ON FEEDBACK CONTROL ALGORITHMS FOR NITROGEN-VACANCY-CAVITY QUANTUM SENSING

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Abstract

Ultrasensitive quantum detection of external weak signals at the nanoscale levels can be implemented in a variety of forms. Here we discuss different feedback control algorithms for the sensing scenario based on the semi-classical Tavis-Cummings model for nitrogen-vacancy (NV) centers located in the diamond. In the frame of this model, the sensing elements are considered as non-interacting two-level quantum systems, distributed in-homogeneously due to heterogeneous local magnetic and strain environments.

The dynamical system of ordinary differential equations corresponding to the model contains the set of control parameters: the detunings between the drive frequency and the cavity frequency and between the drive frequency and NV transition frequency, as well as the relaxation coefficients. Correspondingly, it opens a gate for developing feedback control algorithms for tracking the cavity field, the income signal, and the reflection signal in the model sensing system.

To study the principal features of algorithmic feedback we formulate the simplified 'toy model' for the Tavis-Cummings system and investigate alternative schemes of feedback (gradient methods, target attractor methods) to compare their pros and cons for effective control for nitrogen-vacancy-cavity quantum sensing based on different choices of the control parameter set.

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Key words

Nitrogen–vacancy–cavity system, Tavis–Cummings model, toy model, feedback stabilization.

1 Introduction

Modern sensors cover a wide range of devices [Fraden, 2004], including networks for classical [Sergeenko and Granichin, 2022] and quantum [Coles, 2021] sensing.

1.1 Quantum sensing

Quantum sensors representing the family of quantum devices with some basic features, which make them very specific to compare with classical ones [Degen, Reinhard, et al., 2017]:

1. The sensing system must have discrete energy levels;
2. The sensor must possess a 'turn on and get answer' property, i.e. one can initialize the sensing process and then be able to perform readout;
3. One must be able to manipulate the device coherently;
4. One must be able to engineer the interaction of the sensor with an external physical quantity and to have some response to that quantity.

The development of quantum sensing protocols has come close to practical and even commercial implementation of position sensing, navigation, accelerometers, gyroscopes; timing standardization; electro- and magnetometers; sensing for quantum computing; different kinds of medical sensing and biomagnetic imaging; autonomous vehicles, and consumer electronics [Degen, Reinhard, et al., 2017].

Such features of quantum sensing demand the specific adaptation of control algorithms, as it was done before for other quantum devices [Borisenok, 2021; James, 2021; Koch, Boscain, et al., 2022].

1.2 Nitrogen-vacancy centers in a diamond lattice

Here we focus on the particular type of quantum sensors based on the nitrogen-vacancy (NV) centers in a diamond lattice. This type of sensor became a subject of great interest during the last couple of years due to its following interesting features [Roopini and Radhakrishnan, 2020]:

- It is characterized by extremely high fidelity of sensing. This property is originated in the details of the interaction of the individual nuclear spins of the intrinsic nitrogen atom and proximal carbon nuclei with the electronic spin states, which makes each NV-center act as a small 'quantum register'.
- Its coherent properties persist virtually up to room temperature.

For the details of the quantum sensing protocol, one can refer [Wang, Tiwari, et al., 2024].

The semi-classical Tavis-Cummings model for N NV centers considers the sensing elements as non-interacting two-level quantum systems, distributed inhomogeneously due to heterogeneous local magnetic and strain environments. The dynamical system of ordinary differential equations corresponding to the model contains the set of control parameters: the detunings between the drive frequency and the cavity frequency and between the drive frequency and NV transition frequency, as well as the coefficients of the intrinsic relaxation rate, and of the coupling strength to a microwave probe line.

1.3 The control goal

In this paper, we study the application of different feedback control algorithms (gradient methods, target attractor methods) to compare their pros and cons for effective control for nitrogen-vacancy-cavity quantum sensing based on different choices of the control parameter set. Usually, the sensing characteristic to be controlled is chosen as a quantum fidelity [Jozsa, 1994] or quantum Fisher information [Reilly, Wilson, et al., 2023].

As a control goal, we choose the response to the environmental change:

$$r = \frac{\beta_{out}}{\beta_{\rm in}} \ , \tag{1}$$

where $\beta_{\rm in}$ and β_{out} are the input and output microwave fields.

An alternative control goal option is to drive $\beta_{\rm out}$. In both cases we focus n the stabilization of the control goal.

1.4 Toy model for nitrogen-vacancy-cavity sensing

Here we develop a toy model for quantum sensing as a simplification of the standard Tavis–Cummings system. This model does not cover some principal details of the sensing process: the intricate quantum dynamics inherent to NV centers, relaxation processes, and decoherence.

We focus on the very basic properties of the controlled system to study whether the feedback control algorithms can drive the system successfully, or they must be sufficiently reformulated. It allows us to concentrate on the principle features of different feedback algorithms without discussing many particular details of their realization, including the experimental implementation. We used a similar approach to control qubits, where first we formulated a toy model of feedback algorithm [Borisenok, Fradkov, et al., 2010], and later developed the detailed model [Pechen, Borisenok, et al., 2022].

In the conclusion and discussion part, we make a comparison of alternative feedback algorithms.

2 Toy Model for Nitrogen–Vacancy–Cavity Quantum Sensor

The hybrid quantum sensor based on NV-ensemble of N centers in the doped diamond coupled to a high-quality factor dielectric resonator has been implemented in [Wang, Tiwari, et al., 2024]. In the frame of this model, the sensing elements are considered as non-interacting two-level quantum systems, distributed inhomogeneously due to heterogeneous local magnetic and strain environments.

2.1 Standard Tavis–Cummings system

The energetic structure of the NV-based quantum sensor is quite sophisticated. The standard Tavis–Cummings model for the discussed type of sensor is given by the set of following dynamical equations for the cavity field α, the spin coherence s_j, and the excited-state population p_j:

$$\begin{aligned}
\frac{d\alpha}{dt} &= -\left(i\Delta + \frac{\kappa}{2}\right)\alpha - ig_s\sum_j s_j + \sqrt{\kappa_{c1}}\beta_{\rm in} \ ; \\
\frac{ds_j}{dt} &= -\left(i\Delta_j + \frac{\gamma}{2}\right)s_j - ig_s(1-2p_j)\alpha \ ; \\
\frac{dp_j}{dt} &= -\gamma_p p_j + ig_s\left(s_j\alpha^* - s_j^*\alpha\right) \ .
\end{aligned} \tag{2}$$

Eq.(2) includes the set of frequencies: the driving frequency ω_d, the NV transition frequency ω_j, and the cavity frequency ω_c, they define detunings: $\Delta = \omega_d - \omega_c$ and $\Delta_j = \omega_d - \omega_j$.

The relaxation parameters are represented by: the intrinsic relaxation rate κ_c, the coupling strength to a microwave probe line κ_{c1}, such that $\kappa = \kappa_c + \kappa_{c1}$; the spin relaxation rate γ, the optical polarization rate γ_p (which is considered to be uniform), and the uniform coupling between each j-th spin with the cavity [Wang, Tiwari, et al., 2024].

2.2 Toy model for sensing

NV spins are polarized to the spin ground state using continuous optical pumping [Wang, Tiwari, et al., 2024]. That forms a low entropy configuration, where the spin ensemble serves as a cooling agent for the cavity mode by collectively interacting with the microwave photons.

Based on that, let's choose the case of $\sum_j s_j \simeq$ const, such that the second term in the first Eq.(2) is virtually a constant/ It means that we can eliminate this term by rescaling the field α.

Additionally, we assume the detuning Δ to be close to 0, and neglect the relaxation rate κ.

The control signal is chosen as the coupling parameter to the microwave probe line:

$$u = \sqrt{\kappa_{c1}}. \tag{3}$$

Then our toy model becomes:

$$\begin{aligned} \frac{d\alpha}{dt} &= u\beta_{\text{in}} \; ; \\ \beta_{out} &= u\alpha - \beta_{\text{in}} \; . \end{aligned} \tag{4}$$

We have to confirm that the model (4) does not reflect some important features of the experimental implementation of the nitrogen-vacancy centers in diamond. First, it does not cover the intricate quantum dynamics inherent to NV centers [Ho, Wong, et al., 2021]. We assume here that the typical time scale of such dynamics is much slower in comparison with the sensing process itself [Yuan, Mukherjee, et al., 2024].

Second, different effects of noise and, particularly, decoherence, which is the most dominant magnetic field noise source in diamond [Bauch, Singh, et al., 2020; Park, Lee, et al., 2022], are not included in (4). The theoretical and experimental research demonstrates that decoherence strongly depends on the spin concentration: the decoherence decay is exponential for the high spin concentration in the diamond, but it becomes non-exponential for the low spin concentration [Hayashi, Matsuzaki, et al., 2020]. Another possible way to suppress the quantum spin noise is the inserting so-called π-rotations around the nuclear spin bath into the free evolution [Chen, Qiu, et al., 2023]. In other words, in our toy model, we assume quantum decoherence to be essentially suppressed in one way or another.

3 Alternative Feedback Control Approaches

Among the variety of control algorithms let's focus on two alternative approaches: target attractor, or 'synergetic' method, and on gradient algorithms.

3.1 Target attractor feedback

Target attractor (TA), or 'synergetic', feedback has been proposed in [Kolesnikov, 2012]. It is based on forming in the dynamical system an artificial target attractor, which locks exponentially fast dynamical trajectories in its neighborhood. We developed this approach as a tracking algorithm for the NV-cavity sensor in [Borisenok, 2024], and here we briefly reproduce our main results (for the stabilization case) to compare them later with the gradient methods.

For the control goal based on Eq.(1), let's define the target stabilization level $\beta_{out,t}$, and then engineer the target attractor on the form:

$$\frac{d\beta_{\text{out}}}{dt} = -\frac{(\beta_{out} - \beta_{out,t})}{T} \; ; \tag{5}$$

with the positive constant T.

Then by (1) we obtain:

$$u = \sqrt{\frac{1}{\beta_{\text{in}}}\frac{d\beta_{\text{in}}}{dt}}, \tag{6}$$

where $\beta_{\text{in}} \not\equiv 0$.

By (6) one can observe the main handicap of the TA algorithm: it is valid if

$$\frac{1}{\beta_{\text{in}}}\frac{d\beta_{\text{in}}}{dt} \geq 0 \; . \tag{7}$$

Even if we represent the external fields as a real function (in the original model they are complex), they usually have components of harmonic functions like $\sin t$ or $\cos t$, or their products with exponents like $\exp(-t)$. Thus, for practical implementation, the condition (7) does not look to be realistic.

The reason for such features lies in the structure of the attraction region for the dynamical trajectories. The basin of the target attractor does not cover a full set of the initial conditions and also does not correspond to the entire possible variety of functions describing external fields. All this makes the Kolesnikov synergetic feedback method of limited suitability for controlling quantum sensing processes.

3.2 Gradient Algorithms

The gradient algorithms have already quite a successful history of their application to quantum systems. Particularly, we should mention here the speed gradient (SG) method [Fradkov, 2007], which recently has been applied for the efficient control of qubit energy states [Pechen, Borisenok, et al, 2022].

Nevertheless, although SG is extremely useful for classical applications, at the same time it is quite demanding on the properties of dynamical systems. To guarantee the achievement of the control goal, the control algorithm for the system must satisfy to Fradkov–Pogromsky's theorem [Fradkov and Pogromsky, 1998], which often is not valid for quantum systems. Such cases we observed in the models for quantum batteries and quantum sensors.

The other option for the gradient feedback as quantum control could be the usage of a gradient descent algorithm. Here we adopt a similar approach that we developed for the superconducting qubit-based sensor [Borisenok, 2023].

3.2.1 Gradient stabilization for a constant input field

Let's first study the case of constant β_{in}.

For algebraic computation simplicity, we re-define here the control signal (3) multiplying it by the cavity field as:

$$\eta = u\alpha. \tag{8}$$

The new control signal can be now represented in the form:

$$\eta = \frac{1}{\beta_{\text{in}}} \frac{d}{dt}\left(\frac{\alpha^2}{2}\right) , \tag{9}$$

and the second equation in (4) becomes:

$$\beta_{\text{out}} = \eta - \beta_{\text{in}} . \tag{10}$$

If we drive the outcome field β_{out}, let's define the target $\beta_{out,t}$, and apply the gradient algorithm in the form:

$$\frac{d\beta_{\text{out}}}{dt} = -\Gamma\left(\beta_{\text{out}} - \beta_{\text{out},t}\right) . \tag{11}$$

Here the constant $\Gamma > 0$. Then:

$$\frac{d\eta}{dt} = -\Gamma\left(\eta - \beta_{\text{in}} - \beta_{\text{out},t}\right) , \tag{12}$$

with the solution:

$$\eta(t) = \beta_{\text{in}} + \beta_{\text{out},t} + Ce^{-\Gamma t} , \tag{13}$$

where

$$C = \eta(0) - \beta_{\text{in}} - \beta_{\text{out},t} . \tag{14}$$

Now by (9) we can restore:

$$\alpha(t) = \sqrt{2\beta_{\text{in}}\left[\left(\beta_{\text{in}} + \beta_{\text{out},t}\right)t - \frac{C}{\Gamma}e^{-\Gamma t}\right]} , \tag{15}$$

and

$$u(t) = \frac{\beta_{\text{in}} + \beta_{\text{out},t} + Ce^{-\Gamma t}}{\sqrt{2\beta_{\text{in}}\left[\left(\beta_{\text{in}} + \beta_{\text{out},t}\right)t - \frac{C}{\Gamma}e^{-\Gamma t}\right]}} . \tag{16}$$

RHS(16) is constrained by:

$$\beta_{\text{in}}\left[\left(\beta_{\text{in}} + \beta_{\text{out},t}\right)t - \frac{C}{\Gamma}e^{-\Gamma t}\right] \geq 0, \tag{17}$$

which for the constant chosen as $C = 0$ becomes:

$$\beta_{\text{in}}^2 + \beta_{\text{in}}\beta_{\text{out},t} \geq 0. \tag{18}$$

It is much more realistic to compare with (7).

A similar result can be obtained for the stabilization of r at the target level r_t. We define the control algorithm as:

$$\frac{dr}{dt} = -\Gamma_r(r - r_t) \; ; \; \Gamma_r = const > 0 \, . \tag{19}$$

Then:

$$\frac{d\eta}{dt} = -\Gamma_r\left[\eta - (1 + r_t)\beta_{\text{in}}\right] , \tag{20}$$

and

$$\eta(t) = (1 + r_t)\beta_{\text{in}} + C_r e^{-\Gamma_r t} . \tag{21}$$

That implies:

$$\alpha(t) = \sqrt{2\beta_{\text{in}}\left[(1 + r_t)\beta_{\text{in}}t - \frac{C_r}{\Gamma_r}e^{-\Gamma_r t}\right]} , \tag{22}$$

and

$$u(t) = \frac{(1 + r_t)\beta_{\text{in}} + C_r e^{-\Gamma_r t}}{\sqrt{2\beta_{\text{in}}\left[(1 + r_t)\beta_{\text{in}}t - \frac{C_r}{\Gamma_r}e^{-\Gamma_r t}\right]}} , \tag{23}$$

where

$$C_r = \eta(0) - (1 + r_t)\beta_{\text{in}} . \tag{24}$$

Again, this form looks to be more natural for the real shapes of the sensed fields.

In both cases (16) and (23), the initial conditions are multiplied by the factor $\exp(-\Gamma t)$ or correspondingly $\exp(-\Gamma_r t)$, that makes the control algorithm to be practically non-sensitive to the initial condition set at the time scales $1/\Gamma$ or $1/\Gamma_r$.

3.2.2 Gradient stabilization for a time-dependent input field

In the case of time-dependent $\beta_{\text{in}}(t)$, the profile of the control signal, of course, becomes more complex.

Let's derive it for the stabilization of β_{out}. The same (11) becomes:

$$\frac{d\eta}{dt} = \frac{\beta_{\text{in}}}{dt} - \Gamma\left(\eta - \beta_{\text{in}}(t) - \beta_{\text{out},t}\right) . \tag{25}$$

Then:

$$\eta(t) = e^{-\Gamma t}\left[\eta(0) + \int_0^s f(s)e^{\Gamma s}ds\right] , \tag{26}$$

where

$$f(t) = \frac{d\beta_{\text{in}}}{dt} + \Gamma\left(\beta_{\text{in}}(t) + \beta_{\text{out},t}\right) . \tag{27}$$

Let's put $\eta(0) = 0$, then:

$$\eta(t) = e^{-\Gamma t}\int_0^t \left[\Gamma\left(\beta_{\text{in}}(s) + \beta_{\text{out},t}\right)\right]e^{\Gamma s}ds . \tag{28}$$

After the separation of α and u:

$$\alpha(t) = \sqrt{2\beta_{\text{in}}(t)\eta(t)}\ , \tag{29}$$

and

$$u(t) = \sqrt{\frac{2\beta_{\text{in}}(t)}{\eta(t)}}\ , \tag{30}$$

with $\eta(t)$ given by (28),

We observe here that for the time dependent input field $\beta_{\text{in}}(t)$, the inequality:

$$\frac{\beta_{\text{in}}(t)}{\eta(t)} \geq 0 \tag{31}$$

is not always valid. That could be a sign that our toy model does not cover appropriately the time-dependent case because the evolution of the microwave-driven field stimulates sufficiently the dynamics of the spin variables.

Again, the initial conditions in (26) are multiplied by the decaying exponent $\exp(-\Gamma t)$, which makes the control algorithm practically to be not sensitive to the initial conditions.

4 Conclusions and Discussions

The response to the environmental change in the cavity quantum electromagnetic sensor based on the nitrogen-vacancy centers in the diamond can be efficiently controlled by the application of alternative forms of feedback algorithms.

For the feedback stabilization goal, the gradient methods look to be more appropriate to compare with the target attractor due to the specific features of the attractor basin in the system. Also, the gradient methods are not sensitive to the initial conditions of the system parameters.

Different stages of the working cycles of quantum sensors demand different parameters of optimization in the model systems. That's why it is so important for the following research to investigate tracking rather than stabilization. It opens the gate for tracking different time stages of the sensor operation.

For the following phase of research,the effects of spin dynamics and decoherence must be considered in the control model.

The other focus of our further investigation will be on the problem of computational cost and the details of the practical implementation of feedback algorithms.

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