}, -\bar{u} \geq \beta(\sigma - \sigma_0), \end{cases} \tag{5}$$

where β is a gain factor of input signal.

2.3 Standard form of mathematical model

In order to adduction of mathematical model of magnetic suspension to standard non-dimensional form it is choose scales of variables and time

$$\delta_m = \delta_0, \; I_m = I_0, \; u_m = u_0, \; t_m = \sqrt{\frac{m}{a}} \; .$$

It is introduced non-dimensional variables

$$x_1 = \frac{\delta - \delta_0}{\delta_m}, \; x_2 = \dot{x}_1 t_m, \; x_3 = \frac{I - I_0}{I_m}, \; u' = \frac{u - u_0}{u_m},$$

having sense of relative change a gap, a velocity of body, a current of electromagnet and a controlling voltage (stroke is hereinafter omitted). Selected scales of variables correspond to their interrelations defined by equilibrium conditions of linear system (4)

$$\delta_m = \frac{b}{a} I_m \; , \; u_m = RI_m$$

It is allowed to minimize the number of non-dimensional parameters $h = b^2 / aRt_m$, $T = L / Rt_m$ in mathematical model. These parameters are characterizing transient processes in the electromagnet circuit.

The transition in equations (4), (5) to non-dimensional variable allows converting them to standard linear mathematical model of control system in the matrix form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u(\sigma), \tag{6}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & h/T & -1/T \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 1/T \end{pmatrix}$$

with the function of a limited control

$$u(\sigma) = \begin{cases} +1 & \beta\sigma \geq 1, \\ \beta\sigma & |\beta\sigma| \leq 1, \\ -1 & \beta\sigma \leq -1. \end{cases} \tag{7}$$

Argument of this function is a signal of linear feedback on state variables

$$\sigma = \sum_{k=1}^{3} c_k x_k \quad . \tag{8}$$

The problem consists in finding the parameters $\{c_k\}$ of optimal feedback on state under which is minimized value of square-law functional

$$J = \int_0^\infty (\mathbf{x}^T \mathbf{Q} \mathbf{x} + u^2) dt \tag{9}$$

in a class of functions (7),(8) of a limited control.

3. SYNTHESIS OF CONTROL LAW

3.1 Particularities of object and control system

The particularities magnetic suspension as object of control is defined by the matrixes A, B in equation (6). The object is completely controlled and has one unstable variable y_1^+ corresponding to a single positive eigenvalue $\lambda_1 > 0$ of matrix A. Rest two eigenvalues of matrix A always have negative real parts i.e.

$$\lambda_1^+ > 0, \qquad \text{Re}\,\lambda_k < 0 \quad (k = 2,3) \, . \tag{10}$$

3.2 Optimization of limited control on square-law criterion

Finding of eigenvectors matrix A allows converting equations (6) to Jordan canonical form by canonical variables introducing

$$y = Px \tag{11}$$

by means of non-singular matrix

$$P = \begin{pmatrix} T + \frac{1}{\lambda_1^+} & T\lambda_1^+ + 1 & -T \\ T + \frac{1}{\lambda_2} & T\lambda_2 + 1 & -T \\ T + \frac{1}{\lambda_3} & T\lambda_3 + 1 & -T \end{pmatrix}.$$

In this matrix the first line defines analytical expression of object unstable variable

$$y_1^+ = (T + 1/\lambda_1^+)x_1 + (T\lambda_1^+ + 1)x_2 - Tx_3 . \tag{12}$$

Input signal of a power amplifier under optimal control must depend from unstable variable only

$$\sigma = \lambda_1^+ y_1^+ , \tag{13}$$

i.e. the vector of optimal parameters to feedback on condition will be following

$$C^T = (T + \frac{1}{\lambda_1^+}; T\lambda_1^+ + 1; -T) . \tag{14}$$

As shown in Voronkov, V.S. (2009) minimization square-law functional (9) is achieved for considered object when matrix **Q** in functional has the following elements:

$$q_{11} = q(T + 1/\lambda_1^+)^2 ;\ q_{22} = q(T\lambda_1^+ + 1)^2 ;\ q_{33} = qT^2$$

$$q_{12} = q_{21} = q(T + 1/\lambda_1^+)(T\lambda_1^+ + 1) ;$$

$$q_{13} = q_{31} = -qT(T + 1/\lambda_1^+) ;\ q_{23} = q_{32} = -qT(T\lambda_1^+ + 1) ,$$

$$q \geq 0 \ .$$

This is corresponded to exclusive type square-law criterion (9) in the form

$$J_1 = \int_0^\infty (qy_1^{+2} + u^2)dt , \quad q \geq 0 \ . \tag{15}$$

Its specifics is conditioned the account of limitations control described by functions (7). Depending on choice of parameter q in functional (15) is defined optimal value a gain factor

$$\beta^{opt} = 1 + \sqrt{1 + \frac{q}{\lambda_1^{+2}}} \tag{16}$$

In result got expressions (13), (16) are completely defined optimal on square-law criterion (15) limited control (5) in our case of magnetic suspension.

4. COMPUTER SIMULATION OF ELEMETARY MAGNETIC SUSPENSION WITH OPTIMAL DYNAMICS

The accounting tracks of magnetic suspension dynamics under different gain factors β and concrete values $h = 0$, $T = 2$ of object parameters is considered in order to verify the got analytical results and calculations of values functional (15) for the reason findings of its dependence from gain factor β. Herewith deflections of initial conditions

$$x_{10} = -0.6 ,\ x_{20} = -0.6 ,\ x_{30} = -2.2 \tag{17}$$

from integral manifold

$$\sigma = C^T x = \lambda_1^+ y_1^+ = \lambda_1^+[(T + \frac{1}{\lambda_1^+})x_1 + (T\lambda_1^+ + 1)x_2 - Tx_3] = 0,$$

are chosen alike. Under conditions

$$x_{10} = -0.6 ,\ x_{20} = -0.6 ,\ x_{30} = -1.8$$

the input signal of power amplifier is zero value. In this case moving the system passes under the limits $\bar{u} = +1$ of control function. Results of these calculations with provision for primary finding of control on limit happen to different values of parameter β: equal optimum $\beta^{opt} = 10$, as well as less ($\beta = 8$) and more ($\beta = 12$) its. How follows from these calculations functional (15) really has weakly denominated minimum under found optimum limited control at choice in (15).parameter $q = 80$. Such choice corresponds to the optimal value of a gain factor $\beta^{opt} = 10$ usually used in practice.

5. DYNAMICS OF MAGNETIC SUSPENSION WITH MULTI-DEGREES OF FREEDOM

In order to stabilize a body in magnetic suspension even at its presentation by solid it is required the account of large number degrees of freedom. In instrument applications of magnetic suspension (Morrison, M.M. (1988)) is required the account six degrees of freedom: three onward and three angular. In active magnetic bearings of rotary systems (Somov Ye.I. et all (1991)) are required the account at least fife degrees of freedom: three onward and two angular. Besides number of degrees of freedom for magnetic suspension as object of control increases with provision for real existing additional electric and mechanical degrees of freedom greatly complicating the problem of control law synthesis. Proposed approach to decision of the synthesis problem in magnetic suspension is founded on account only unstable variables of object the number of which as a rule are greatly less then full dimensionality of its mathematical model.

6. MAIN RESULTS OF STUDY AND DESINING MAGNETIC SUSPENSION AT CONCRETE DEVICES

The qualitative stabilization magnetic suspension of sensitive element in measuring instruments has allowed using its advantages in contrast with contact types of suspension. In balance with magnetic suspension (Voronkov, V.S., Sigun'kov S.A. (1996)) there has reached a relative error equal $0.8 \cdot 10^{-5}$. In pendulous gyrocompasses with magnetic suspension (Voronkov, V.S. , Pozdeev O.D. (1995)) there has reached level of harmful torque equal 10^{-10} Nm.

7. CONCLUSIONS

The obtained results show the possibility of analytical synthesis of limited control in magnetic suspension providing minimum square-law functional exclusive type (15). The difficulties of analytical decision of the problem are overcome in this instance as offered in Voronkov, V.S. (2009). This approach allowing by means of unstable variable separation of object to reduce the order optimized mathematical system model before first. Moreover minimization square-law criterion (15) provides the coincidence of system controllability area with area of attraction of stabilized equilibrium. This allows validly choosing the levels of limits on control.

8. ACKNOWLEDGMENTS

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