ROBUSTNESS ANALYSIS OF A MODEL-REFERENCE ADAPTIVE VOLTAGE CONTROLLER WITH RESPECT TO NO-LOAD VOLTAGE DISTURBANCE IN POWER SYSTEMS

Giuseppe Fusco * **Mario Russo** *

* *Università degli Studi di Cassino*
via G. Di Biasio 43, 03043 Cassino (FR), Italy
{fusco,russo}@unicas.it

Abstract: In a previous paper, the authors have developed the design of a voltage controller in power systems, which ensures tracking of a reference voltage signal while accounting for unknown variations of the steady-state operating conditions of the system. The design is based on the use of the discrete-time model-reference adaptive system theory. In view of this research topic and starting from the obtained results, the present paper studies the closed-loop stability and robustness properties of the adaptive voltage control scheme in presence of no-load voltage disturbance. After recalling the main background material, the paper illustrates this study with reference to a system model with all zeros inside the unit circle.

Keywords: Discrete-time systems, Model-reference adaptive control, Power system control, Robustness, Voltage control.

1. INTRODUCTION

In power systems voltage control is a fundamental task (Cigre, 1992). It aims at ensuring adequate system voltage profile in presence of unknown and unexpected variations of the normal operating conditions of the system such as variations and/or disconnection of loads, variations of generations, transmission lines opening/closing. These variations are classified as small disturbances (Kundur, 1994; Sauer and Pai, 1998).

Usually, voltage control is organized in a three-level hierarchy (Corsi *et al.*, 2004; Iliĉ *et al.*, 1995). The primary control level, which is based on local voltage regulation; the secondary control level, which is based on regional/area voltage regulation (RVR) and the tertiary control level, which is system centralized.

The objective of the primary control level is local control action: considering a single node, the busbar voltage amplitude can be controlled to follow a reference signal by acting on the reactive power injection at that node. Direct nodal voltage control is performed at generation nodes by synchronous generators and at some key nodes of the transmission system by synchronous and static compensators, such as static VAr systems. Such devices rapidly vary their reactive power injection to control the voltage amplitude of the busbar at which they are connected following a reference signal, which is determined by secondary voltage regulation.

In addition, primary voltage control, which responds very rapidly, may be involved in rejecting some slow transient phenomena, such as voltage amplitude fluctuations caused by specific loads (e.g. arc furnaces), or in improving the damping of electromechanical oscillations and the small signal stability of synchronous generators (Larsen *et al.*, 1996). In both cases, some additional time-varying reference signals are added to the RVR reference signal sent to primary voltage controllers of nearby compensators.

Recalling the objective of the primary control level, an effective nodal voltage controller must be able to counteract the effects of small disturbances while ensuring robustness with respect to the no-load voltage (Kundur, 1994) and giving closed-loop stability. A suitable technique for the design of voltage controller is based on the use of the adaptive control theory (Åström and Wittenmark, 1989; Tao, 2003; Wellstead and Zarrop, 1991). Several papers have shown the application of a such theory in the framework of power system control, see among others (Chaudhuri *et al.*, 2004; Chen and Malik, 1995; Fusco and Russo, 2003; Kothari *et al.*, 1996; Soós and O.P, 2001; Wang *et al.*, 1994). In particular, in (Fusco and Russo, 2006*a*) a nodal voltage controller based on the use of the discrete-time model-reference adaptive theory has been proposed. The adaptive laws were designed on the basis of a gradient approach and their properties studied employing Lyapunov analysis. Following this results, this paper aims to analyze the robustness of this adaptive laws carried out with reference to unknown no-load voltage disturbance. This study is developed by assuming that the discrete-time power system model is not corrupted by noise and it has all zeros inside the unit circle. The latter hypothesis represents a common assumption in the voltage regulation problem. Regarding the former one, some short comments will be given in the case of noisy model.

2. POWER SYSTEM MODELING

In voltage control problem the power system dynamics seen from the regulation node can be approximated neglecting noise terms by means of the following discrete-time linear model (Fusco and Russo, 2006*a*; Soós and O.P, 2001)

$$A(z^{-1})\Big(v(t_{c,k})-v_0(t_{c,k})\Big)=z^{-d}\,B(z^{-1})\,u(t_{c,k}) \quad (1)$$

in which

$$A(z^{-1})=1+a_1\,z^{-1}+\ldots\ldots+a_{n_A}\,z^{-n_A}$$

$$B(z^{-1})=b_0+b_1\,z^{-1}+\ldots\ldots+b_{n_B}\,z^{-n_B}$$

are algebraic polynomials in the delay operator z^{-1} with $b_0\neq 0$, where $t_{c,k}=k\,T_c$, being T_c the sampling period and k integer. In model (1) $v(t_{c,k})$ is the controlled nodal voltage amplitude at the fundamental frequency, $u(t_{c,k})$ is the controller output, d is a known delay due to the presence of electronic actuator, and $v_0(t_{c,k})$ the no-load voltage representing the nodal voltage when $u(t_{c,k})=0$ (Kundur, 1994). In the remainder it will be assumed that polynomial $B(z^{-1})$ has only stable roots and it can be written as

$$B(z^{-1})=b_0\,B^{+}(z^{-1}) \quad (2)$$

where the sign of b_0 is known and $|b_0|\leq b_0^M$ with $b_0^M>0$. Usually it is realistic to assume that (2) is verified except for some specific cases, such as voltage regulation at midpoint of a long transmission line (Padiyar and Kulkarni, 1997), which yield to a non-minimum phase model.

The no-load voltage $v_0(t_{c,k})$ can be thought as generated from the dynamical system

$$A_d(z^{-1})\,v_0(t_{c,k})=(1-z^{-1})\,v_0(t_{c,k})=D\,\delta(t_{c,k}) \quad (3)$$

where $D\delta(t_{c,k})$ is a pulse. At this point, embedding model (3) in model (1) one has

$$A(z^{-1})\,v(t_{c,k})=z^{-d}\,b_0\,B^{+}(z^{-1})\,u(t_{c,k}) + \frac{A(z^{-1})}{A_d(z^{-1})}\,D\,\delta(t_{c,k}). \quad (4)$$

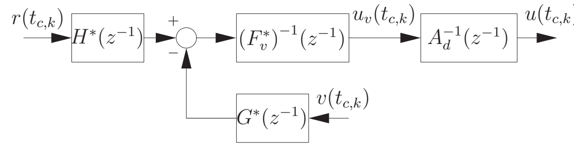

Fig. 1. Block scheme corresponding to control law (7) with factorization (8).

3. MODEL-REFERENCE DESIGN

The model-reference design has the objective of finding an output feedback control signal $u(t_{c,k})$ for the power system model (4) with n_A, n_B, d, a_i, b_j and D known, such that $v(t_{c,k})$ tracks a given reference output $v_m(t_{c,k})$ so that the error

$$e(t_{c,k})=v(t_{c,k})-v_m(t_{c,k}) \quad (5)$$

is small. The reference signal $v_m(t_{c,k})$ is generated from a reference model system

$$A_m(z^{-1})\,v_m(t_{c,k})=z^{-d}\,B_m(z^{-1})\,r(t_{c,k}) \quad (6)$$

where $A_m(z^{-1})$ and $B_m(z^{-1})$ are assigned polynomials, $b_{m,0}\neq 0$, and $r(k)$ is the command signal. A classical choice for the polynomials $A_m(z^{-1})$ and $B_m(z^{-1})$ leads to the following model-reference: $v_m(t_{c,k})=r(t_{c,k-d})$, that is, the output $v_m(t_{c,k})$ assumes the values of the reference $r(t_{c,k})$ with d steps of delay (Tao, 2003). In the remainder, polynomials will be displayed omitting z^{-1}.

The voltage control law assumes the form

$$F^*u(t_{c,k})=-G^*v(t_{c,k})+H^*r(t_{c,k}). \quad (7)$$

In particular, since the controller embeds an integral action polynomial F^* is factorized as

$$F^*=F_v^*\,A_d. \quad (8)$$

In addition since the controller cancels the roots of B^+ polynomial F_v^* is factorized as

$$F_v^*=B^+\,\widetilde{F}^*. \quad (9)$$

The block scheme of control law (7) with factorization (8) is shown in Figure 1. The polynomials

$$\widetilde{F}^*=1+\widetilde{f}_1^*z^{-1}+\ldots+\widetilde{f}^*_{n_{\widetilde{F}}}\,z^{-n_{\widetilde{F}}}$$

$$G^*=g_0^*+g_1^*z^{-1}+\ldots+g^*_{n_G}\,z^{-n_G}$$

are solutions of the Diophantine equation

$$A\,A_d\,\widetilde{F}^*+z^{-d}\,b_0\,G^*=C \quad (10)$$

in which

$$C=A_0\,A_m=1+c_1z^{-1}+\ldots+c_{n_C}\,z^{-n_C}$$

and where A_0 is an assigned observer polynomial. Equation (10) has a unique solution if polynomials A and B are co-prime and the following compatibility conditions are satisfied (Åström and Wittenmark, 1989):

$$\begin{aligned} n_{A_0} &\geq 2\,n_A-n_{B^+}-n_{A_m}\\ n_G &< n_A+n_{A_d}=n_A+1 \\ n_{\widetilde{F}} &\geq d-1 \end{aligned} \quad (11)$$

Looking at first constraint in (11) it is easy to recognize that $n_C=n_{A_0}+n_{A_m}>2\,n_A$. Finally

F^* is obtained via (8) and (9) while H^* is given by $H^* = h_0^* A_0 B_m = A_0 B_m / b_0$.

4. ADAPTIVE LAW DESIGN

When the operating points differ from the one corresponding to the model-reference design, the parameters a_i, b_j and D are unknown. Thus let us consider the following adaptive version of law (7)

$$F_k\, u(t_{c,k}) = -G_k\, v(t_{c,k}) + H_k\, r(t_{c,k}) \qquad (12)$$

in which

$$F_{v,k} = 1 + f_{v,1}(t_{c,k}) z^{-1} + \ldots + f_{v,n_{F_v}}(t_{c,k}) z^{-n_{F_v}}$$
$$G_k = g_0(t_{c,k}) + g_1(t_{c,k}) z^{-1} + \ldots + g_{n_G}(t_{c,k}) z^{-n_G}$$
$$H_k = h_0(t_{c,k})\, A_0\, B_m$$

where, according to (8), it results

$$F_k = F_{v,k}\, A_d. \qquad (13)$$

Moreover, defining the following vector

$$\boldsymbol{\theta} = \Big[f_{v,1} \ldots f_{v,n_{F_v}}\ g_0 \ldots g_{n_G}\ h_0 \Big]^{\mathrm{T}} \in \mathbb{R}^{n_t}$$

with $n_t = n_{F_v} + n_G + 2$ and changing the parameters in the direction of the negative gradient of the normalized quadratic cost function given by

$$J(t_{c,k}) = \frac{1}{2} \frac{\epsilon^2(t_{c,k})}{m^2(t_{c,k})}$$

one obtains (Fusco and Russo, 2006*b*)

$$\boldsymbol{\theta}(t_{c,k+1}) = \boldsymbol{\theta}(t_{c,k}) + \frac{\mathrm{sign}\{b_0\}\, \boldsymbol{\Gamma}\, \epsilon(t_{c,k})\, \boldsymbol{\varphi}_f(t_{c,k-d})}{m^2(t_{c,k})}$$
$$\rho(t_{c,k+1}) = \rho(t_{c,k}) - \frac{\gamma\, \epsilon(t_{c,k}) \xi(t_{c,k})}{m^2(t_{c,k})} \qquad (14)$$

with $\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_i\} \in \mathbb{R}^{n_t}$, γ and γ_i positive gains, and

$$\boldsymbol{\varphi}_f(t_{c,k-d}) = \frac{1}{C} \Big[\Big\{ u_v(t_{c,k-d-i}) \Big\}, \Big\{ v(t_{c,k-d-j}) \Big\}, -B_m\, A_0\, r(t_{c,k-d}) \Big]^{\mathrm{T}}$$

$$\xi(t_{c,k}) = \Big[\boldsymbol{\theta}(t_{c,k-d}) - \boldsymbol{\theta}(t_{c,k}) \Big]^{\mathrm{T}} \boldsymbol{\varphi}_f(t_{c,k-d}) \qquad (15)$$

$$\epsilon(t_{c,k}) = e(t_{c,k}) + \rho(t_{c,k})\, \xi(t_{c,k}) \qquad (16)$$

$$e(t_{c,k}) = b_0 \Big(\theta^* - \theta(t_{c,k-d}) \Big)^{\mathrm{T}} \varphi_f(t_{c,k-d}) + d(t_{c,k}) \qquad (17)$$

$$m^2(t_{c,k}) = k_1 + \boldsymbol{\varphi}_f(t_{c,k-d})^{\mathrm{T}} \boldsymbol{\varphi}_f(t_{c,k-d}) + \xi^2(t_{c,k}) \qquad (18)$$

where $\rho(t_{c,k})$ is the estimate of b_0 and $k_1 > 0$. In (17) the disturbance term

$$d(t_{c,k}) = \frac{A\widetilde{F}^*}{C}\, D\, \delta(t_{c,k})$$

represents the contribution of the no-load voltage. This term coincides with the impulse response of filter $A\widetilde{F}^*/C$; such a response is such that

$$\lim_{k \to \infty} d(t_{c,k}) = 0.$$

The adaptive design in (Fusco and Russo, 2006*b*) has been developed assuming that power system model is noise free. If this assumption is removed, a white noise term $\nu(t_{c,k})$ can be added at the right-hand side of (4). In this case (17) becomes

$$e(t_{c,k}) = b_0 \Big(\theta^* - \theta(t_{c,k-d}) \Big)^{\mathrm{T}} \varphi_f(t_{c,k-d}) + d(t_{c,k}) + d_\nu(t_{c,k})$$

in which the term $d_\nu(t_{c,k})$ takes the from

$$d_\nu(t_{c,k}) = \frac{A_d\, \widetilde{F}^*}{C}\, \nu(t_{c,k}). \qquad (19)$$

However, having in mind that $\nu(t_{c,k})$ is mainly due to measurement noise and commutation in the electronic devices, it is quite realistic to assume that (19) represents a bounded disturbance not necessarily in L^2. To handle such a circumstance, adaptive laws (14) can be suitably modified by adding at the right-hand side of them two modification terms that can be designed, for example, using a dead-zone (Kreisselmeier and Anderson, 1986). The convergence and robustness analysis in presence of the term $d_\nu(t_{c,k})$ is not presented here motivated by the limited space allowed. In (Fusco and Russo, 2006*b*) it has been also demonstrated that adaptive laws (14) and (14) have the following properties: $\boldsymbol{\theta}(t_{c,k}) \in L^\infty$, $\rho(t_{c,k}) \in L^\infty$, $\epsilon(t_{c,k})/m(t_{c,k}) \in L^2 \bigcap L^\infty$ and $\boldsymbol{\theta}(t_{c,k+\ell_0}) - \boldsymbol{\theta}(t_{c,k}) \in L^2$ for any finite integer ℓ_0.

5. ROBUSTNESS ANALYSIS

The robustness analysis of the designed adaptive voltage controller with respect to the unknown no-load voltage disturbance $v_0(t_{c,k})$ will prove that controller (12) and laws (14) guarantee that all signals in the closed-loop system are bounded and

$$\lim_{k \to \infty} e(t_{c,k}) = 0.$$

To proceed, multiplying the Diophantine equation (10) by $v(t_{c,k})$, using (8) and

$$u_v(t_{c,k}) = A_d\, u(t_{c,k}) \qquad (20)$$

see Figure 1, solving with respect to $u_v(t_{c,k})$ gives

$$u_v(t_{c,k}) = -\sum_{i=1}^{n_{F_v}} f_{v,i}^*\, u_v(t_{c,k-i}) - \sum_{j=0}^{n_G} g_j^*\, v(t_{c,k-j}) + \frac{1}{b_0} \Big(v(t_{c,k+d}) + \sum_{q=1}^{n_C} c_q\, v(t_{c,k+d-q}) - \frac{A\, F_v^*}{B^+}\, D\, \delta(t_{c,k+d}) \Big) \qquad (21)$$

in which c_q denotes the coefficients of C. At this point, defining the polynomial A_e as

$$A_e = A_d\, A = 1 + a_{e,1} z^{-1} + \ldots + a_{e,n_A+1}\, z^{-(n_A+1)}$$

from model (4) and using (20) it is possible to write

$$v(t_{c,k+d}) = -\sum_{i=1}^{n_A+1} a_{e,i}\, v(t_{c,k+d-i}) + \sum_{j=0}^{n_B} b_j\, u_v(t_{c,k-j}) + A\, D\, \delta(t_{c,k+d}). \quad (22)$$

Substituting in (22) the expression for $u_v(t_{c,k})$ given by (21) one has

$$v(t_{c,k+d}) = -\sum_{i=1}^{n_A+1} a_{e,i}\, v(t_{c,k+d-i}) + \sum_{j=1}^{n_B} b_j\, u_v(t_{c,k-j})$$

$$-b_0 \sum_{i=1}^{n_{F_v}} f^*_{v,i}\, u_v(t_{c,k-i}) - b_0 \sum_{j=0}^{n_G} g^*_j\, v(t_{c,k-j}) + v(t_{c,k+d})$$

$$+\sum_{q=1}^{n_C} c_q\, v(t_{c,k+d-q}) - \frac{A\, F^*_v}{B^+}\, D\, \delta(t_{c,k+d})$$

$$+A\, D \delta(t_{c,k+d}) \quad (23)$$

To cast (23) in compact form let us introduce the vector

$$\boldsymbol{n}(t_{c,k}) = \Big[\underbrace{u_v(t_{c,k-1}) \ldots u_v(t_{c,k-n_{F_v}})}_{n_{F_v}} \; \underbrace{v(t_{c,k+d-1}) \ldots v(t_{c,k})}_{d} \; \underbrace{v(t_{c,k-1}) \ldots v(t_{c,k-\mu})}_{\mu} \Big]^{\mathrm{T}} \in \mathbb{R}^{n_n}$$

where $n_n = n_{F_v} + d + \mu$, $\mu = \max\{n_G, n_C - d\}$ with $n_C > 2\, n_A \geq n_A + 1$. Equation (23) can then be expressed in matrix form as

$$\boldsymbol{n}(t_{c,k+1}) = \boldsymbol{N}^* \boldsymbol{n}(t_{c,k}) + \boldsymbol{b}^*_n \left(v(t_{c,k+d}) - \frac{A\, F^*_v}{B^+}\, D\, \delta(t_{c,k+d}) \right) + \boldsymbol{d}^*_n\, A\, D \delta(t_{c,k+d}) \quad (24)$$

where

$$\boldsymbol{N}^* = \begin{bmatrix} \boldsymbol{n}_1^{*\mathrm{T}} & \boldsymbol{n}_2^{*\mathrm{T}} \\ \boldsymbol{N}^*_1 & \mathbf{0}_{(n_{F_v}-1)\times(d+\mu)} \\ \boldsymbol{n}_3^{*\mathrm{T}} & \boldsymbol{n}_4^{*\mathrm{T}} \\ \mathbf{0}_{(d+\mu-1)\times(n_{F_v})} & \boldsymbol{N}^*_2 \end{bmatrix} \in \mathbb{R}^{(n_n \times n_n)}$$

$$\boldsymbol{b}^*_n = \Big[\frac{1}{b_0} \; \underbrace{0 \ldots\ldots\ldots 0}_{n_{F_v}-1} \; 1 \; \underbrace{0 \ldots\ldots\ldots 0}_{d+\mu-1} \Big]^{\mathrm{T}}$$

$$\boldsymbol{d}^*_n = \Big[\underbrace{0 \ldots\ldots\ldots\ldots 0}_{n_{F_v}} \; 1 \; \underbrace{0 \ldots\ldots\ldots 0}_{d+\mu-1} \Big]^{\mathrm{T}}$$

with

$$\boldsymbol{N}^*_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & 1 & 0 \end{bmatrix} \in \mathbb{R}^{(n_{F_v}-1)\times(n_{F_v})}$$

$$\boldsymbol{N}^*_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & 1 & 0 \end{bmatrix} \in \mathbb{R}^{(d+\mu-1)\times(d+\mu)}$$

$$\boldsymbol{n}^*_1 = -\Big[f^*_{v,1} \ldots f_{v,n_{F_v}} \Big]^{\mathrm{T}} \in \mathbb{R}^{n_{F_v}}$$

$$\boldsymbol{n}^*_2 = \Big[\frac{c_1}{b_0} \ldots \frac{c_{d-1}}{b_0} \quad \big(\frac{c_d}{b_0} - g^*_0\big) \ldots \ldots \big(\frac{c_{d+\mu}}{b_0} - g^*_\mu\big) \Big]^{\mathrm{T}} \in \mathbb{R}^{d+\mu}$$

$$\boldsymbol{n}^*_3 = b_0 \Big[\big(-f^*_{v,1} + \frac{b_1}{b_0}\big) \ldots \big(-f^*_{v,n_B} + \frac{b_{n_B}}{b_0}\big) - f^*_{v,n_B+1} \ldots - f_{v,n_{F_v}} \Big]^{\mathrm{T}} \in \mathbb{R}^{n_{F_v}}$$

$$\boldsymbol{n}^*_4 = \Big[(c_1 - a_{e,1}) \ldots (c_{d-1} - a_{e,d-1}) \; (c_d - a_{e,d} - b_0\, g^*_0) \ldots \ldots (c_\mu - a_{e,\mu} - b_0\, g^*_\mu) \Big]^{\mathrm{T}} \in \mathbb{R}^{d+\mu}$$

The eigenvalues of matrix $\boldsymbol{N}^*$ are inside the unit circle; to show this property, consider the following equality:

$$A_e\, z^{-n_{F_v}}\, u_v(t_{c,k}) = \boldsymbol{c}^*_n \left(z\mathbf{I}_{n_n} - \boldsymbol{N}^*\right)^{-1} \boldsymbol{b}^*_n\, B\, u_v(t_{c,k}) = \boldsymbol{c}^*_n \frac{\mathrm{Adj}\big(z\mathbf{I}_{n_n} - \boldsymbol{N}^*\big)}{\det\big(z\mathbf{I}_{n_n} - \boldsymbol{N}^*\big)} \boldsymbol{b}^*_n\, B\, u_v(t_{c,k}) \quad (25)$$

where

$$\boldsymbol{c}^*_n = \Big[\underbrace{0 \ldots\ldots\ldots}_{n_{F_v}-1} \; 1 \; \underbrace{0 \ldots\ldots\ldots 0}_{d+\mu} \Big]$$

obtained using model (4) and (24) with $D = 0$.

Due to the structure of $\boldsymbol{b}^*_n$ and $\boldsymbol{c}^*_n$ one has

$$\boldsymbol{c}^*_n\; \mathrm{Adj}\big(z\mathbf{I}_{n_n} - \boldsymbol{N}^*\big)\, \boldsymbol{b}^*_n = \frac{1}{b_0}\, (-1)^{n_{F_v}+1} \det\big(\boldsymbol{\mathcal{N}}_{1,n_{F_v}}\big)$$

$$+ (-1)^{2\, n_{F_v}+1} \det\big(\boldsymbol{\mathcal{N}}_{n_{F_v}+1, n_{F_v}}\big) = \frac{1}{b_0}\, z^{\mu+d}\, A_e \quad (26)$$

where $\boldsymbol{\mathcal{N}}_{1,n_{F_v}}$ and $\boldsymbol{\mathcal{N}}_{n_{F_v}-1,n_{F_v}}$ denote the $(n_n-1)\times(n_n-1)$ matrices obtained by deleting respectively the first row and (n_{F_v})th column and the $n_{F_v}-1$th row and the (n_{F_v})th column of $\big(z\mathbf{I}_{n_n} - \boldsymbol{N}^*\big)$. Substituting (26) in (25) one has

$$\det\big(z\mathbf{I}_{n_n} - \boldsymbol{N}^*\big) = z^{n_n - n_B}\, B^+ \quad (27)$$

which demonstrates that $\boldsymbol{N}^*$ has n_B eigenvalues coincident with the roots of B^+ and the remaining ones in the origin.

Now, replacing in (16) both the expression for $\xi(t_{c,k})$ given by (15) and the error $e(t_{c,k})$ defined in (5), solving with respect to $v(t_{c,k})$ and substituting in (24) yields

$$\boldsymbol{n}(t_{c,k+1}) = \boldsymbol{N}^*\, \boldsymbol{n}(t_{c,k}) + \boldsymbol{b}_n^*\Big(v_m(t_{c,k+d}) + g_n(t_{c,k})$$

$$-\frac{A\, F_v^*}{B^+}\, D\delta(t_{c,k+d})\Big) + \boldsymbol{d}_n^*\, A\, D\delta(t_{c,k+d}) \quad (28)$$

where

$$g_n(t_{c,k}) = \rho(t_{c,k+d})\,\Big(\boldsymbol{\theta}(t_{c,k+d}) - \boldsymbol{\theta}(t_{c,k})\Big)^{\mathrm{T}}\, \boldsymbol{\varphi}_f(t_{c,k})$$

$$+\,\epsilon(t_{c,k+d})$$

Since $\rho(t_{c,k}) \in L^\infty$, $\epsilon(t_{c,k})/m(t_{c,k}) \in L^\infty$, and recalling the expression for $m^2(t_{c,k})$ given by (18) and for $\xi(t_{c,k})$ given by (15) one has

$$|g_n(t_{c,k})| \le \frac{|\epsilon(t_{c,k+d})|}{|m(t_{c,k+d})|}\, |m(t_{c,k+d})|$$

$$+k_3 \Big\|\boldsymbol{\theta}(t_{c,k+d}) - \boldsymbol{\theta}(t_{c,k})\Big\|_2\, \big\|\boldsymbol{\varphi}_f(t_{c,k})\big\|_2 \le$$

$$\sqrt{k_1}\, \frac{|\epsilon(t_{c,k+d})|}{|m(t_{c,k+d})|} + \frac{|\epsilon(t_{c,k+d})|}{|m(t_{c,k+d})|}\, \big\|\boldsymbol{\varphi}_f(t_{c,k})\big\|_2$$

$$+\left(\frac{|\epsilon(t_{c,k+d})|}{|m(t_{c,k+d})|} + k_3\right) \Big\|\boldsymbol{\theta}(t_{c,k+d}) - \boldsymbol{\theta}(t_{c,k})\Big\|_2\, \big\|\boldsymbol{\varphi}_f(t_{c,k})\big\|_2$$

$$\le k_4 + x_n(t_{c,k})\, \big\|\boldsymbol{\varphi}_f(t_{c,k})\big\|_2$$

in which

$$x_n(t_{c,k}) = \frac{|\epsilon(t_{c,k+d})|}{|m(t_{c,k+d})|} + k_5\, \Big\|\boldsymbol{\theta}(t_{c,k+d}) - \boldsymbol{\theta}(t_{c,k})\Big\|_2 \quad (29)$$

for some positive constants k_3, k_4 and k_5. Based on (27), it is possible to affirm that there exists a nonsingular matrix $\mathbf{T}^*$ such that

$$\|\, \mathbf{T}^*\, \boldsymbol{N}^*\, \mathbf{T}^{*-1}\, \|_2 < 1.$$

With matrix $\mathbf{T}^*$ let us define the vector norm

$$\|\, \boldsymbol{n}^*\, \| = \|\, \mathbf{T}^*\, \boldsymbol{n}^*\, \|_2$$

that enables us to rewrite (28) as

$$\big\|\boldsymbol{n}(t_{c,k+1})\big\| \le \Big\|\mathbf{T}^*\, \boldsymbol{N}^*\, \boldsymbol{n}(t_{c,k})\Big\|_2 +$$

$$\Big\|\mathbf{T}^*\, \boldsymbol{b}_n^*\, v_m(t_{c,k+d})\Big\|_2 + \Big\|\mathbf{T}^*\, \boldsymbol{b}_n^*\, g_n(t_{c,k})\Big\|_2 +$$

$$\Big\|\mathbf{T}^*\, \boldsymbol{b}_n^*\, \frac{A\, F_v^*}{B^+}\, D\, \delta(t_{c,k+d})\Big\|_2 + \Big\|\mathbf{T}^*\, \boldsymbol{d}_n^*\, A\, D\, \delta(t_{c,k+d})\Big\|_2$$

At this point using (29) one obtains

$$\big\|\boldsymbol{n}(t_{c,k+1})\big\| \le \big(k_6 + k_7\, x_n(t_{c,k})\big)\, \big\|\boldsymbol{n}(t_{c,k})\big\| + k_8 \quad (30)$$

Now, since $x_n(t_{c,k}) \in L^2$, application of the Hölder inequality gives

$$\sum_{k=k_0}^{k_0+k_f} x_n(t_{c,k}) \le \sqrt{k_f+1}\, \sqrt{\sum_{k=k_0}^{k_0+k_f} x_n^2(t_{c,k})}$$

$$\le k_9 \sqrt{k_f+1} \quad (31)$$

for any $k_f \ge 1$ and some positive constant k_9.

Using (31) one has

$$\prod_{k=k_0}^{k_0+k_f} \big(k_6 + k_7\, x_n(t_{c,k})\big)$$

$$\le \left(k_6 + \frac{k_7}{k_f+1} \sum_{k=k_0}^{k_0+k_f} x_n(t_{c,k})\right)^{k_f+1}$$

$$\le \left(k_6 + \frac{k_9\, k_7}{\sqrt{k_f+1}}\right)^{k_f+1}$$

$$\le k_6^{k_f+1} \left(1 + \frac{k_9\, k_7}{k_6\, \sqrt{k_f+1}}\right)^{k_f+1}$$

with $k_9\, k_7/k_6 > 0$. Since

$$\lim_{k_f\to\infty} \left(1 + \frac{k_9\, k_7}{k_6\, \sqrt{k_f+1}}\right)^{\sqrt{k_f+1}} = e^{\frac{k_9\, k_7}{k_6}}$$

monotonically, one has

$$k_6^{k_f+1} \left(1 + \frac{k_9\, k_7}{k_6\, \sqrt{k_f+1}}\right)^{k_f+1}$$

$$\le k_6^{k_f+1}\, e^{\frac{k_9\, k_7}{k_6} \sqrt{k_f+1}}$$

from which it follows that

$$\lim_{q\to\infty} \sum_{k_f=1}^{q} \prod_{k=k_0}^{k_0+k_f} \big(k_6 + k_7\, x_n(t_{c,k})\big) < \infty \quad (32)$$

Using (30) and (32) it is possible to affirm that $\boldsymbol{n}(t_{c,k})$ is bounded and then $\epsilon(t_{c,k}) \in L^2$, $\big(\boldsymbol{\theta}(t_{c,k+1}) - \boldsymbol{\theta}(t_{c,k})\big) \in L^2$ so that

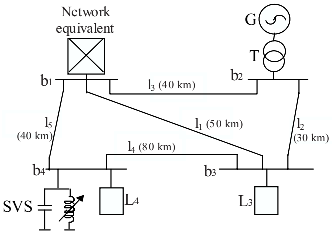

Fig. 2. Test power system.

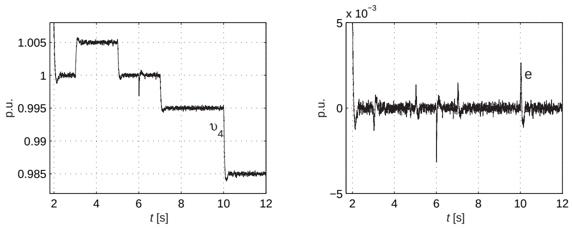

Fig. 3. Time evolution of $v_4(t)$ (left) and $e(t)$ (right).

$$\lim_{k\to\infty} \epsilon(t_{c,k}) = 0$$

$$\lim_{k\to\infty} \big(\boldsymbol{\theta}(t_{c,k+1}) - \boldsymbol{\theta}(t_{c,k})\big) = 0$$

$$\lim_{k\to\infty} \xi(t_{c,k}) = 0.$$

Finally from (16) we obtain

$$\lim_{k\to\infty} e(t_{c,k}) = \lim_{k\to\infty} \big(v(t_{c,k}) - v_m(t_{c,k})\big) = 0$$

which shows that the adaptive voltage regulator ensures robustness with respect to the disturbance $v_0(t_{c,k})$.

6. CASE STUDY

The test power system is shown in Figure 2. All details can be found in (Fusco and Russo, 2006*a*). The controller law (12) has been implemented starting from $n_A = 4$, $n_B = 2$ and $d = 4$. The nodal voltage at node 4 and the tracking error are shown in Figure 3. The first 2 s of simulation have not been reported because they do not represent actual system operation due to the fact that the variables of the power system are initialized far from their steady-state values. In particular at $t = 6$ s a 20% step increase of the load $Q1$ is experienced.

7. CONCLUSIONS

This paper has shown that model-reference adaptive system theory can be employed in nodal voltage control problem. It has been demonstrated that a voltage control scheme designed according to such a theory is robust against the no-load voltage disturbance while all signals in the closed-loop are bounded. This properties are guaranteed if the power system model has all zeros inside the unit circle, as it is realistic to assume.

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