INVERSE OPTIMAL CONTROL APPROACH FOR PINNING COMPLEX DYNAMICAL NETWORKS

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Abstract

In this paper, a new control strategy is proposed for pinning complex weighted networks based on the Inverse Optimal Control approach. A control law is developed for stabilization of the network and minimization of an associated cost functional.

Key words

Complex Networks, Synchronization, Pinning Control, Inverse Optimal Control.

1 Introduction

Complex Networks, also called complex systems, are interesting due to their possible applications on diverse fields, from biological and chemical systems to electronic circuits and social networks. Complex Networks are studied to model and analyze process and phenomena consisting of interacting elements named nodes, and to control their global and individual behavior [Aström et al. 2001], [Bocaleti, 2006] and [Chen, Wang and Li 2012]. Since new discoveries about their structural characteristics were underline on seminal papers [Barabási and Albert 1999], [Erdös and Rnyi 1959], [Strogatz 2001] and [Watts and Strogatz 1998] , intensive research has been developed on this field. The models used to describe complex networks in continuous time derive from graph theory and from the Kuramoto model of linear coupling oscillators [Strogatz 2000]. Many models have been developed with different structures and coupling characteristics like the small world model [Strogatz 2001], the E-R random graph model [Erdös and Rnyi 1959] and the Barabási-Albert power law degree distribution model [Barabási and Albert 1999].

Synchronization is a desirable feature [Pikovsky, Rosenblum and Kurths 2001]; examples are the identical oscillators in cardiac peacemaker cells or the waves propagation in a brain [Strogatz 2001]. Results have showed that synchronization takes place only if structural and coupling restrictions are fulfilled. One example is the master stability function; another result is the derived from Wu-Chua conjecture in [Li, Wang and Chen 2004] which correlates the coupling strength with structural Laplacian matrix. In order to guarantee synchronization, efficient control techniques must be applied and developed [Chen, Wang and Li 2012].

The basic idea of Pinning Control is to use the network structure to contribute in its regulation; with this end a local control action is applied to a small number of nodes, fixing it dynamics at a desired equilibrium point [Ramirez 2009]. How many and which nodes to select is an open problem yet. The contrast between random and specific pinning have been investigated for different topologies [Li, Wang and Chen 2004]. Measures like degree distribution, clustering coefficient, average shortest path length, efficiency, betweness, coreness and asorativity have been enounced to characterize nodes importance and their surroundings [Bocaleti, 2006].

Techniques like proportional control have been implemented [Chen, Liu and Lu 2007]; other advanced techniques like geometric control [Solis-Perales, Rodriguez and Obregon-Pulido 2010] or adaptable control [Jin and Yang 2012] and [Zhou, Lu and Lü 2008], have been applied with good results. To minimize the control effort and to ensure stability margins are important issues; in this paper we use the inverse optimal control approach to obtain those desirable feature for Pinning Control.

The present paper is organized as follows: in section II required preliminaries are presented followed by inverse optimal control application to nodes in a network is presented in section III, finally numerical simulations are included in section IV.

2 Mathematical Preliminaries

Definition 1 [Li, Wang and Chen 2004]. The Kronecker product of two matrices A and B is defined as:

$$A \otimes B = \begin{pmatrix} a_{11}B \cdots a_{1m}B \\ \vdots \quad \ddots \quad \vdots \\ a_{n1}B \cdots a_{nm}B \end{pmatrix}$$

where if A is an $n \times m$ matrix and B is a $p \times q$ matrix, then $A \otimes B$ is an $np \times mq$ matrix.

Definition 2 [Li, Wang and Chen 2004]. The product $A \otimes f(x_i, t)$ is defined as:

$$A \otimes f(x_i, t) = \begin{pmatrix} a_{11}f(x_1,t) + a_{11}f(x_i,t) + \cdots + a_{1m}f(x_m,t) \\ \vdots \\ a_{n1}f(x_1,t) + a_{11}f(x_i,t) + \cdots + a_{nm}f(x_m,t) \end{pmatrix}$$

where if A is an $n \times m$ matrix and f is a $p \times 1$ function then $A \otimes f(x_i, t)$ is an $np \times 1$ vector.

Definition 3 [Krstić and Deng 1998]. The Legendre-Fenchel Transform denote by ℓ is defined as:

$$\ell\gamma(r) = r(\gamma')^{-1}(r) - \gamma((\gamma')^{-1}(r)) \quad (1)$$

where γ, γ' are class K_∞ [Krstić and Deng 1998] and $(\gamma')^{-1}(r)$ stands for the inverse function of $\frac{d\gamma(r)}{dr}$. The Legendre-Fenchel Transform has the next property:

Property 1 [Krstić and Deng 1998]. If a function γ and its derivative γ' are class K_∞ then $\ell\gamma$ is a class K_∞ function.

Definition 4 [Li, Wang and Chen 2004]. Given a square matrix V, a function $\phi : R^n \times R \to R^n$ is V-uniformly increasing if:

$$(x-y)^T V(\phi(x,t) - \phi(y,t)) \geq \sigma\|x-y\|^2 \quad (2)$$

The function ϕ is V-uniformly decreasing if $-\phi$ is V-uniformly increasing; in other words ϕ is V-uniformly decreasing if:

$$(x-y)^T V(\phi(x,t) - \phi(y,t)) \leq -\sigma\|x-y\|^2 \quad (3)$$

Definition 5 [Khalil 2002]A function $f(x) : R^n \to R$ is radially unbounded if:

$$\|x\| \to \infty \Rightarrow f(x) \to \infty \quad (4)$$

2.1 Complex Networks

The General Complex Dynamical Weighted Network Model [Li, Wang and Chen 2004] is described as:

$$\dot{x}_i = f(x_i) + \sum_{j=1, j\neq i}^{N} c_{ij}a_{ij}\Gamma(x_j - x_i) \quad i = 1, 2, \ldots, N \quad (5)$$

where $x_i = (x_1, x_2, ..., x_n)^T \in R^n$ is the state vector variables of node i, the constant $c_{ij} > 0$ represent the coupling strength between node i and j; $\Gamma \in R^{n\times n}$ is a matrix linking coupled variables; the corresponding dynamical function $f(x_i)$ is the same for all $i = 1, 2 \ldots, N$. We omit the time dependence for simplicity.

The matrix $A = (a_{ij}) \in \Re^{N\times N}$ represent the topology or structure of the network; examples of topologies are the small world [Watts and Strogatz 1998], the scale free networks [Barabási and Albert 1999] and the homogeneous random network [Erdös and Rnyi 1959]; it is assumed the network (5) to be undirected network [Bocaleti, 2006] for which $a_{ij} = a_{ji} = 1$ for connected nodes and $a_{ij} = a_{ji} = 0$ otherwise.

Let the diagonal elements of A be $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$, then $-A$ is symmetric positive definite.

We suppose the network is connected in the sense of having no insolated clusters; then A is irreducible and $-A$ has the eigenvalues $eig(A) = \{0 = \lambda_1 < \lambda_2 \ldots \leq \lambda_N\}$; this properties guarantee an undirected, full connected, diffusively coupled network with not self-connected nodes [Li, Wang and Chen 2004].

Definition 6 [Li, Wang and Chen 2004]. The degree of a node i denoted by k_i is defined as the number of its connections, and is expressed by:

$$k_i = \sum_{j=1, j\neq i}^{N} a_{ij} = \sum_{j=1, j\neq i}^{N} a_{ji} \quad (6)$$

Assumption 1. The coupling strengths $c_{i,j}$ fulfill the next diffusive property:

$$c_{ii}a_{ii} + \sum_{j=1, j\neq i}^{N} c_{ij}a_{ij} = 0 \quad (7)$$

A pin controlled complex dynamical network of the form (5) which fulfils (7) can be expressed as follows:

$$\begin{aligned} \dot{x}_i &= f(x_i) + \sum_{j=1}^{N} c_{ij} a_{ij} \Gamma x_j + u_i \\ & i = 1, 2, \ldots l \\ \dot{x}_i &= f(x_i) + \sum_{j=1}^{N} c_{ij} a_{ij} \Gamma x_j \\ & i = l+1, \ldots, N \end{aligned} \tag{8}$$

where where $l = [\delta N]$ is the fraction of nodes to be controlled taking the nearest integer to δN with $0 < \delta << 1$ and $u_i \in R^m$ is a control vector.

Let be $G = (g_{ij}) \in R^{N \times N}$ where $g_{ij} = -c_{ij} a_{ij}$, and $D' \in R^{N \times N}$ defined as $D' = diag(c_{11} d_1, c_{22} d_2, \ldots c_{ll} d_l, 0, \ldots, 0)$, where $d_{ij} > 0$ are control gains, and define the matrix $(G + D')$.

Condition (7) guarantees that G is a zero sum row matrix; considering $c_{ij} > 0$ and $c_{ij} = c_{ji}$, then G is a irreducible, symmetric and semi-positive definite matrix, and consequently $(G + D') \geq 0$, implying that $\lambda_{min}(G + D') \geq 0$. We can express (8) as:

$$\dot{X} = I_N \otimes [f(x_i)] - [(G+D') \otimes \Gamma] X + (D' \otimes \Gamma) \bar{X} \tag{9}$$

where $X \in R^{Nn \times 1}$ is defined as $X = (x_1^T, x_2^T, \ldots, x_N^T)^T$ and $\bar{X} = (x_s^T, x_s^T, \ldots, x_s^T)^T$.

Taking:

$$\alpha = \frac{C}{\lambda_{min}(\Gamma)}$$

if the next condition is fulfilled by (9)

$$\lambda_{min}(G + D') = \alpha + \delta = \frac{C}{\lambda_{min}(\Gamma)} + \delta \tag{10}$$

where $0 < \delta \in R$, then local asymptotically stability at the homogeneous stationary state x_s and synchronization in the entire network is guaranteed. For $\dot{x}_i = f(x_i)$ as a chaotic system C will be the maximum Lyapunov exponent h_{max} [Li, Wang and Chen 2004].

Definition 7 [Wang and Chen 2003]. A dynamical network is said to be asymptotically synchronized if:

$$x_1(t) = x_2(t) = \ldots = x_N(t) = x_s(t) \quad as \quad t \to \infty \tag{11}$$

where $x_s(t) \in R^n$ is a solution of an isolated node.

2.2 Optimal Control

Optimal Control has as its main objective the gain assignment in a feedback control loop which minimize a cost functional. In the direct approach, it has to be solved the so-called Hamilton-Jacobi-Bellman (HJB) equation, which is not an easy task. This fact motivates to solve the inverse optimal control stabilization; in the inverse approach, a stabilizing feedback is designed first and then it is establish that it optimizes a cost functional.

Consider a system of the form:

$$\dot{x} = f(x) + g_1(x) d + g_2(x) u \tag{12}$$

where $u \in \Re^m$ is a control input, d stands for a addition disturbance and $f(0) = 0$, omitting the time dependence of f, g_1 and g_2, for notational simplicity.

Definition 8 [Khalil 2002]. The Lie derivative of h along $f(x)$ is defined as:

$$L_f h(x) = \frac{\partial h}{\partial x} f(x) \tag{13}$$

where $f : D \to R^n$ and $h(x) : D \to R$.

Theorem 1 [Krstić and Deng 1998]. If a system of the form (12) is input-to-state stabilizable then the inverse optimal problem is solvable.

Theorem 2 [Krstić and Deng 1998]. Consider the auxiliary system

$$\dot{x} = f(x) + g_1(x)[\ell\gamma(2|L_{g_1}V|)\frac{(L_{g_1}V)^T}{|L_{g_1}V|^2}] + g_2(x) u \tag{14}$$

where $V(x)$ is a Lyapunov function candidate and γ is a class $\mathcal{K}_\infty$ function whose derivative γ' is also a class $\mathcal{K}_\infty$ function. Suppose that there exist a matrix-valued function $R_2(x) = R_2(x)^T > 0$ such that the control law $u = \alpha(x) = -R_2^{-1}(L_{g_2}V)^T$ globally asymptotically stabilizes (14) with respect to $V(x)$. Then the control law

$$u = \alpha^*(x) = \beta\alpha(x) = -\beta R_2^{-1}(L_{g_2}V)^T \tag{15}$$

with any $\beta \geq 2$, solves the inverse optimal gain assignment problem for system (12) by minimizing the cost functional

$$\begin{aligned} J(u) = \sup_{d \in \mathcal{D}} \{ \lim_{t \to \infty} [2\beta V(x(t)) + \\ \int_0^t (l(x) + u^T R_2{}^{-1} u - \beta\lambda\gamma(\frac{|d|}{\lambda})\, \mathrm{d}\tau)] \} \end{aligned} \tag{16}$$

for any $\lambda \in (0, 2]$ where $\mathcal{D}$ denote the set of locally bounded functions, and

$$\begin{aligned} l(x) = & -2\beta[L_f V + \ell\gamma(2|L_{g_1} V|) \\ & -L_{g_2} V R_2^{-1} (L_{g_2} V)^T] + \\ & +\beta(2-\lambda)\ell\gamma(2|L_{g_1} V|) \\ & +\beta(\beta-2) L_{g_2} V R_2^{-1} (L_{g_2} V)^T \end{aligned} \quad (17)$$

where $2\beta V(x(t))$ and $l(x)$ must be positive definite, radially unbounded functions.

3 Inverse Optimal Pin Control applied to a Complex Network

Let consider a General Complex Dynamical Network as in (8), and assume it is input-to-state stabilizable; with each node as a chaotic system, and assume $f(x_i)$ uniformly decreasing (3) for all pinned nodes.

Let suppose that there is a homogeneous stationary state for all nodes in the network such as:

$$x_1 = x_2 = \cdots = x_N = x_s \quad (18)$$

where

$$f(x_s) = 0 \quad (19)$$

then we state the next theorem.

Theorem 3. The Pinning control input:

$$u = -2\mu g_{ii} \Gamma x_{ei} \quad (20)$$

for the General Complex Dynamical Network (8), locally asymptotically stabilize the entire network; moreover u minimize the next cost functional:

$$\begin{aligned} J(u) = \sup_{d \in \mathcal{D}} \{ & \lim_{t\to\infty} [4\|x_{ei}\|^2 \\ & + \int_0^t (4(\sigma_i + 4\mu g_{ii})\|x_{ei}\|^2 \\ & -4([x_{ei}^T[\sum_{j=1}^N x_j]]^{\frac{4}{3}}) - \frac{27}{64})\mathrm{d}\tau)]\} \end{aligned} \quad (21)$$

if (10) is fulfilled, and if

$$\mu > \frac{((\sigma_i + 2g_{ii})|x_{ei}^T[\sum_{j=1}^N x_j]|)^{\frac{4}{3}}}{g_{ii}\|x_{ei}\|^2} - \frac{\sigma_i}{g_{ii}} \quad (22)$$

is also fulfilled, where σ_i is the uniformly decreasing constant, $g_{ii} = -\sum_{j=1, j\neq i}^N g_{ij} > 0$, $x_{ei} = x_i - x_s$ and μ is a control parameter. Hence u is an inverse optimal control for the Network (8).

Proof. Let define for pinned nodes the error related to the stationary state as $x_{e_i} = x_i - x_s$; notice that $\dot{x}_{e_i} = \dot{x}_i - \dot{x}_s = f(x_i) - f(x_s) = f(x_i)$, then the error dynamics is:

$$\begin{aligned} \dot{x}_{e_i} &= f(x_i) + \sum_{j=1}^N c_{ij} a_{ij} \Gamma x_j + u_i \\ & i = 1, 2, \ldots l \end{aligned} \quad (23)$$

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Analyzing the pinned nodes ($i = 1, 2, \ldots l$) as in the Inverse Optimal Control approach, we express (23) in terms of (14), and get:

$$\dot{x}_{e_i} = f(x_i) + [-\sum_{j=1}^N g_{ij} \Gamma x_j][1] + [1]u_i \quad (24)$$

where $g_{ij} = -c_{ij} a_{ij}$, and in reference to (14) $g_1(x) = -\sum_{j=1}^N g_{ij} \Gamma x_j \in R^n$, $d = 1 \in R$, $g_2(x) = 1 \in R$, $u = u_i \in R^n$. Notice that the influence of other nodes of the network in the node i is considered as a disturbance.

By assuming that pinned nodes (23) fulfills Theorem 1; then to apply (15) to those nodes, we select a Lyapunov candidate function as:

$$\begin{aligned} V &= \frac{1}{2}(x_i - x_s)^T (x_i - x_s) \\ &= \frac{1}{2} x_{ei_1}^2 + \frac{1}{2} x_{ei_2}^2 + \ldots + \frac{1}{2} x_{ei_n}^2 \end{aligned} \quad (25)$$

The partial derivative of (25) with respect to x_{ei} is:

$$\frac{\partial V}{\partial x_{ei}} = x_{ei}^T \quad (26)$$

We calculate

$$L_f V = \frac{\partial V}{\partial x_{ei}}(f(x_{ei})) = x_{ei}^T (f(x_{ei})) \quad (27)$$

$$L g_1 V = \frac{\partial V}{\partial x_{ei}}(g_1(x)) = -x_{ei}^T [\sum_{j=1}^N g_{ij} \Gamma x_j] \quad (28)$$

$$L g_2 V = \frac{\partial V}{\partial x_{ei}} g_2(x) = x_{ei}^T \quad (29)$$

The auxiliary system (14) for (23) is:

$$\dot{x}_{eia} = f(x_{ei}) + [-\sum_{j=1}^{N} g_{ij}\Gamma x_j][\ell\gamma(2|x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j]|) \times \frac{(-x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j])^T}{|x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j]|^2}] + u_{ia} \quad (30)$$

We calculate the derivative of the Lyapunov function (25) along the trajectories of the auxiliary system:

$$\dot{V} = \frac{\partial V}{\partial x_{ei}}\dot{x}_{eia} = x_{ei}^T[f(x_{ei}) + [\sum_{j=1}^{N} g_{ij}\Gamma x_j][\ell\gamma(2|x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j]|)\times \frac{(x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j])^T}{|x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j]|^2}] + u_{ia}]$$

We define $\dot{V}$ as $\dot{V} = \Delta_1 + \Delta_2 + \Delta_3$:

$$\begin{aligned} \Delta_1 &= x_{ei}^T(f(x_{ei})) \\ \Delta_2 &= \ell\gamma(2|x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j]|) \\ \Delta_3 &= x_{ei}^T u_{ia} \end{aligned} \quad (31)$$

We will establish that $\dot{V}$ is negative definite. By (3), $\Delta_1 \leq -\sigma_i\|x_{ei}\|^2 < 0$. To calculate Δ_2, we use the Legendre-Fenchel transform of $\gamma \in K^\infty$ as:

$$\ell\gamma(2r) = \int_0^r (\gamma(x)')^{-1}(s)\,\mathrm{d}s = r^{\frac{4}{3}} \quad (32)$$

Then we determine

$$\begin{aligned} \ell\gamma(2|L_g V_c|) &= \ell\gamma(-x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j]) \\ &= [x_{ei}^T[\sum_{j=1}^{N} g_{ij}\Gamma x_j]]^{\frac{4}{3}} \end{aligned} \quad (33)$$

We select for (30) the input for the auxiliary system u_{ia} as:

$$u_{ia} = (R_2)^{-1}(Lg_2 V) = -\mu g_{ii}\Gamma x_{ei} \quad (34)$$

where $0 < \mu \in R$ is a parameter to be determined and $R_2 = \frac{1}{\mu g_{ii}}\Gamma^{-1} > 0$.

Taking $\Gamma = I_n$ and substituting (34) in Δ_3, we obtain:

$$\begin{gathered} \dot{V} = \Delta_1 + \Delta_2 + \Delta_3 = \\ -\sigma_i\|x_{ei}\|^2 + [x_{ei}^T[\sum_{j=1}^{N} g_{ij}x_j]]^{\frac{4}{3}} - \mu g_{ii}x_{ei}^T\Gamma^{-1}x_{ei} < 0 \end{gathered} \quad (35)$$

In order to have $\dot{V} < 0$ we obtain direct from (35) the next condition for asymptotic stability of the auxiliary system (30):

$$\mu > \frac{((\sigma_i + 2g_{ii})|x_{ei}^T[\sum_{j=1}^{N} x_j]|)^{\frac{4}{3}}}{g_{ii}\|x_{ei}\|^2} - \frac{\sigma_i}{g_{ii}} \quad (36)$$

Finally from (15) we calculate the control input which locally asymptotically stabilize the original dynamics of the pinned nodes (23) as:

$$\begin{aligned} u = \alpha^*(x) = \beta\alpha(x) = -\beta R_2^{-1}(L_{g_2}V)^T = \\ = -2\mu g_{ii}\Gamma x_{ei} \end{aligned} \quad (37)$$

where $\beta = 2$.

$l(x_{ie})$ is defined as in (17), taking $\lambda = 2$:

$$l(x_{ie}) = -4x_{ei}^T f(x_{ei}) - 4[x_{ei}^T[\sum_{j=1}^{N} x_j]]^{\frac{4}{3}} + 4\mu g_{ii}\|x_{ei}\|^2] \quad (38)$$

$l(x_{ie})$ has a lower bound $L_l(x_{ei})$; this is obtained doing (36) an equality, substituting μ in (38) and using the V-uniformily decreasing property of $f(x)$; this bound is given by:

$$\begin{gathered} l(x_{ie}) \geq 3\sigma_i\|x_{ei}\|^2 \\ -4[x_{ei}^T[\sum_{j=1}^{N} x_j]]^{\frac{4}{3}} + 4[(\sigma_i + 2g_{ii})|x_{ei}^T[\sum_{j=1}^{N} x_j]|^{\frac{4}{3}} \\ = L_l(x_{ei}) > 0 \end{gathered} \quad (39)$$

In (39) as $\|x_{ei}\| \to \infty$ then $L_l(x_{ei}) \to \infty$, which is also true for $l(x_{ie})$; Then $l(x_{ie})$ is positive definite and radially unbounded.

The cost functional minimized by (37) is given by (16); if we substitute the Lyapunov Function bound $x_{ei}^T x_{ei} \geq \|x_{ei}\|^2$ and $l(x)$ we obtain (21). Notice that $4\|x_{ei}\|^2$ is radially unbounded in (21) as required. Hence (37) is an inverse optimal control law for the pinned nodes.

We now analyze the non pinned nodes error dynamics (23). Comparing our result in (37) with the one obtained in [Li, Wang and Chen 2004], $u_{i_k} = -c_{i_k} di_k \Gamma(xi_k - x_s)$, we constat they have the same structure with $c_{i_k i_k} = g_{ii}$, $\Gamma = I_m$ and $di_k = 2\mu$. By virtual control [Li, Wang and Chen 2004] if (10) is fulfilled, then the non pinned nodes error dynamics (8) is locally asymptotically stable at the homogeneous state x_s. □.

4 Simulations

Simulations are done using a 50-node scale free network with degree distribution $\delta(k_i) \approx 2$. The coupling strengths c_{ij} for their connections fulfill the diffusive property (7) and are randomly assigned. Each node is selected as a chaotic Chens oscillator [Li, Wang and Chen 2004]. A single Chen's oscillator is describe by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= (c - a)x_1 - x_1 x_3 + c x_2 \\ \dot{x}_3 &= x_1 x_2 - b x_3 \end{aligned} \quad (40)$$

The parameters in (40) are taken as $a = 35$, $b = 3$ and $c = 28$; with this parameters a unstable equilibrium point exists at $x_s = [7.9373, 7.9373, 21]$, this point is selected as the synchronization state. The parameter C in (10) is taken as the maximum positive Lyapunov exponent $h_{ie} \approx 2.01745$. The Γ matrix is given as I_3. In this simulation $\mu = 1000$ calculated by (36) by taking a boundary in the states of the network $\|x_j\| < 50$.

For Figure 1 the control law uses $\mu = 1000$ and the weights c_{ij} are randomly assigned as $0 < c_{ij} < 6$; with this parameters the synchronization is not achieve. Finally in Figure 2 the optimal control law is implemented with $\mu = 1000$ and the weights c_{ij} are randomly assigned as $0 < c_{ij} < 20$; for this case the states of the entire network synchronize to x_s.

5 CONCLUSIONS

In this paper we have established a new control strategy for pinning weighted complex networks, based on the Inverse Optimal Control approach. The control law obtained minimizes the control efforts and synchronize the entire network as proposed at an homogeneous state. Simulations illustrate the application of the proposed scheme.

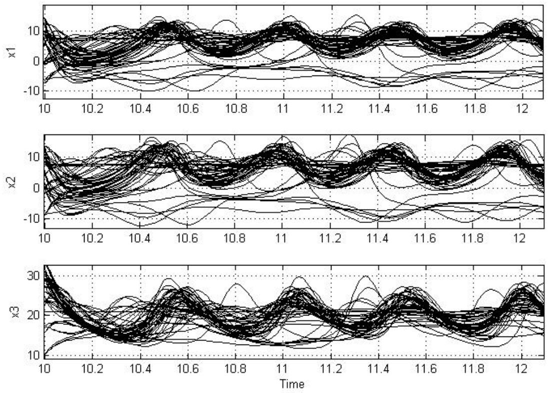

Figure 1. Scale free network with c_{ij} randomly assigned as $0 < c_{ij} < 6$ and $\mu = 1000$

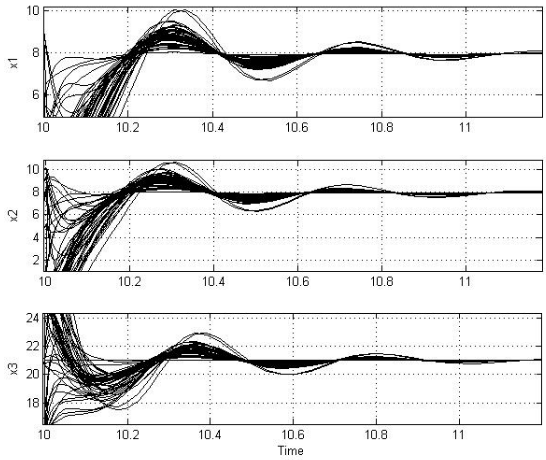

Figure 2. Scale free network with c_{ij} randomly assigned as $0 < c_{ij} < 20$ and $\mu = 1000$

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