Diagnosis and Reconfiguration of the Spacecraft Fault Tolerant Gyromoment Control Systems*

Ye.I. Somov [*,†], **V.P. Makarov** [*,†], **and V.M. Matrosov** [†]

[*] *Samara Scientific Center, Russian Academy of Sciences*
3a Studenchesky Lane, Samara 443001 Russia
(e-mail: e_somov@mail.ru)
[†] *Stability and Nonlinear Dynamics Research Center,*
Mechanical Engineering Research Institute (IMASH),
Russian Academy of Sciences (RAS),
5 Dm. Ul'yanov Str. Moscow 119333 Russia

Abstract: Methods for modelling and detection of the causes of anomalous functioning the automatic control systems and practical methods for analysis of the anomalous situations, are presented. Methods for consecutive classification of onboard equipment failures and control loop reconfiguration are considered. Some results on analysis of the gyromoment attitude control systems for the Earth sensing spacecraft, are represented.

Keywords: fault tolerance, spacecraft, attitude control system, nonlinear dynamics

1. INTRODUCTION

The problem of fault-tolerance and dynamic reliability is actual for wide class of automatically controlled mechanical systems in machine building, power engineering, aerospace industry etc. The failure of any instrument in a control loop changes system structure in principle and can lead to arising a contingency situation. Over the last two decades, the basic research on fault diagnosis, fault detection and isolation (FDI) at control systems have received much attentions. The main trends on the FDI and reconfiguration problems were analyzed:

- a model-based approach using the parameter estimation and parity methods;
- a knowledged-based approach including the AI-methods, fuzzy logic and neural networks,

having in mind a control aerospace practice. The model-based approach is now recognized as an important and efficient method, the trends in extending that methodology to nonlinear control systems today are practically realized at space engineering by most modern motto: from *Artificial Intelligence* to *Natural Tricks*. During the recent 30 years authors have been accumulated a substantial experience in modelling, dynamic research and designing the spacecraft (SC) attitude control systems (ACS) with high fault-tolerance, survivability and autonomy at the expense of functional excessibility. The dynamic requirements to the ACS for the communication SC are:

- continuous precision 3-axis orientation of the SC body under the conditions of possible ACS onboard equipment failures, disturbances on optical devices etc., and also at executing a SC orbit correction;
- possibility of the SC body re-orientation for its orbit correction, as well autonomous orientation of the solar array panels (SAPs) and each high-gain receiving-transmitting antenna (RTA) with respect to the SC body;
- robustness to variations of the SC inertial and rigidity characteristics under minimum mass, size and power expenditures.

For the remote sensing SC there are the need:

- to orient the line-of-sight to a predetermined part of the Earth surface with the scan in designated direction;
- to compensate a image motion at the onboard optical telescope focal plane.

Moreover, for the remote sensing spacecraft these requirements are expressed by rapid angular manoeuvring and spatial compensative motion with a variable vector of angular rate.

Increased requirements to such information satellites (lifetime up to 10 years, exactness of spatial rotation manoeuvers with the effective damping of the SC flexible construction oscillations, fault-tolerance, reliability as well as to reasonable mass, size and energy characteristics) has motivated intensive development the gyro moment clusters (GMCs) based on excessive number of reaction wheels (RWs), gyrowhells (GWs) and gyrodines (GDs) — single-gimbal control moment gyros.

For SC close-loop control the principal meter has been represented by a strapdown inertial navigation system (SINS) based on the fine gyros and optoelectronic sensors (for example, a fine fixed-head star sensor with a wide field

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of view), which are intended for correction of the SINS. For increasing the SINS accuracy and reliability a certain redundancy on measuring channels is usually introduced at any inertial gyroscopic assembly based on both a precise float single-axis gyro and an electrostatic gyro with a rigid spherical rotor.

2. STATEMENT OF GENERAL PROBLEM

Let be given the nonlinear generalized controlled object $\mathcal{O}$ for a time $t \in \mathrm{T}_{t_0} \equiv [t_0, \infty)$:

$$D^{+}\mathrm{x}(t) = \mathcal{F}(\mathrm{x}(t), \mathrm{u}, \mathrm{p}(t,\mathrm{x}), \gamma_\nu^f(t)),\ \mathrm{x}(t_0) = \mathrm{x}_0; \quad (1)$$

$$\begin{aligned} \mathrm{y}(t) &= \psi^o(\mathrm{x}(t), \gamma_\nu^f(t)); \\ \mathrm{z}^o(t) &= \phi^o(\mathrm{x}(t), \mathrm{y}(t), \mathrm{p}(t,\mathrm{x})), \end{aligned} \quad (2)$$

where $\mathrm{x}(t) \in \mathcal{H} \subset \mathbb{R}^{n_\nu}$; $\mathrm{x}_0 \in \mathcal{H}_0 \subseteq \mathcal{H}$; $\mathrm{y}(t) \in \mathbb{R}^{r_\nu^s}$ is a output vector for measurement and diagnosis of object's state, and $\mathrm{z}^o(t) \in \mathbb{R}^{r_\nu^f}$ is a vector for description of its failure conditions; $\mathrm{u} = \{u_j\} \in \mathrm{U} \subset \mathbb{R}^{r_\nu^c}$ is a control vector, and $\mathrm{p}(t,\mathrm{x}) \in \mathcal{P}$ is the vector-function of disturbances in class $\mathcal{P}$; D^+ is symbol of a right derivative with respect to time, and $\gamma_\nu^f(t) \in \mathcal{B}^m$ under $\mathcal{B}^m \equiv \mathcal{B}\times\mathcal{B}\cdots\times\mathcal{B}$, $\mathcal{B} = \{0,1\}$ is vector of logic variables, which are outputs of a "fault's" asynchronous logic automaton (ALA) $\boldsymbol{\mathcal{A}}^f$ for its time $\nu \in \mathbb{N}_0 \equiv [0, 1, 2, \dots)$,

$$\gamma_\nu^f = \delta^f(\kappa_\nu^f, \mathrm{l}_\nu^f);\ \kappa_{\nu+1}^f = \lambda^f(\kappa_\nu^f, \mathrm{l}_\nu^f), \kappa_0^f = \kappa^f(0), \quad (3)$$

with memory, where logic vectors of object's state $\kappa_\nu^f = \kappa^f(\nu)$ and input $\mathrm{l}_\nu^f = \mathrm{l}^f(\nu) = \mathrm{g}^f(\mathrm{z}^o(t_\nu^f))$, which are used for representing fault occurrences and damage development depending on the automaton time ν, bound up with the continuous time as $t = t_\nu^f + (\tau^f - t_\nu^f)$; $\tau^f \in \tau_\nu^f \equiv [t_\nu^f, t_{\nu+1}^f)$, $\nu \in \mathbb{N}_0$. Moreover, $\mathrm{l}_\nu^f(t) = \mathrm{const}\, \forall t \in \tau_\nu^f$ and change of the logic vector γ_ν^f in general case leads to variation of *dimensions* for vectors $\mathrm{x}(t)$ and $\mathrm{y}(t)$ under mappings in time moments $t = t_\nu^f$:

$$\mathrm{x}(t_{\nu+}^f) = \mathcal{P}_\nu^x(\mathrm{x}(t_{\nu-}^f)); \quad \mathrm{y}(t_{\nu+}^f) = \mathcal{P}_\nu^y(\mathrm{y}(t_{\nu-}^f)).$$

Let $\mathrm{T}_u, \mathrm{T}_q \le \mathrm{T}_u$ and $\mathrm{T}_r \ge \mathrm{T}_u$ are fixed sampling periods of control, state measurement and the control reconfiguration, moreover, multiplicity conditions must be satisfied for these periods, and

$$\begin{aligned} x_k = x(t_k);\ t_k = k\mathrm{T}_u,\ t_s = s\mathrm{T}_q,\ t_\mu = \mu\mathrm{T}_r; \\ x_k^{\mathrm{f}} = \boldsymbol{\mathcal{F}}_{\mathrm{T}_u}(x_s); \quad x_\mu^{\mathrm{f}} = \boldsymbol{\mathcal{F}}_{\mathrm{T}_r}(x_k), \end{aligned}$$

where x_k^{f} is the value of the variable x_s measured with the sampling period T_q, which is filtered out at the time $t = t_k$; $\boldsymbol{\mathcal{F}}_{\mathrm{T}_y}(\cdot)$ is the *digital* filtering operator with the sampling period $\mathrm{T}_y, y = u, r$.

Let be also given subsystem of discrete measurement of the object state and digital filtering:

- for diagnostics of the object $\mathcal{O}$

$$\mathrm{y}_s^d = \psi^d(\mathrm{y}_s);\ \mathrm{z}_k^{\mathrm{df}} = \mathcal{F}_{\mathrm{T}_u}(\mathrm{y}_s^d),\ k, s \in \mathbb{N}_0; \quad (4)$$

- for forming the control and its reconfiguration

$$\begin{aligned} \mathrm{y}_s^u &= \psi^u(\mathrm{y}_s); \quad \mathrm{y}_k^{\mathrm{f}} = \mathcal{F}_{\mathrm{T}_u}(\mathrm{y}_s^u); \\ \mathrm{z}_\mu^{\mathrm{f}} &= \mathcal{F}_{\mathrm{T}_r}(\mathrm{z}_k^{\mathrm{df}}), \quad \mu, k, s \in \mathbb{N}_0. \end{aligned} \quad (5)$$

Principal problems are contained in synthesis of:

- the synchronous logic automaton (SLA) $\boldsymbol{\mathcal{A}}^d$ with memory for the structural state diagnosis

$$\gamma_k^d = \delta^d(\kappa_k^d, \mathrm{l}_k^d);\ \kappa_{k+1}^d = \lambda^d(\kappa_k^d, \mathrm{l}_k^d),\ \kappa_0^d = \kappa^d(t_0), \quad (6)$$

with logic vectors of state κ_k^d, input $\mathrm{l}_k^d = \mathrm{g}^d(\mathrm{z}_k^{\mathrm{df}})$ and output γ_k^d;

- the SLA $\boldsymbol{\mathcal{A}}^r$, also with memory, for description of damage's block-keeping and reconfiguration

$$\gamma_\mu^r = \delta^r(\kappa_\mu^r, \mathrm{l}_\mu^r);\ \kappa_{\mu+1}^r = \lambda^r(\kappa_\mu^r, \mathrm{l}_\mu^r),\ \kappa_0^r = \kappa^r(t_0), \quad (7)$$

with logic vectors of state κ_μ^r, input $\mathrm{l}_\mu^r = \mathrm{g}^r(\mathrm{z}_\mu^{\mathrm{f}}, \gamma_\mu^{\mathrm{df}})$, where $\gamma_\mu^{\mathrm{df}} = \mathcal{F}_{\mathrm{T}_r}(\gamma_k^d)$, and output γ_μ^r;

- the nonlinear control law (CL) with its reconfigurations due to the SLA $\boldsymbol{\mathcal{A}}^r$ routine

$$\begin{aligned} \mathrm{u}_k &= \mathcal{U}(\hat{\mathrm{x}}_{e\,k}, \mathrm{y}_{e\,k}^{\mathrm{f}}, \mathrm{y}_{o\,k}, \gamma_\mu^r); \\ \hat{\mathrm{x}}_{e\,k+1} &= \hat{\mathcal{F}}_e(\hat{\mathrm{x}}_{e\,k}, \mathrm{y}_{e\,k}^{\mathrm{f}}, \mathrm{y}_k^o, \mathrm{u}_k, \gamma_k^d, \gamma_\mu^r), \\ \hat{\mathrm{x}}_{e\,0} &= \hat{\mathrm{x}}_e(t_0); \quad k, \mu \in \mathbb{N}_0, \end{aligned} \quad (8)$$

where $\mathrm{y}_{e\,k}^{\mathrm{f}} = \mathcal{F}_{\mathrm{T}_u}(\psi_e^u(\mathrm{y}_{e\,s}))$; $\mathrm{y}_{e\,s} = \psi_e^o(\mathrm{x}_{e\,s}, \gamma_k^d)$, and $\mathrm{x}_{e\,s} = \mathrm{x}_e(t_s) \in \mathbb{R}^{n_\mu^e}$ is the state vector of a simplified discrete object's model

$$\mathrm{x}_{e\,s+1} = \mathcal{F}_e(\mathrm{x}_{e\,s}, \mathrm{u}_k, \gamma_k^d, \gamma_\mu^r),\ \mathrm{x}_{e\,0} = \mathrm{x}_e(t_0), \quad (9)$$

and $\hat{\mathrm{x}}_{e\,k} = \hat{\mathrm{x}}_e(t_k) \in \mathbb{R}^{n_\mu^e}$ is its estimation; $n_\mu^e \le n = \max\{n_\nu\}$, and y_k^o is a programmed vector.

Feedback loops (4)–(9) are intended for fault-tolerant control of the object (1)–(3).

3. SYNTHESIS OF LOGIC AUTOMATA

For the FDI a three-level logic-digital system is generally applied onboard Russian spacecraft for remote sensing, communication and navigation:

- on the lower level — the *integral local* SLAs $\boldsymbol{\mathcal{A}}_d^d$ with memory for automatic monitoring of the relevant device status by measurement of available physical variables (currents, movements, rates etc.)
- on the middle level — the *local loop* SLAs $\boldsymbol{\mathcal{A}}_c^d$ with memory for automatic monitoring of the control loop status (the roll, yaw and pitch channels, the SAP control loop etc.);
- on the higher "system" level — a SLA $\boldsymbol{\mathcal{A}}^d$, also with memory, for the *global* functional diagnostics of the main control loop by comparison of outputs for normal and emergency models of the ACS operation.

At two last levels the functional diagnostics is executed with using reference models – by comparison of output signals of models and measured values of system state coordinates. Results of the ACS state diagnosis, carried out by specialists of the spacecraft mission control center, indicate high performance of the methods based on applying detailed information about instruments, control algorithms, control laws and set of other options of the SC functioning, and also some invariant relations between system state variables. For high fail-safe operation of the ACS, maximum employment of functional redundancy has been provided by using the SLA to apply all the reverse complete sets of devices or their electric circuits. At synthesis of the diagnosis SLAs $\boldsymbol{\mathcal{A}}_d^d, \boldsymbol{\mathcal{A}}_c^d$ and $\boldsymbol{\mathcal{A}}^d$ (6) and

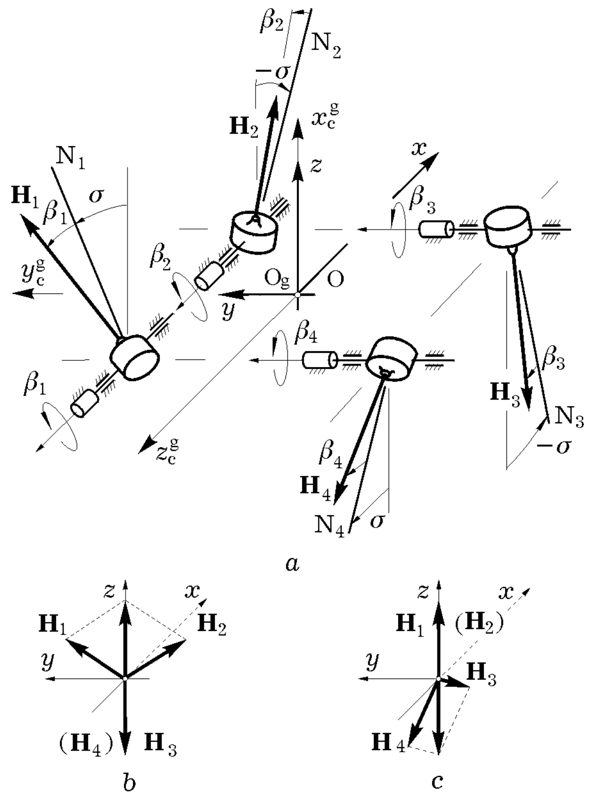

Fig. 1. The fault-tolerant *2-SPE* scheme of the GMC

also of a damage block-keeping and reconfiguration SLA $\boldsymbol{\mathcal{A}}^r$ (7), the *Natural Tricks* are used. They are based on both well-known physical invariant relations (for example, general momentum invariant for the "SC+GMC"-system) and engineering inventiveness, presented at a perfect logic-uncontraditory form.

4. THE MODIFIED WALD CRITERION

Implemented at programmed level (in the SC onboard computer) any plan for localization of system's failures is related with necessity to solve a problem of informative parameters choice. As a rule, in practice number of the system's controlled parameters includes all significant coordinates of its state, which characterize the basic dynamic indexes and determine the quality of functioning the highest level of hierarchy.

The onboard algorithm for the ACS diagnosis is based on its reference model work in a background regime, i.e. at the SC mission control in real time. Thus at first, for detection of an anomalous situation on each control period the vector of discrepancies between measured ($\mathbf{X} = \{x_i\}$) and model ($\mathbf{X}_\mathrm{m} = \{x_{\mathrm{m}i}\}$) parameters is computed: $\mathbf{E} = \{e_i\} = \mathbf{X} - \mathbf{X}_\mathrm{m}$. Then, the obtained data are analyzed concerning their conformity to chosen criteria, in the elementary case – their coincidence with limits of possible modification of controlled parameters a priori defined from design performances. The main disadvantage of the plan of automatic diagnosis of a current state (in real time) is difficulty of obtaining (the a priori assignment) evaluation of decision making credibility about a failure of a system structural element and its dependence on quantity of control periods.

More effective approach to a system diagnosis and making a decision on a failure consists in the following. Temporal behavior of control parameters ej(t), j=1,2 is possible to be considered as a random process which performances depend on set of factors. These are measurement errors: inaccuracy of control actions optimization and an object motion modelling as a result of its model simplification; inexactness of knowledge of spacecraft design data, perturbation actions etc. In this case classification can be conducted not on instantaneous values of discrepancies ej(t) in the end of each control period T_k, but according to random process presented by discrete sequence of values $\mathrm{ej}_k = \mathrm{ej}(t_k)$, where $k = 1, 2, 3\ldots.$ Classification of such random process is implemented by a mathematical apparatus of a consecutive analysis of hypothesis in the form of the modified consecutive criterion by the ration of probabilities (MCCRP) of *Wald*. In this criterion the limits of a controlled parameters modification depend on time (or numbers of the control periods), and also on taken value inaccuracy. Generally the MCCRP possesses following important properties:

- convergence with probability 1, and by alignment of threshold values α and β it is possible to supply with flexible tracing of classification inaccuracy levels;
- does not demand independence and equality of probability distributions of classified casual vectors;
- supplies minimization of average number of the observations necessary for reaching the given level of reliability of the value, and minimization of average volume of information stored for classification, that considerably simplifies its implementation in the SC onboard software.

Procedure for the analysis by the modified Wald criterion is implemented as follows. For each parameters discrepancy value the vector of the log – likely-hood ratio is computed: $\lambda \mathrm{j}_k = -\ln(P(\mathrm{ej}_k/W_1)/P((\mathrm{ej}_k/W_2)$, where ej_k is a value of vector ej on k-th step of computation; $P(\mathrm{ej}_k/W_j)$ is a function of the conventional density of probabilities ej_k at the fixed event, consisting of the fact that ej_k belongs to the class j=1,2. Value $\lambda \mathrm{j}_k$ is also random. Therefore, for independent allocation of ej_k the summarized log - likehood criterion L after n observations is equal

$$L = -\ln\{P[(\mathrm{e1}_1, \ldots \mathrm{e1}_\mathrm{n})/W_1]/P[(\mathrm{e2}_1, \ldots \mathrm{e2}_\mathrm{n})/W_2]\} = -\sum_{k=1}^{\mathrm{n}} \ln\{P[(\mathrm{e1}_k)/W_1]/P[(\mathrm{e2}_k)/W_2]\} = \sum_{k=1}^{\mathrm{n}} \lambda \mathrm{j}_k$$

where $k = 1, 2, \ldots \mathrm{n}$ is the number of a control step, and W_1 and W_2 are classes of a system state (accordingly "norm" and "not the norm"). The MCCRP decision rule is presented as

$$L \le \alpha_k \quad \to \mathrm{ej} \in W_1;$$
$$\alpha_k < L < \beta_k \to \text{(to prolong processing of measuring)};$$
$$\beta_k \le L \to \mathrm{ej} \in W_2.$$

In essence this rule consists in comparison of the L value with aligned limits α_k and β_k. These limits are constant values (as in classical Wald criterion) or monotone decreasing functions of current discrete time k. It allows to build the consecutive classifier in such a way that it is possible to align the average number of indications processing necessary for final decision as well as the probability of a false discerning.

5. A GYRO MOMENT CLUSTER

For information spacecraft it is important to minimize the GMC mass and provide the possibility for reconfiguration of its structure and control algorithms for 2–3 possible faults in any electro-mechanical executive device of the GMC. Authors have been executed multilateral analysis of schemes for constructing the small-mass GMC based on the RWs, the GWs and the GDs with both the gear stepping drives (GSDs) and the moment gearless drives (MGDs) on their precession axes, in combination with unloading loops of accumulated angular momentum (AM) by reaction trusters and/or the magnetic torquers.

The following *minimal*-excessible GMC structure is most rational for providing fault-tolerance: *2-SPE* scheme based on 4 GDs , see Fig. 1. Sometimes for the main mode of the spacecraft attitude control only 3 executive devices are used — the fourth executive device is in the "cold" reserve. For example, let the point O be the spacecraft mass center and Oxyz is the body reference frame (BRF), see Fig. 1a. In the GMC canonical reference frame $\mathrm{O_g}x_c^{\mathrm g}y_c^{\mathrm g}z_c^{\mathrm g}$ the angular momentum projections of the first (GD-1 & GD-2) and the second (GD-3 & GD-4) pairs of gyrodines always are summed up along the axis $\mathrm{O_g}x_c^{\mathrm g}$. The gyrodine neutral positions $N_p, p=1:4$ are directed at the angles $\pm\sigma$ with respect to positive (for the 1st GD's pair) and to negative (for the 2nd GD's pair) directions of the axis $\mathrm{O_g}x_c^{\mathrm g}$, see Fig. 1$a$. Under the GMC Z -arrangement on the spacecraft body, when the axis $\mathrm{O_g}x_c^{\mathrm g}$ is the same as the axis Oz of BRF, for $\sigma=\pi/6$ and $\beta_p\in[-\pi/2,\pi/2]$ the following 4 efficient (for 3-axis spacecraft attitude control) GMC configurations are possible on the basis of *only* 3 *active* gyrodines:

- the configurations Z-I, I=1:4 — the GMC without GD-I, represented at the nominal state in Fig. 1b (configurations Z-4 or Z-3) and in Fig. 1c, (configurations Z-2 or Z-1).

So, the gyrocomplex scheme in Fig. 1a is *fault-tolerant* under diagnostics of the faulted GD and the GMC reconfiguration by *passages* between configurations Z - I under *specific logic conditions*.

6. MATHEMATICAL MODELS

The body reference frame (BRF) attitude with respect to the inertial reference frame (IRF) is defined by quaternion $\mathbf{\Lambda}=(\lambda_0,\boldsymbol{\lambda}),\boldsymbol{\lambda}=(\lambda_1,\lambda_2,\lambda_3)$. Assume that $\mathbf{\Lambda}^p(t)$ is a quaternion, and $\boldsymbol{\omega}^p(t)=\{\omega_i^p(t)\}$ and $\dot{\boldsymbol{\omega}}^p(t)$ are angular rate and acceleration vectors of the programmed SC body's motion in IRF. The error quaternion is $\mathbf{E}=(e_0,\mathbf{e})=\tilde{\mathbf{\Lambda}}^p(t)\circ\mathbf{\Lambda}$, the *Euler* parameters' vector is $\boldsymbol{\mathcal{E}}=\{e_0,\mathbf{e}\}$, and the attitude error's matrix is $\mathbf{C}_e\equiv\mathbf{C}(\boldsymbol{\mathcal{E}})=\mathbf{I}_3-2[\mathbf{e}\times]\mathbf{Q}_e$, where $\mathbf{Q}_e\equiv\mathbf{Q}(\boldsymbol{\mathcal{E}})=\mathbf{I}_3e_0+[\mathbf{e}\times]$ with $\det(\mathbf{Q}_e)=e_0$. Here symbols $\langle\cdot,\cdot\rangle$, $\times$, $\{\cdot\}$, $[\cdot]$ for vectors and $[\mathbf{a}\times]$, $(\cdot)^{\mathrm t}$ for matrixes are conventional denotations. The BRF's attitude with respect to orbital reference frame (ORF) O$x^oy^oz^o$ is defined by quaternion $\mathbf{\Lambda}^o=\tilde{\mathbf{\Lambda}}_o(t)\circ\mathbf{\Lambda}$, where $\mathbf{\Lambda}_o$ is known quaternion of the ORF attitude with respect to the IRF, by angles of yaw ψ, roll φ and pitch θ for the rotational sequence {1-3-2}, and also by the matrix $\mathbf{C}_e^o=[\varphi]_2\,[\theta]_3\,[\psi]_1$, where $[\alpha]_i$ is the matrix of elementary rotation, and also by vector of *Euler*'s parameters $\boldsymbol{\mathcal{E}}^o$, moreover the matrix $\mathbf{C}_e^o=\mathbf{C}(\boldsymbol{\mathcal{E}}^o)$. For a fixed position of *flexible* structures on the SC body with some simplifying assumptions and $t\in\mathrm{T}_{t_0}=[t_0,+\infty)$ a SC angular motion model appears as:

$$\dot{\mathbf{\Lambda}}=\mathbf{\Lambda}\circ\boldsymbol{\omega}/2;\,\mathbf{A}^o\{\dot{\boldsymbol{\omega}},\ddot{\mathbf{q}},\ddot{\boldsymbol{\beta}},\dot{\mathbf{\Omega}}\}=\{\mathbf{F}^{\omega},\mathbf{F}^{q},\mathbf{F}^{\beta},\mathbf{F}^{h}\},\qquad(10)$$

$$\mathbf{F}^{\omega}=\mathbf{M}^{\mathrm g}-\boldsymbol{\omega}\times\mathbf{G}+\mathbf{M}_d^o+\mathbf{Q}^o;\,\mathbf{M}^{\mathbf g}=-\dot{\boldsymbol{\mathcal{H}}}=-\mathbf{A}_{\mathrm h}\dot{\boldsymbol{\beta}};$$
$$\mathbf{F}^{q}=\{-a_{jj}^q((\delta^q/\pi)\Omega_j^q\dot{q}_j+(\Omega_j^q)^2q_j)+\mathrm{Q}_j^q(\boldsymbol{\omega},\dot{q}_j,q_j)\};$$
$$\mathbf{F}^{\beta}=\mathbf{A}_{\mathrm h}^{\mathrm t}\boldsymbol{\omega}+\mathbf{M}_c^g+\mathbf{M}_d^g+\mathbf{M}_b^g+\mathbf{M}_f^g+\mathbf{Q}^g(\cdot);$$
$$\mathbf{F}^{h}=\mathbf{M}_c^h+\mathbf{M}_d^h+\mathbf{M}_f^h+\mathbf{Q}^h(\cdot);\;\mathbf{M}_c^h=\mathbf{M}^h+\mathbf{M}^{ha};$$

$$\mathbf{A}^o=\begin{bmatrix}\mathbf{J}^o & \mathbf{D}_q & \mathbf{D}_g & \mathbf{D}_h\\ \mathbf{D}_q^{\mathrm t} & \mathbf{A}^q & \mathbf{0} & \mathbf{0}\\ \mathbf{D}_g^{\mathrm t} & \mathbf{0} & \mathbf{A}^g & \mathbf{0}\\ \mathbf{D}_h^{\mathrm t} & \mathbf{0} & \mathbf{0} & \mathbf{A}^h\end{bmatrix};\;\begin{array}{l}\mathbf{M}_c^g=\mathbf{M}^g+\mathbf{M}^{gd}+\mathbf{M}^{ga};\\ \mathbf{G}=\mathbf{G}^o+\mathbf{D}_q\dot{\mathbf{q}}+\mathbf{D}_g\dot{\boldsymbol{\beta}};\\ \mathbf{G}^o=\mathbf{J}^o\,\boldsymbol{\omega}+\boldsymbol{\mathcal{H}}\,(\boldsymbol{\beta});\\ \mathbf{A}_{\mathrm h}=[\partial\boldsymbol{\mathcal{H}}\,(\boldsymbol{\beta})\,/\,\partial\boldsymbol{\beta}];\end{array}$$

$$\mathbf{H}=\{\mathrm{H}_p\};\;\mathbf{\Omega}=\{\Omega_p\};\;\boldsymbol{\beta}=\{\beta_p\};\;\boldsymbol{\omega}=\{\omega_i\};$$
$$\mathbf{q}=\{q_j\};\;\mathbf{H}_p(\beta_p)=\mathrm{H}_p\mathbf{h}_p;\;\boldsymbol{\mathcal{H}}(\boldsymbol{\beta})=\textstyle\sum\mathbf{H}_p(\beta_p);$$

torques $\mathbf{M}_d^g$ and $\mathbf{M}_d^h$ of a physical damping, and also the electro-magnetic damper (EMD) torques $\mathrm{M}_{dp}^g(k_d^g,\dot{\beta}_p)$ with gain k_d^g are nonlinear continuous functions; vectors of the rolling friction torques in bearings on the gyrorotor (GR) axes $\mathbf{M}_f^h$ and on GD's precession axes $\mathbf{M}_f^g$, and also in general case the torque's vector $\mathbf{M}_b^g$ describing the influence of limiting supports on GD's precession axes, are *nonlinear discontinuous* functions.

The components of the GMC control vectors $\mathbf{M}_c^g$ and $\mathbf{M}_c^h$ with regard for the possible *faults* in electric circuits of MGDs or GSDs as well as the EMDs on the GD precession axes, and also that of the electric drives on the GD's rotor axes and arresters (cages) are described by *hybrid functions*

$$\mathrm{M}_p^x=\sum_{l=1}^{2}\gamma_p^{fxl}(\nu)\;\gamma_p^{rxl}(\mu)\;a_p^x\;i_p^{xl},\qquad(11)$$

where $x=g,gd,ga,h,ha$, coordinates $\gamma_p^{yxl},y=f,r$ are *logic* variables $\gamma_p^{yxl}\in\{0,1\}$; $\gamma_p^{yx1}\wedge\gamma_p^{yx2}=0$; $\gamma_p^{yx1}\vee\gamma_p^{yx2}=1$, $p=1:4$; i_p^{xl} are the control currents and currents at the GD electro-magnetic arresters in main ($l=1$) and in reserve ($l=2$) circuits, and a_p^x are constants.

The functions $\gamma_p^{fxl}(\nu)$ are outputs of an ALA $\boldsymbol{\mathcal{A}}^f$ with memory used for representing fault occurrences and damage development depending on the automaton time $\nu\in\mathbb{N}_0$. Functions $\gamma_k^{rxl}(\mu)$ are outputs of a SLA $\boldsymbol{\mathcal{A}}^r$, also with memory, for description of damage's or fault's blockkeeping and the reconfiguration sequence depending on the automaton time $\mu\in\mathbb{N}_0$. The currents in GD's control circuits $i_p^{gl}(t)$ for $\gamma_p^{rgl}=1$ and $i_p^{hl}(t)$ for $\gamma_p^{rhl}=1$ are proportional to GD's digital control voltages $u_p^x(t)=\mathrm{Zh}[\mathrm{Sat}(\mathrm{Qntr}(u_{pk}^x,b_u^x),B_u^x),\mathrm{T}_u]$, where $u_{pk}^x,x=g,h$ are the outputs of NCLs on the GDs precession and GRs axes, and functions $\mathrm{Sat}(x,a)$ and $\mathrm{Qntr}(x,a)$ are general-usage ones, while the holder model with the period T_u is of the type: $y(t)=\mathrm{Zh}[x_k,\mathrm{T}_u]=x_k\quad\forall t\in[t_k,t_{k+1})$.

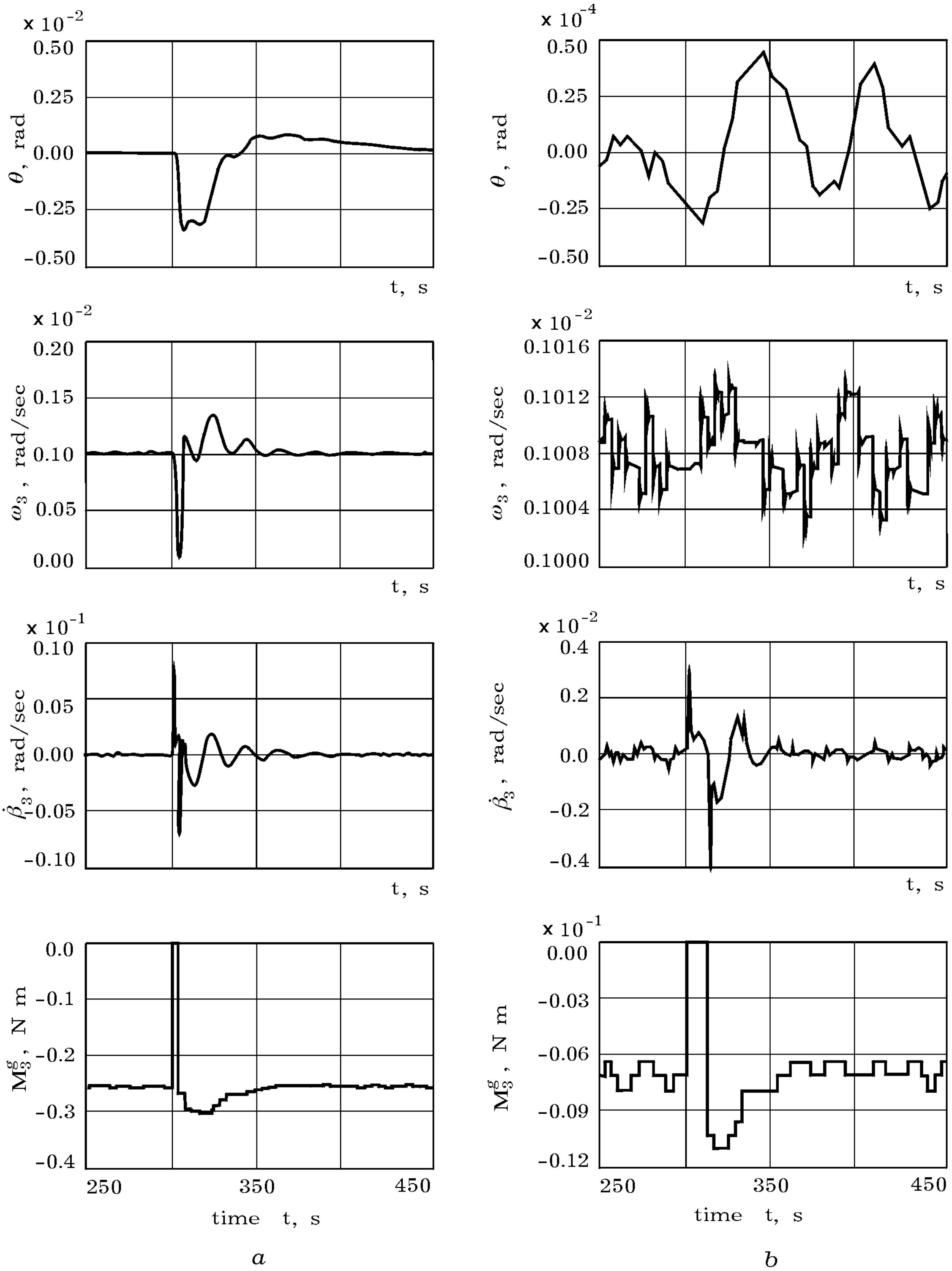

Fig. 2. The dynamic processes under fault in the control current circuit of the GD-3 torque driver

7. PROVISION OF FAULT-TOLERANCE

The *verbal* description of the provision of fault-tolerance of a spacecraft ACS with gyrocomplex in Fig. 1*a*, for its initial configuration Z-4, when $\mathrm{H}_p = \mathrm{h}_g, p = 1:3$ and

$$\gamma_p^{fxl} = \gamma_p^{rx1} = 1, x = \mathrm{g}, gd, \mathrm{h};\ \gamma_p^{rx1} = 0, x = ga, ha,$$

and GD-4 in the stopping state: $\mathrm{H}_4 = \beta_4 = 0$ and

$$\gamma_4^{rxl} = 0, x = \mathrm{g}, gd, \mathrm{h};\ \gamma_4^{rxl} = 1, x = ga, ha,$$

see (11), is as follows. In the normal mode, the magnetic unloading loop ensures the condition $\mathbf{G}^o \approx \mathbf{0}$ under formation of the magnetic control torque vector

$$\mathbf{M}_{mc}^o = \mathbf{L}_m(t) \times \mathbf{B}_{\oplus};\ \mathbf{L}_m(t) = \mathbf{Zh}[\mathbf{L}_{mk}, \mathrm{T}_u],$$

where $\mathbf{B}_{\oplus}$ is a magnetic displacement vector of geomagnetic field and

$$\mathbf{L}_{mk} = -l_m^o \phi_m^o(\mathrm{R}_0, \lambda_m, b_m, \mathrm{R}_k)\, \mathbf{e}_{mk};\quad \mathbf{e}_{mk} = \mathbf{c}_k/\mathrm{c}_k;$$

prosper-tour.tex $\phi_m^o(a,\lambda_m,b_m,x) = \{(1\,\forall x > \lambda_m b_m) \vee (0\,\forall x < b_m)\}$;

$\mathbf{c}_k = \mathbf{R}_k \times \mathbf{B}^{\mathrm{f}}_{\oplus k}; \;\; \mathbf{R}_k = \mathbf{J}^o(\boldsymbol{\gamma}_k^p)\, \boldsymbol{\omega}_{ok}^{\mathrm{ef}} + \boldsymbol{\mathcal{H}}(\boldsymbol{\beta}_k^{\mathrm{f}})$;

l_m^o is the modulus of the magnetic driver dipole torque, $\phi_m^o(a,\lambda_m,b_m,a) = a,\ a \in \{0,1\}$ is a scalar relay hysteresis function with threshold of operation b_m and coefficient of return $0 < \lambda_m < 1$.

Let the fault of the torque gearless driver current circuit in GD-3 occurred at any moment

$$t = t_\nu^f \in [t_{k_*-1}, t_{k_*});\ \nu = 1,\ \gamma_3^{fg1}(1) = 0.$$

Then by SLAs $\boldsymbol{\mathcal{A}}^d_{\mathrm{GD-3}}$ or $\boldsymbol{\mathcal{A}}^d$, and by SLA $\boldsymbol{\mathcal{A}}^r$ in the result of circuits switching ($\gamma_3^{rg1} = 0; \gamma_3^{rg2} = 1$) is guaranteed for the discrete time $k = k^* = k_*$ or $k = k^* = k_* + 1$, respectively. Moreover, the intensity of dynamic processes for the attitude control channels is essentially dependent not only on the time interval duration $\delta t^f_{k_*} = t_{k^*} - t^f_\nu$, when there is no control, but also on the potentialities of the gyrodines, which remained operable in the aspect of compensation of disturbing influence of the angular rate vector $\boldsymbol{\omega}_{\mathrm{o}}$ because of the spacecraft orbital motion.

After such an isolation of the fault, the scheduled reconfiguration of Z-4 ⇒ Z-3 process starts:

- $\gamma_4^{rha1} = 0$ and $\gamma_4^{rh1} = 1$ with speeding-up from the rest state of GD-4 rotor,
- at unloading loop the magnetic driver operates with $\mathbf{R}_k = \boldsymbol{\mathcal{H}}(\boldsymbol{\beta}_k^{\mathrm{f}})$,
- at achieving of a small neighborhood for the GMC "park" state $\mathrm{H}_p = \mathrm{h}_g;\ \beta_p = 0,\ p = 1:4$, there takes place *simultaneous*:
  - · the GD-3 caging ($\gamma_3^{rga1} = 1$),
  - · GD-4 uncaging ($\gamma_4^{rga1} = 0$) and switching ($\gamma_4^{rg1} = 1$) on-line the control closed-loop.

On the final stage of this process:

- the magnetic unloading loop is returned into the nominal mode,
- the SLA's $\boldsymbol{\mathcal{A}}^r$ output $\gamma_3^{rh1} = 0$ and the GD-3 rotor is speeding-down to the rest state,
- finally, after reaching the condition $\mathrm{H}_3 \approx 0$, the GD-3 is caged ($\gamma_3^{rha1} = 1$).

Thus, the GMC restores its redundancy with respect to control circuits of torque gearless drivers for the on-line gyrodines, and it is prepared for the rapid isolation of any new gyrodine fault and for new reconfiguration.

As discussed above, the intensity of dynamic processes is essentially dependent on the potentialities of the gyrodines which remained operable in the aspect of compensation of the spacecraft orbital motion. This fact is illustrated in Fig. 2, where the sampling period values $\mathrm{T}_q = 0.25\,s$ and $\mathrm{T}_u = 4\,s$. Such processes are presented with respect to the pitch channel of ACS under the orbital stabilization for configurations Y-4 (Fig. 2*a*) and Z-4 (Fig. 2*b*), when the GD-3 fault takes place under $t = 300.1\,s$.

In the Y-4 case, there are no operable gyrodines needed for creating control torques along the axis Oz, so despite the "fast" fault diagnostics by the SLA $\boldsymbol{\mathcal{A}}^d_{\mathrm{GD-3}}$ and switching the torque gearless driver's reserve circuit into on-line the control closed-loop at the time $t = t_{k^*} = 304\,s$, there take place substantial overshoots of attitude errors.

Such overshoots are absent for similar fault in GD-3 within the GMC according to the configuration Z-4 , since GD-1 and GD-2 in this case remain operable for creating control torques along the axis Oz, see Fig. 1*b*. So, even for the "slow" GD-3 fault diagnosis with the aid of SLA $\boldsymbol{\mathcal{A}}^d$ and switching the torque gearless driver's reserve circuit in GD-3 by the time $t = t_{k^*} = 308\,s$, the precision angular stabilization with respect to pitch remains the same.

8. CONCLUSIONS

Contemporary methods were presented, which closelly connected to designing the precise robust and fault-tolerant attitude control systems applied at Russian information spacecraft. With the aid of these methods and software the authors have been conducted dynamic research and designing such spacecraft ACSs, including those in accordance with international projects.