ADAPTIVE MODEL TRACKING WITH MITIGATED PASSIVITY CONDITIONS

Itzhak Barkana [1]

Abstract: Feasibility of nonlinear and adaptive control methodologies in multivariable linear time-invariant systems with state space realization $\{A, B, C\}$ has apparently been limited by the standard strict passivity (or positive realness) conditions that imply that the product CB must be positive definite symmetric. A recent paper has managed to mitigate the symmetry condition, requiring instead that the positive definite and not necessarily symmetric matrix CB be diagonalizable. Although the mitigated conditions were useful in proving pure stabilizability with Adaptive Controllers, the Model Tracking question has remained open. This paper further extends the previous results, showing that the new passivity conditions can be used to guarantee stability of the adaptive control system and asymptotically perfect model tracking.

Keywords: Control systems, stability, passivity, uncertain systems, almost strict passivity (ASP), adaptive control

1. INTRODUCTION

Consider the square system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

Here, x is the n-dimensional state vector, u is the m-dimensional input vector and y is the m-dimensional output vector, and A, B, and C are matrices of corresponding dimensions. Because in various methodologies of nonstationary control the stability analysis concerns both the state and the dynamical gains, stability of the control system has been treated with positive definite quadratic Lyapunov functions of the form

$$V(t) = x^T(t)Px(t) + tr[(K(t) - \widetilde{K})\Gamma^{-1}(K(t) - \widetilde{K})^T]. \tag{3}$$

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Here, $K(t)$ is the adaptive gain used with the controller $u(t) = K(t)y(t)$ and $\widetilde{K}$ represents an ideal output feedback gain. Define

$$A_K = A - B\widetilde{K}C \tag{4}$$

Although the proofs of stability using (3) do not require the original system to be strictly positive real (SPR), they require the existence of a constant output feedback gain $\widetilde{K}$ (unknown and not needed for implementation) that could make the fictitious closed-loop system $\{A_K, B, C\}$ is SPR. The common state-space definition of the strictly positive-realness property in linear time invariant systems is:

Definition 1. A linear time-invariant system with a state-space realization $\{A_K, B, C\}$, where $A_K \in \mathbf{R}^{n,n}$, $B \in \mathbf{R}^{n,m}$, $C \in \mathbf{R}^{m,n}$, with the $m * m$ transfer function $T(s) = C(sI - A_K)^{-1}B$, is called 'strictly passive (SP)' and its transfer function 'strictly positive real (SPR)' if there exist two positive definite symmetric (PDS) matrices, P and Q, such that the following two relations are simultaneously satisfied:

$$PA_K + A_K^T P = -Q \tag{5}$$

$$PB = C^T. \tag{6}$$

Relations (5)-(6) have been shown to be very useful in nonlinear control applications and in particular in control with uncertainty or in adaptive control (Fradkov, 1976), (Sobel *et al.*, 1982), (Steinberg and Corless, 1985), (Barkana and Kaufman, 1985), (Zeheb, 1986), (Fradkov and Hill, 1998). The original system that only needs a constant output feedback to become strictly positive real has been called 'almost strictly positive real (ASPR)' (Barkana and Kaufman, 1985), (Barkana, 1987), also called 'feedback passive' or 'passifiable'. For quite a long time, the meaning and practical implications of ASPR systems has remained rather obscure within the adaptive control community, although even as early as 1976 has been shown (Fradkov, 1976) that any minimum-phase system with a positive definite *symmetric* matrical product CB can be rendered SPR via constant output feedback, and many other works have re-invented and further developed the idea since (see (Barkana, 2004*a*) and the references therein for a brief history and direct proof of this important statement). The importance of this specific class of systems has gradually gained more and more acceptance in the control community (Kokotovic and Arcak, 2001). Moreover, the class of ASPR systems proved to be more general that initially thought when Huang et al. (Huang *et al.*, 1999) showed that if a system cannot be made SPR via constant output feedback, no dynamic feedback can render it SPR.

However, from relation (6) and its transpose one gets

$$B^T PB = B^T C^T = CB > 0. \tag{7}$$

The non-singularity of CB implies that the transfer function $T(s)$ has n poles and $n-m$ zeros, yet (7) also implies that the customary SPR relations can be applied only to systems where the product CB is *positive definite symmetric* (PDS).

While the implied positivity of CB could be expected and understood as a natural extension of the sign condition in SISO systems, the symmetry condition seemed to limit the applicability of adaptive control techniques, as its satisfaction in uncertain systems may be difficult to guarantee, in general. Although both requirements seemed to be needed for the proof of stability with adaptive controllers, a recent publication (Barkana, Teixeira and Hsu, 2006) showed that the symmetry condition can be mitigated and that the positive definite matrix CB must only be diagonalizable.

2. MITIGATION OF THE SYMMETRY CONDITION

While investigating ways that would possibly mitigate the symmetry assumption on the product CB and thus extend the feasibility of the SPR concept to larger classes of systems, it was intuitive (Barkana, Teixeira and Hsu, 2006) to try the new Lyapunov function

$$\begin{aligned} V(t) &= x^T(t)Px(t) \\ &\quad + tr[S(K(t)-\widetilde{K})\Gamma^{-1}(K(t)-\widetilde{K})^T S^T]. \end{aligned} \tag{8}$$

Note that any nonsingular matrical factor S (unknown and not needed for implementation) would allow the matrical product in the second term in (8) to be positive definite symmetric and thus the trace to be positive definite. Borrowing a definition that has been introduced by Fradkov and his colleagues (Fradkov, 2003), (Peaucelle, Fradkov and Andrievski, 2005), the applicability of passivity conditions was extended (Barkana, Teixeira and Hsu, 2006) via the following definition:

Definition 2. Under the assumption of Definition 1, the state-space realization $\{A_K, B, C\}$ is called W-Strictly Passive (WSP) and its transfer function $T(s) = C(sI - A_K)^{-1}B$ is called W-Strictly Positive Real (WSPR) if there exist three positive definite symmetric matrices, P, Q, and $W = S^T S$ such that the following two relations are simultaneously satisfied:

$$PA_K + A_K^T P = -Q \tag{9}$$

$$PB = C^T W \tag{10}$$

It is important to note that the symmetry condition for $W = S^T S$ has initially originated in the requirement that S in the Lyapunov function (8) be nonsingular and the second term in (8) be positive definite.

Furthermore, it was shown (Barkana, Teixeira and Hsu, 2006) that a system can become WSP via constant output feedback if it is minimum-phase and if the positive definite and not necessarily symmetric product CB is diagonalizable. Finally, it was also shown that the WSP conditions (9)-(10) are sufficient conditions that can guarantee stability with adaptive output feedback controllers (Barkana, Teixeira and Hsu, 2006).

The development in (Barkana, Teixeira and Hsu, 2006) had thus finally ended with a straightforward result that managed to mitigate a symmetry condition that had been around for more than 40 years.

Note: Although it mitigates a long established symmetry condition, Definition 2 still excludes those systems where the positive definite CB

product has a regular Jordan, rather than diagonal, form. While recommending (Barkana, Teixeira and Hsu, 2006) for publication, one reviewer suggested that maybe the symmetry condition on W could also be eliminated. As this suggestion reflected this author's own long unfulfilled desire, this renewed challenge has "almost" led to the next mitigation of the passivity conditions. The attempted next step is based on the observation that, although the product of two positive definite matrices M and N is is not necessarily positive definite, the trace of the product, tr(MN), *is* positive definite if at least one matrix is also symmetric. With this observation, one can use the new Lyapunov function

$$\begin{aligned} V(t) &= x^T(t)Px(t) \\ &+ tr[W(K(t)-\widetilde{K})\Gamma^{-1}(K(t)-\widetilde{K})^T]. \end{aligned} \tag{11}$$

The second term in (11) (i.e., the trace) is thus positive definite even if the positive definite matrix W is *not* necessarily symmetric. This could apparently lead to further relaxation of the passivity conditions, because one can show that the existence of such a W that is PD and not necessarily symmetric is then guaranteed if the product CB has just all eigenvalues in the right half plane, even if CB is *neither* symmetric *nor* Positive Definite. However, as we show in this paper, although some examples may show stability, at least at this stage this attempt finally proved to be an exercise in futility because, ultimately, the Lyapunov *derivative* does require W to be Positive Definite Symmetric.

To avoid any eventually misleading interpretation of the new definition, we again emphasize here that the (fictitious) matrix W is not needed or used, that conditions (9)-(10) represent properties of the *original* plant $\{A, B, C\}$, and the controller controls the output $y(t) = Cx(t)$ of this original plant. Still, it is interesting to mention that these WSP relations are equivalent to requiring that an *associated* (fictitious) system $\{A_K, B, WC\}$, with the output given by $z(t) = Wy(t)$, be Strictly Passive and its *associated* transfer function, $T_a(s) = WC(sI - A_K)^{-1}B$, be SPR, in plain accord with the customary Definition 1.

We will show that this simple result allows the applications of the useful passivity properties without requiring the customary CB symmetry condition.

3. APPLICATION OF THE WASP PROPERTY TO ADAPTIVE CONTROL

As most systems are not WSP, we called WASP those systems that only require an (assumably unknown) constant, positive definite, gain $\widetilde{K}_e$ to render the fictitious closed-loop system WSP. In other words, given the system (1)-(2), if one can assume that the (possibly unstable) plant is minimum-phase and that all eigenvalues of CB are located in the right half-plane, the fictitious control

$$u(t) = \widetilde{K}_e y(t) \tag{12}$$

would result in the closed-loop system

$$\dot{x}(t) = [A - B\widetilde{K}_e C]x(t) + Bv(t) \tag{13}$$

$$y(t) = Cx(t) \tag{14}$$

that satisfies the WSP relations

$$P[A - B\widetilde{K}_e C] + [A - B\widetilde{K}_e C]^T P = -Q \tag{15}$$

$$PB = C^T W^T \tag{16}$$

The new conditions and definitions are important only if one can show that they are as useful as the customary SPR conditions for the proofs of stability with adaptive controllers. Here we note that with the publication of (Barkana, Teixeira and Hsu, 2006), some colleagues have expressed their concern that the uncertainty in W may actually eliminate the usefulness of the WASP relations in the tracking case. Therefore, instead of the simple adaptive stabilizing illustration of (Barkana, Teixeira and Hsu, 2006), in this paper we decided to present a full adaptive model tracking case that uses the Simple Adaptive Control (SAC) methodology (Kaufman, Barkana and Sobel, 1998), (Barkana, 2007). Specifically, we assume that the plant output is required to track the output of a 'model'.

3.1 Model following with SAC

In SAC methodology, the so-called 'model' is simply a stable plant that only serves to generate the trajectory that the plant should follow, and for this reason it is also called 'command generator' and the methodology is sometimes called 'command generator tracking'. Otherwise, the 'model' is not required to reproduce the plant or to use any prior knowledge about the plant and can also be of any (lower or larger) order insofar as it generates the desired trajectory. As usual in adaptive control, one first assumes that the underlying fully deterministic output model tracking problem is solvable. A recent publication (Barkana, 2005) shows that if the Model Reference uses a step input in order to generate the desired trajectory, the underlying tracking problem is *always* solvable. If, instead, the model input command is itself generated by an unknown system of order n_u, the model is required to be sufficiently large to accommodate this command (Barkana, 1983), (Kaufman, Barkana and Sobel, 1998), or

$$n_m + m \geq n_u \tag{17}$$

The model is:

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) \tag{18}$$

$$y(t) = C_m x_m(t) \tag{19}$$

Here, x_m is the n_m-dimensional state vector, u_m is the m-dimensional input vector and y_m is the m-dimensional output vector, and A_m, B_m, and C_m are matrices of corresponding dimensions.

The simple adaptive control (SAC) algorithm (Sobel *et al.*, 1982), (Barkana and Kaufman, 1985) monitors the tracking error

$$e_y(t) = y_m(t) - y(t) \tag{20}$$

and the available model variables, x_m and u_m, and uses the following reference vector

$$r^T(t) = [e_y(t)\ x_m(t)\ u_m(t)]^T \tag{21}$$

to generate the adaptive control gains

$$K(t) = [K_e(t)\ K_x(t)\ K_u(t)] \tag{22}$$

through the procedure

$$\dot{K}(t) = e_y(t) r^T(t) \Gamma \tag{23}$$

and the adaptive control signal

$$\begin{aligned} u(t) &= K(t) r(t) \\ &= K_e(t) e_y(t) + K_x(t) x_m(t) + K_u(t) u_m(t). \end{aligned} \tag{24}$$

Here, Γ is a positive definite scaling matrix that regulates the rate of adaptation. The underlying deterministic tracking problem assumes that there exists an ideal control

$$u^*(t) = \widetilde{K}_x x_m(t) + \widetilde{K}_u u_m(t). \tag{25}$$

that could keep the plant along an ideal trajectory $x^*(t)$ that would asymptotically perform perfect tracking. In other words, the ideal plant

$$\dot{x}^*(t) = A x^*(t) + B u^*(t) \tag{26}$$

$$y^*(t) = C x^*(t) \tag{27}$$

moves along "ideal trajectories" such that

$$y^*(t) = y_m(t) \tag{28}$$

A recent work (Barkana, 2005) has given a thorough treatment to the existence of the underlying ideal control gains. It was shown that such ideal control gains always exist under the minimum-phase assumption. Therefore, here we can assume that the underlying problem is solvable and thus, that some ideal gains $\widetilde{K}_x$ and $\widetilde{K}_u$ exist. Because the plant and the model can have different dimensions, the 'following error' $e_x(t)$ is defined to be the difference between the ideal and the actual plant state

$$e_x(t) = x^*(t) - x(t) \tag{29}$$

and correspondingly

$$e_y(t) = y_m(t) - y(t) = y^*(t) - y(t) = C e_x(t) \tag{30}$$

Differentiating (29) gives:

$$\begin{aligned} \dot{e}_x(t) &= \dot{x}^*(t) - \dot{x}(t) \\ &= A x^*(t) + B u^*(t) - A x(t) - B u(t) \\ &= A e_x(t) - B(u(t) - u^*(t)) \end{aligned} \tag{31}$$

Adding and subtracting $B\widetilde{K}_e e_y(t)$ above gives

$$\dot{e}_x(t) = (A - B\widetilde{K}_e C) e_x(t) - B\left[K(t) - \widetilde{K}\right] r(t) \tag{32}$$

where for convenience we denoted

$$\widetilde{K} = [\widetilde{K}_e\ \widetilde{K}_x\ \widetilde{K}_u]. \tag{33}$$

This long introduction allows the proof of the following theorem of stability:

Theorem 1. Under the WASP conditions and the assumptions of this subsection, all gains and state variables of the Adaptive Control system represented by (23) and (32) are bounded and the system performs asymptotically perfect tracking.

PROOF. The positive definite Lyapunov function (11) applied to the adaptive system (23) and (32) is

$$\begin{aligned} V(t) &= e_x^T(t) P e_x(t) \\ &\quad + tr[W(K(t) - \widetilde{K}) \Gamma^{-1} (K(t) - \widetilde{K})^T]. \end{aligned} \tag{34}$$

At this stage, we do not require W to be symmetric. The derivative of $V(t)$ is

$$\begin{aligned} \dot{V}(t) &= \dot{e}_x^T(t) P e_x(t) + e_x^T(t) P \dot{e}_x(t) \\ &\quad + tr[W \dot{K}(t) \Gamma^{-1} (K(t) - \widetilde{K})^T] \\ &\quad + tr[W (K(t) - \widetilde{K}) \Gamma^{-1} \dot{K}^T(t)] \end{aligned} \tag{35}$$

$$\begin{aligned} \dot{V}(t) &= e_x^T(t) P (A - B\widetilde{K}C) e_x(t) \\ &\quad + e_x^T(t) (A - B\widetilde{K}C)^T P e_x(t) \\ &\quad - e_x^T(t) P B [K(t) - \widetilde{K}] r(t) \\ &\quad - r^T(t) [K(t) - \widetilde{K}]^T B^T P e_x(t) \\ &\quad + tr[W e_y(t) r^T(t) \Gamma \Gamma^{-1} (K(t) - \widetilde{K})^T] \\ &\quad + tr[W (K(t) - \widetilde{K}) \Gamma^{-1} \Gamma r(t) e_y^T(t)] \end{aligned} \tag{36}$$

Recalling that $tr(AB) = tr(BA)$, $x^Ty = y^Tx$, and $tr(x^Ty) = x^Ty$ and using the WASP relations gives

$$\begin{aligned} &\dot{V}(t) = \\ &e_x^T(t)[P(A - B\widetilde{K}C) + (A - B\widetilde{K}C)^T P]e_x(t) \\ &\quad - e_x^T(t)C^T W^T [K(t) - \widetilde{K}]r(t) \\ &\quad - r^T[K(t) - \widetilde{K}]^T WCe_x(t) \\ &\quad + e_x^T(t)C^T W[K(t) - \widetilde{K}]r(t) \\ &\quad + r^T(t)[K(t) - \widetilde{K}]^T WCe_x(t)] \end{aligned} \tag{37}$$

One of the last two terms in (37), originating in the derivative of the adaptive gain terms in V(t), cancels a previous, possibly troubling, non-positive, term and thus lead to the Lyapunov derivative

$$\begin{aligned} &\dot{V}(t) = \\ &e_x^T(t)[P(A - B\widetilde{K}C) + (A - B\widetilde{K}C)^T P]e_x(t) \\ &\quad + e_x^T(t)C^T(W - W^T)[K(t) - \widetilde{K}]r(t) \end{aligned} \tag{38}$$

At this stage, one can see that the *symmetry* of W is obviously *needed* to finally get

$$\dot{V}(t) = -e_x^T(t)Qe_x(t). \tag{39}$$

The Lyapunov derivative $\dot{V}(t)$ in (39) is thus negative definite with respect to $e_x(t)$, although only negative semidefinite with respect to the entire state-space $\{e_x(t), K(t)\}$. Although the direct result of Lyapunov stability theory is only that all dynamic values are bounded, according to LaSalle's invariance principle (Kaufman, Barkana and Sobel, 1998), all state-variables and adaptive gains are bounded and the system ultimately ends within the domain defined by $\dot{V}(t) \equiv 0$. Because $\dot{V}(t)$ is negative definite in e_x, the system thus ends with $e_x(t) \equiv 0$, that in turn implies $e_y(t) \equiv 0$. In other words, the adaptive control system demonstrates asymptotic convergence of the state and output error and boundedness of the adaptive gains. Furthermore, it has been recently shown (Barkana, 2005) that the adaptive control gains ultimately reach a set of stabilizing constant values at the end of a steepest descent minimization of the tracking error. QED.

Finally, it was shown that, while its assumed existence indeed facilitates the proof of stability and asymptotically perfect tracking of the adaptive control system without requiring the symmetry of CB, the *fictitious* symmetric matrix W and its assumed uncertainty play no role in implementation and have no (negative) effect whatsoever on the asymptotic tracking properties of the adaptive control system.

4. COUNTEREXAMPLES TO STANDARD MODEL REFERENCE ADAPTIVE CONTROL

For an illustration of the stability properties of Simple Adaptive Control, in this section we use some "counterexamples" that lead to divergence when standard gradient-based MRAC techniques are applied (Hsu and Costa, 1999). In these examples, a 2*2 stable plant with CB positive definite is required to follow the behavior of a stable model of same order. Both the plant and the model have diagonal system matrices and same negative eigenvalues, and only the input-output matrix differentiates between the two. The plant, a 2D adaptive robotic visual servoing with uncalibrated camera, is defined by the system matrices

$$A = \begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix} B = \begin{bmatrix} cos\varphi & sin\varphi \\ -hsin\varphi & hcos\varphi \end{bmatrix} C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{40}$$

The simple model is defined by the matrices

$$A_m = \begin{bmatrix} -a & 0 \\ 0 & -a \end{bmatrix} B_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} C_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{41}$$

It is shown (Hsu and Costa, 1999) that the standard MRAC system becomes unstable with $a = \varphi = 1, h = 0.5$, and $u_m = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$ or with $a = 9, \varphi = 1, h = 0.5$, and $u_m = \begin{bmatrix} 10sint \\ 10cost \end{bmatrix}$. The divergence is treated in detail in (Hsu and Costa, 1999) and we only mention it here because instability occurred in both cases although there was no "unmodeled dynamics," there was "sufficient excitation," and the required "sufficient" passivity conditions were also satisfied. It is worth mentioning that this was a theoretical analysis that resulted in a diverging equation. Here, we revisit the examples and attempt to use the simplicity of SAC, so we first use the same slow adaptation rate as (Hsu and Costa, 1999), $\gamma = 1$, with all adaptive gains. However, because the rate of adaptation is theoretically unlimited with SAC, we will later show that higher adaptation rates not only do not affect the stability but they also definitely result in superior performance.

1) First case: $h = 0.5,\ a = 1,\ \varphi = 1,\ u_m(t) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$. Note that CB is positive definite, yet not symmetric. One can see that the SAC system indeed shows a stable behavior. Because all initial adaptive gains are zero and the rate of adaptation is slow, one can see an expected large transient before all values converge and the tracking error vanishes.

2) A second case was run with two sinusoidal input commands: $a = 9,\ u_m(t) = \begin{bmatrix} 10sint \\ 10cost \end{bmatrix}$. SAC again

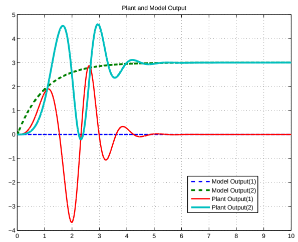

Fig. 1. Case 1: Plant and Model Output

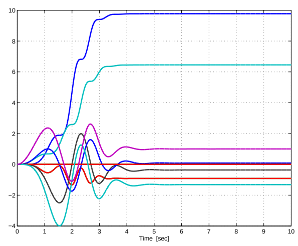

Fig. 2. Case 1: Adaptive Gains

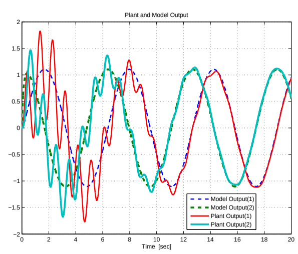

Fig. 3. Case 2: Plant and Model Output

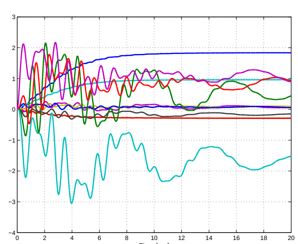

Fig. 4. Case 2: Adaptive Gains

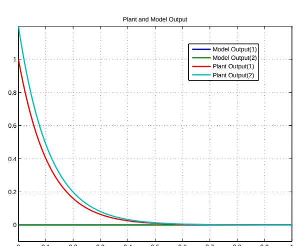

Fig. 5. Case 3: Plant and Model Output

shows stable behavior while the standard MRAC diverged.

3) Third case: Here, we run the second case after we eliminate any input command in order to avoid any impression that the SAC might need input excitation: $a = 9,\ u_m(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

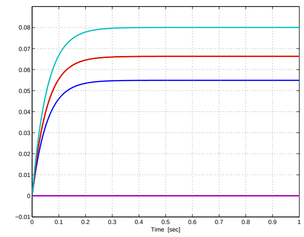

Fig. 6. Case 3: Adaptive Gains

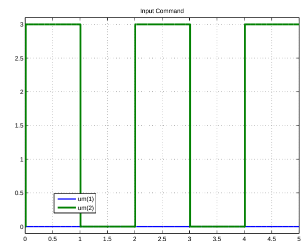

Fig. 7. Case 4: Input Commands

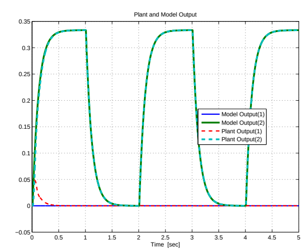

Fig. 8. Case 4: Plant and Model Output

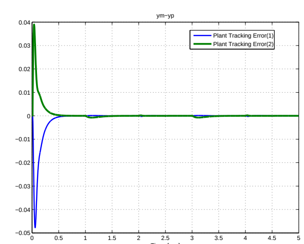

Fig. 9. Case 4: Output Tracking Errors

Note that if the plant starts at zero initial conditions, it would remain there. Therefore, we just gave it some initial conditions $\begin{bmatrix} 1 \\ 1.2 \end{bmatrix}$. With no input command, while all model states are zero, K_x and K_u remain zero, while K_e reaches some stabilizing value. Here, one can see that the SAC system remains stable with no dependence on the existence of a model or input command.

4) For an illustration of SAC performance, we again run the second case, yet the step input becomes a square wave input. Here we use high adaptation coefficients $\gamma_e = 1e4,\ \gamma_x = 1e2,\ \gamma_u = 1e2$

As seen below, the plant follows the model so closely that most of the time, except for an initial adaptation transient, the model and plant positions practically coincide.

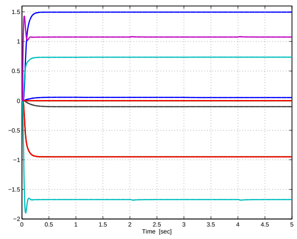

Fig. 10. Case 4: Adaptive Gains

Note that all adaptations start with zero initial gain values. In practice, any initial use with the specific plant would allow selection of initial gains that would reduce the transient response even more (Kaufman, Barkana and Sobel, 1998).

4) In all tests above, the matrix product CB was Positive Definite and not symmetric, yet diagonalizable (having distinct eigenvalues). Such case guarantees the existence of a symmetric W, as required by the proof of stability. In spite of that, we here decided to test the stability of the adaptive system when CB is not Positive Definite or symmetric. Here we again ran the previous case with $\varphi = 1.25$. The input-output matrix product is now

$$CB = \begin{bmatrix} 0.3153 & 0.9490 \\ -0.4745 & 0.1577 \end{bmatrix} \tag{42}$$

and one can easily see that CB is not positive definite or symmetric, yet its eigenvalues, 0.2365 - 0.6664i and 0.2365 - 0.6664i, are located in the right half plane. As it may happen with nonlinear control applications, the results of simulation test under the same conditions as in test 3 above showed exactly the same performance, although the sufficient condition is not satisfied.

5. CONCLUSIONS

This paper eliminates an apparent limitation of Adaptive Model tracking and thus extends the applicability of the important passivity properties from minimum-phase systems that have a positive definite symmetric CB product to the larger class of systems where the Positive Definite CB is only diagonalizable and not necessarily symmetric.

The results of this paper are important for the extension of feasibility of adaptive and nonlinear control to real-world systems that are not necessarily WSP or WASP and therefore may not satisfy even the new, relaxed, WASP conditions. For such systems has been shown that if a controller H stabilizes the system G, then the augmented system $G + H^{-1}$ is minimum-phase (Barkana, 1987). This way, with proper selection of the relative degree of H^{-1}, basic stabilizability properties of systems could be used to implement WASP configurations. However, because the symmetry of CB of the (basically unknown) augmented system cannot be guaranteed, this paper facilitates the use of parallel feedforward by eliminating a fundamental limitation of the approach. In various design environments, one can use available prior knowledge to either devise a stabilizing controller first (Barkana, 1987), (Kaufman, Barkana and Sobel, 1998), or directly the 'parallel feedforward configuration (PFC)' or 'shunt' (Iwai and Mizumoto, 1992), (Fradkov, 1994), (Betser and Zeheb, 1995).

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