SIGNAL PROCESSING IN ASTROINERTIAL ATTITUDE DETERMINATION SYSTEM FOR THE SPACE ROBOTS

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Abstract
We consider some principal problems on precise attitude determination of the free-flying robots and maneuvering land-survey mini-satellites operated on the orbit altitudes from 550 up to 1000 km. We present onboard discrete algorithms for nonlinear signal processing, calibration, alignment and verification of an astroinertial attitude determination system which is a part of the strapdown inertial navigation system.

Key words
space robot, attitude determination, signal processing

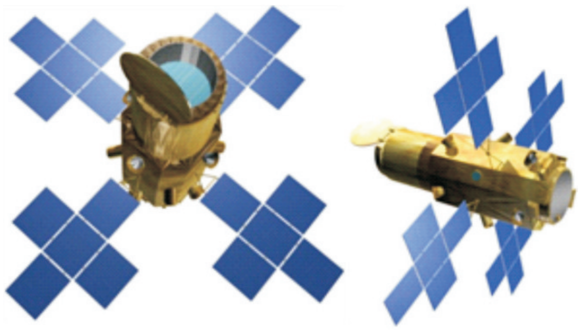

Figure 1. The land-survey mini-satellite, two views

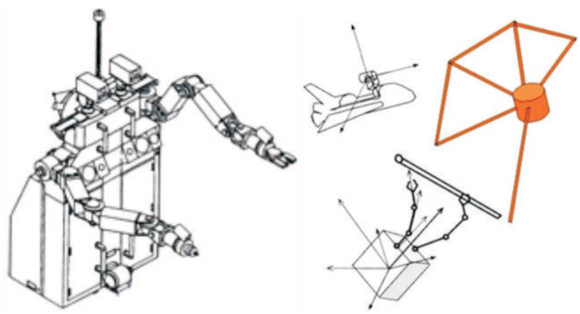

Figure 2. The free-flying SMR at the LSS assembling

1 Introduction

Problems of a spatial guidance, navigation and control are actual for contemporary small information spacecraft (SC) and the free-flying space manipulation robots (SMRs). The information mini-satellites today are applied for communication, geodesy, radio- and opto-electronic observation of the Earth (Fig. 1) et al. The satellites are applied at the orbit altitudes from 550 up to 1000 km, they have mass up to 500 kg and their structure contains the large-scale solar array panels (SAPs) for an energy supplying of the electro-reaction engines.

The free-flying SMRs are designed for service of manned orbital space station (SS), for instance International Space Station, and for assembly of the large space structures, Fig. 2. The SMRs are applied for servicing some space experiments which are performed outside of the SS, inspection and repair operations et al. For example, actual problem is the capturing of an inoperative SC by the SMR. To achieve the mission, the chaser SMR must fly around the target SC so as to track its docking port whose position and attitude generally change with respect to inertial reference frame (IRF), according to a tumbling motion of the target SC. When the position and attitude relative errors become sufficiently small, the chaser SMR can safely to capture the target, and then combined mechanical system on the whole is replaced to the SS (Ikeda *et al.*, 2008).

Global space projects required a designing of the large space structures. In order to replace the decreasing resources of energy carriers, it is planned to construct the large solar power stations provided with solar cell panels sizes up to a football ground. There exists also a project on using the large orbital reflectors to illuminate some towns in the northern regions by solar light during the polar nights. Such on-orbit space structures will be realized by free-flying SMRs, see Fig. 2, which have own mass up to 300 kg, and additional mass of their mechanical flexible payload may be up to 150 kg. Unlike the usual mooring manoeuvres, very often the SMR relative initial velocities can be about zero at stage of short-range guidance. Therefore an acceleration is required at initial stage of SMR flight. For closing and mooring regimes other peculiarity is recurrence of these manoeuvres. This peculiarity increases probability of arising the dangerous collisions with SS surface and requires very careful research of safety mooring, especially at its final phase and a docking. For increasing a safety of the SMR functioning nearby the SS it is foreseen: (i) formation of the safety corridors on the trajectory's intervals which are tangential to the SS surface; (ii) a hovering on final interval of the trajectory before docking to the SS surface or in the joining points of corridors; (iii) a "soft" docking to the SS surface from the hover state at expense of a manipulator's mobility. The safe corridors are formed by beacons mounted onto SS surface (Somov *et al.*, 2014). For every beacon a carrier frequency is different from other in order to identify situations when SMR enters into corridor of next beacon. A corridor's conic surface bounds an admissible spatial motion of SMR nearby the SS. Required safety is defined by fulfilment of "no" intersections of the corridors with guaranteed distance to nearest boundary of SS surface. For SMR trajectory control the important problem consists in SMR transfer flight from one point onto the SS surface to another one which can't be seen from first point.

We have studied the SC attitude determination system (ADS) with an inertial measurement unit (IMU) based on the gyro sensors and an astronomical system (AS) based on star trackers (STs), the that are fixed to the SC body. The ADS is a part of the strapdown inertial navigation system (SINS) which solves the general navigation problem determine both orientation and location of a satellite (Ishlinsky, 1976; Titterton and Weston, 2004).

The problems of the ADS signal processing are connected with integration of kinematic equations in using the information only on the quasi-coordinate increment vector obtained by the IMU at availability of noises, calibration (identification and compensation for the IMU bias $\mathbf{b}^{\mathrm{g}}$ and variation m of the measure scale coefficient by the angular rate vector $\boldsymbol{\omega}$) and alignment (identification and compensation of errors on a mutual angular position of the IMU $\mathbf{G}$ and AS $\mathbf{A}$ reference bases) (Branetz and Shmyglevsky, 1992; Pittelkau, 2001) by its signals with the main period T_o. Many authors applied quaternion $\boldsymbol{\Lambda} = (\lambda_0, \boldsymbol{\lambda})$ with $\boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \lambda_3\}$, an orientation matrix $\mathbf{C}$, Euler vector $\boldsymbol{\phi} = \mathbf{e}\Phi$, terminal rotation vector $\boldsymbol{\rho} = 2\mathbf{e}\,\mathrm{tg}(\Phi/2)$ etc. Moreover, for the SC low angular motion with a small variation of angle Φ during period T_o and almost fixed Euler unit $\mathbf{e}$, integrating kinematic relation for Euler vector $\boldsymbol{\phi}(t)$ with calculation of values $\Lambda_k \equiv \Lambda(t_k)$ is carried out by the scheme:

$$\delta\boldsymbol{\phi}_k = \mathbf{i}_k^{\omega} = \int_{t_k}^{t_{k+1}} \boldsymbol{\omega}(\tau)d\tau \equiv \mathbf{Int}(t_k, T_o, \boldsymbol{\omega}(t));$$
$$\boldsymbol{\phi}_k + \delta\boldsymbol{\phi}_k = \boldsymbol{\phi}_{k+1} \Rightarrow \mathbf{C}_{k+1} \Rightarrow \boldsymbol{\Lambda}_{k+1},$$
$$\delta\boldsymbol{\phi}_k = \delta\Phi_k\mathbf{e}_k,\ t_{k+1} = t_k + T_o,\ k \in \mathbb{N}_0 \equiv [0, 1, 2, ...).$$

Angular movements of a maneuvering land-survey SC are performed on sequence of the time intervals for the observation scanning routes (SRs) and quick rotational maneuvers (RMs) with variable direction of angular rate vector $\boldsymbol{\omega}$ and its module ω up to 5 deg/s. The same requirements apply to the SMRs' mobility. For Euler vector $\boldsymbol{\phi}(t)$ differential equation has the form (Bortz, 1971) $\dot{\boldsymbol{\phi}} = \boldsymbol{\omega} + \frac{1}{2}\boldsymbol{\phi} \times \boldsymbol{\omega} + f(\Phi)\,\boldsymbol{\phi} \times (\boldsymbol{\phi} \times \boldsymbol{\omega})$ where scalar function $f(\Phi) = [1 - \Phi S_\Phi/(2(1 - C_\Phi))]/\Phi^2$ with notations $S_\Phi \equiv \sin\Phi$ and $C_\Phi \equiv \cos\Phi$. Even for small values of angle Φ function $f(\Phi) \approx 1/12$, therefore there arises a coning – known effect of non-commutative rotation due to changing namely direction of vector $\boldsymbol{\omega}$ (Bortz, 1971; Gusinsky *et al.*, 1997).

In development of our papers (Somov, 2009; Somov and Butyrin, 2010; Somov *et al.*, 2013) here the measured information is applied at intermediate points with a period T_q multiple of the main sampling period T_o; polynomial approximation and interpolation are used and integration of the kinematic equation for the vector of modified Rodrigues parameters (MRP) (Yong *et al.*, 2012) is carried out. Quaternion $\boldsymbol{\Lambda}$ is connected with MRP vector $\boldsymbol{\sigma} = \mathbf{e}\,\mathrm{tg}(\Phi/4)$ by straight $\boldsymbol{\sigma} = \boldsymbol{\lambda}/(1 + \lambda_0)$ $(\boldsymbol{\Lambda} \Rightarrow \boldsymbol{\sigma})$ and reverse $\boldsymbol{\lambda} = 2\boldsymbol{\sigma}/(1 + \sigma^2);\ \lambda_0 = (1 - \sigma^2)/(1 + \sigma^2)$ $(\boldsymbol{\sigma} \Rightarrow \boldsymbol{\Lambda})$ relations. For vector $\boldsymbol{\sigma}$, the kinematic equations have the form:

$$\dot{\boldsymbol{\sigma}} = \mathbf{F}^{\sigma}(\boldsymbol{\sigma}, \boldsymbol{\omega}) \equiv \tfrac{1}{4}(1 - \sigma^2)\boldsymbol{\omega} + \tfrac{1}{2}\boldsymbol{\sigma} \times \boldsymbol{\omega} + \tfrac{1}{2}\boldsymbol{\sigma}\langle\boldsymbol{\sigma}, \boldsymbol{\omega}\rangle;$$
$$\boldsymbol{\omega} = 4[(1 - \sigma^2)\dot{\boldsymbol{\sigma}} - 2(\boldsymbol{\sigma} \times \dot{\boldsymbol{\sigma}}) + 2\boldsymbol{\sigma}\langle\dot{\boldsymbol{\sigma}}, \boldsymbol{\sigma}\rangle]/(1 + \sigma^2)^2.$$

2 The Problem Statement

We have introduced the inertial basis $\mathbf{I}$; basis $\mathbf{B}$ and the body reference frame (BRF) connected with the SC body; standard orbital basis $\mathbf{O}$ and the orbit reference frame (ORF); the sensor basis $\mathbf{S}$ (by a telescope); virtual basis $\mathbf{A}$ which is calculated by processing an accessible measurement information from AS, and the IMU virtual basis $\mathbf{G}$ computed by processing the measurement information from the integrating gyros. The BRF attitude with respect to basis $\mathbf{I}$ is defined by quaternion $\boldsymbol{\Lambda}$ and with respect to the ORF – by angles of roll ϕ_1, yaw ϕ_2 and pitch ϕ_3 in sequence 312.

For simplicity, assume that basis $\mathbf{B}$ is coaxial to basis $\mathbf{S}$. We also assume that the measurement information is processed in the IMU with a frequency of about 3 kHz and, as a result, the measured values of the quasi-coordinate increment vector $\mathbf{i}^{\mathrm{g}\,\omega}_{\mathrm{m}\,s}$, $s \in \mathbb{N}_0$ enter from the IMU with period $T_q \ll T_o$ and the quaternion measured values $\boldsymbol{\Lambda}^{\mathrm{a}}_{\mathrm{m}\,k}$ enter from AS:

$$\mathbf{i}^{\mathrm{g}\,\omega}_{\mathrm{m}\,s} = \mathbf{Int}(t_s, T_q, \boldsymbol{\omega}^{\mathrm{g}}_{\mathrm{m}}(t)) + \boldsymbol{\delta}^{\mathrm{n}}_s;\ \boldsymbol{\Lambda}^{\mathrm{a}}_{\mathrm{m}\,k} = \boldsymbol{\Lambda}_k \circ \boldsymbol{\Lambda}^{\mathrm{n}}_k;$$
$$\boldsymbol{\omega}^{\mathrm{g}}_{\mathrm{m}}(t) \equiv (1 + m)\mathbf{S}^{\Delta}(\boldsymbol{\omega}(t) + \mathbf{b}^{\mathrm{g}}). \quad (1)$$

Here $\boldsymbol{\omega}^{\mathrm{g}}_{\mathrm{m}}(t)$ is the measured SC angular rate vector in base $\mathbf{G}$ taking into account the unknown small and slow variations of the IMU bias vector $\mathbf{b}^{\mathrm{g}} = \mathbf{b}^{\mathrm{g}}(t)$; orthogonal matrix $\mathbf{S}^{\Delta}(t)$ describes errors on a mutual angular position of the IMU and AS reference

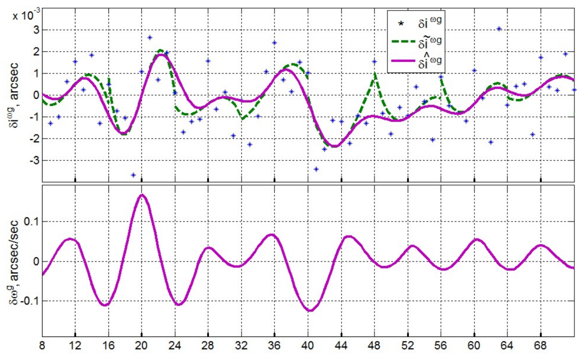

Figure 3. The IMU measurement filtering by two-pass technology

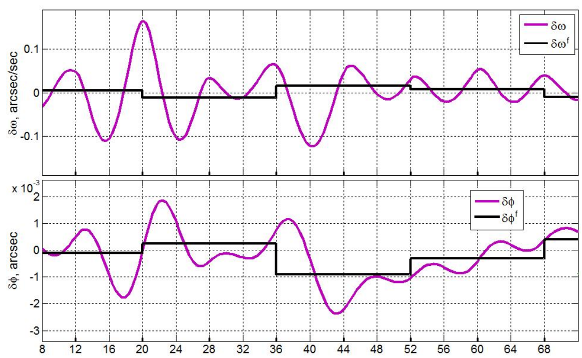

Figure 4. Estimations and digital filtering of the IMU signals

frames, moreover matrix $\mathbf{S}^{\Delta} \approx \mathbf{I}_3 + [\boldsymbol{\Delta}\times]$ where vector $\boldsymbol{\Delta} = \{\Delta_x, \Delta_y, \Delta_z\}$ presents the alignment error; scalar function $m = m(t)$ presents an unknown slow variation of the IMU scale factor. We take into consideration the Gaussian noises $\boldsymbol{\delta}_s^{\mathrm{n}}$ with RMS deviation σ^{b} and $\boldsymbol{\Lambda}_k^{\mathrm{n}}$ with RMS deviation σ^{a} in the IMU and AS output signals, accordingly. We also assume small variation of the IMU scale factor, for example, $|m(t)| \leq 0.01$, when relation $1 - m^2 \cong 1$ is satisfied. The problem consists in developing algorithms for obtaining the quaternion estimation $\hat{\boldsymbol{\Lambda}}_l$, $l \in \mathbb{N}_0$ with period $T_p = t_{l+1} - t_l$ multiple to period T_o, in a general case, with a fixed delay T_d with respect to the time moments t_k, and discrete algorithms for the ADS calibration and alignment with derivation of estimates $\hat{\mathbf{b}}_k^{\mathrm{g}}$, $\hat{\mathbf{S}}_k^{\Delta}$ and $\hat{m}_k$ for considered maneuvering SC with a variable direction of its angular rate vector.

The problems on identification of "alignment" matrix $\mathbf{S}_k^{\Delta}$ and variation of a measure scale factor m are the most complicated ones. This is due to a multiplicative character of the interconnected parametric disturbances indicated. Suggested principal ideas are that: there is needed to define the $\hat{\boldsymbol{\Delta}}$ and $\hat{m}$ estimations only on the *whole* for virtual bases $\mathbf{A}$ and $\mathbf{G}$ with respect to main base $\mathbf{S} = \mathbf{B}$, without concrete details on errors of individual onboard measuring devices and to integrate the kinematic equations with a small computing drift; an idea is being developed to use approximation and interpolation of the measured information in the intermediate points with period T_q multiple to the main sampling period T_{o}; identification of the IMU drift vector $\mathbf{b}^{\mathrm{g}}$ is ensured by nonlinear discrete Luenberger observer.

3 The IMU Drift Calibration

We will provide a forming of digital estimations $\hat{\mathbf{b}}_k^{\mathrm{g}}$, $\hat{\mathbf{S}}_k^{\Delta}$ and $\hat{m}_k$ fixed on period T_o when estimations $\hat{\mathbf{b}}_k^{\mathrm{g}}$ is updated on-line, i.e. in each time moment t_k, and estimations $\hat{\mathbf{S}}_k^{\Delta}$, $\hat{m}_k$ are regularly formed off-line, i.e. based on the processing of available measurement data, accumulated during long-term time intervals. For discrete filtering the measured values of the quasi-coordinate increment vector $\mathbf{i}_{\mathrm{m}\,s}^{\mathrm{g}\,\omega}$ we first used the original two-pass filtering technology – combination of approximation of the measured data $\mathbf{i}_{\mathrm{m}\,s}^{\mathrm{g}\,\omega}$ by the vector polynomial $\tilde{\mathbf{i}}_{\mathrm{m}\,k}^{\mathrm{g}\,\omega}(\tau)$ of the 3rd order in the sliding window 9 measurements on method of least squares and the spline interpolation on the centers of two adjacent sliding windows by the vector spline $\check{\mathbf{i}}_{\mathrm{m}\,k}^{\mathrm{g}\,\omega}(\tau)$ of the 5th order for local time $\tau = t - kT_o \in [0, T_o])$. The technology is illustrated by scheme in Fig. 3. Here errors $\delta\mathbf{i}^{\omega\,\mathrm{g}}$ of the a measured quasi-coordinate are marked by blue "stars" for time moments t_s (index s is shown only), green dotted lines are given polynomials $\tilde{\mathbf{i}}^{\omega\,\mathrm{g}}(\tau)$ of 3rd order and burgundy line is presented the smoothly conjugate splines $\hat{\mathbf{i}}^{\omega\,\mathrm{g}}(\tau) = \check{\mathbf{i}}^{\omega\,\mathrm{g}}(\tau)$ of the 5th order. Error $\delta\boldsymbol{\omega}^{\mathrm{g}}(\tau)$ for estimation $\hat{\boldsymbol{\omega}}^{\mathrm{g}}(\tau)$ on the angular rate is presented in lower part of the figure. The estimation $\hat{\boldsymbol{\omega}}^{\mathrm{g}}(\tau)$ strongly agreed with estimation $\hat{\mathbf{i}}^{\omega\,\mathrm{g}}(\tau)$ as it is carried out by explicit analytical relations.

At compensation of errors on the drift vector, a mutual angular position of the IMU and AS reference frames and on a scale coefficient, the continuous vector estimation $\hat{\mathbf{i}}_k^{\omega}(\tau)$ in base $\mathbf{A}$ is computed by relation $\hat{\mathbf{i}}_k^{\omega}(\tau) = (1 - \hat{m}_k)(\hat{\mathbf{S}}_k^{\Delta})^{\mathrm{t}}(\check{\mathbf{i}}_k^{\mathrm{g}\,\omega}(\tau) - \hat{\mathbf{b}}^{\mathrm{g}}\tau)$ on k-th time interval $\mathrm{T}_k \equiv [t_k, t_{k+1}]$, moreover $\hat{\mathbf{i}}_{k+1}^{\omega} = \hat{\mathbf{i}}_k^{\omega}(T_o)$.

Identification of IMU bias $\mathbf{b}^{\mathrm{g}}$ is carried out with period T_o by the Luenberger observer (Kwakernaak and Sivan, 1972). At the time interval T_k an estimation of the SC attitude is attained by integration of the vector differential equation $\dot{\hat{\boldsymbol{\sigma}}}_k(\tau) = \mathbf{F}^{\sigma}(\hat{\boldsymbol{\sigma}}_k(\tau), \hat{\boldsymbol{\omega}}_k(\tau))$ using ODE45 method (Shampine, 1986; Mathews and Fink, 1999) with a forming of an estimation of the MRP

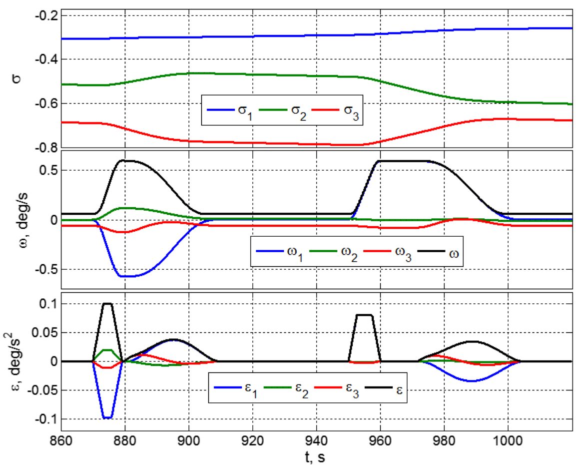

Figure 5. The SMR attitude vector spline guidance law in the IRF

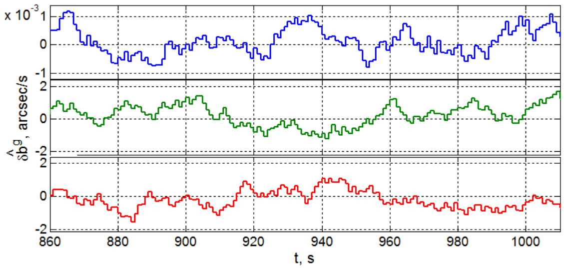

Figure 6. Errors on estimation of the IMU drift vector

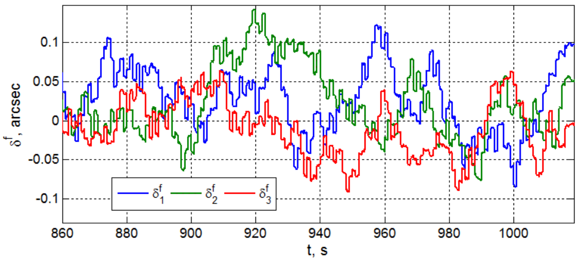

Figure 7. Errors in the IMU digital signals on the SC orientation

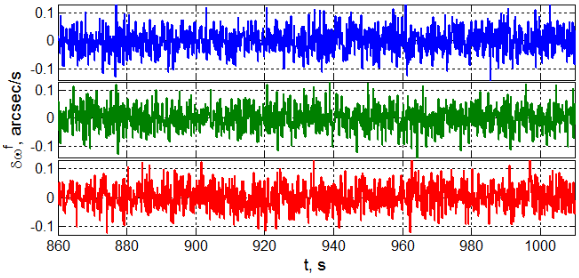

Figure 8. Errors in the IMU output digital signals on angular rate

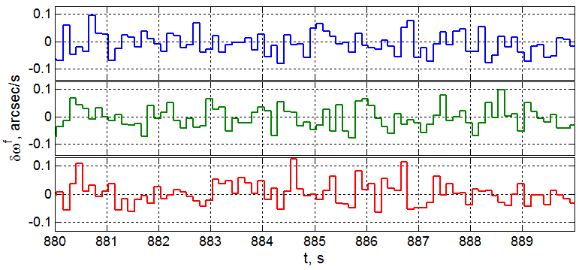

Figure 9. Errors in IMU signals on angular rate, fragment of Fig. 8

vector $\hat{\boldsymbol{\sigma}}_k(\tau)$. For the vector equation an initial condition is formed by AS signals in the time moment t_0 only (at the ADS switch on), at another cases the initial conditions are calculated by signals of the Luenberger observer.

Assume that at the time moment $t = t_k$ we have the AS information on the SC attitude in the form of quaternion $\boldsymbol{\Lambda}^{\rm a}_{{\rm m}\,k}$, the correcting vector $\Delta\mathbf{p}_k(g_2^{\rm o}, \mathbf{Q}_k)$ and quaternion $\Delta\mathbf{P}_k(g_1^{\rm o}, \mathbf{Q}_k)$ were formed, where quaternion $\mathbf{Q}_k \equiv (q_{0k}, \mathbf{q}_k) \equiv (C_{\frac{\varphi_k}{2}}, \mathbf{e}_k^q\, S_{\frac{\varphi_k}{2}}) \equiv \mathbf{Q}_k(\mathbf{e}_k^q, \varphi_k) = \tilde{\boldsymbol{\Lambda}}^{\rm a}_{{\rm m}\,k} \circ \hat{\boldsymbol{\Lambda}}_k$. At the same time moment t_k by a transformation $\hat{\Lambda}_k \Rightarrow \hat{\boldsymbol{\sigma}}_k$ the initial condition $\hat{\boldsymbol{\sigma}}_k(0) \equiv \hat{\boldsymbol{\sigma}}_k$ is defined for calculation of the MRP vector estimate $\hat{\boldsymbol{\sigma}}_k(\tau)$ on the k-th interval by numerical integration of the differential equation.

After such integration one can obtain the MRP vector's value $\hat{\boldsymbol{\sigma}}_{k+1} = \hat{\boldsymbol{\sigma}}_k(T_o)$ for the local time moment $\tau = T_o$. The value of quaternion $\hat{\mathbf{R}}_k$ is calculated by a transformation $\hat{\boldsymbol{\sigma}}_{k+1} \Rightarrow \hat{\mathbf{R}}_k$.

The developed discrete nonlinear Luenberger observer has the form (Somov and Butyrin, 2016):

$$\hat{\boldsymbol{\Lambda}}_{k+1} = \hat{\mathbf{R}}_k \circ \Delta\mathbf{P}_k(g_1^{\rm o}, \mathbf{Q}_k);$$

$$\hat{\mathbf{b}}^{\rm g}_{k+1} = \hat{\mathbf{b}}^{\rm g}_k + \Delta\mathbf{p}_k(g_2^{\rm o}, \mathbf{Q}_k);$$

$$\Delta\mathbf{P}_{k+1} = \mathbf{Q}_{k+1}(\mathbf{e}^q_{k+1}, g_1^{\rm o}\varphi_{k+1}); \Delta\mathbf{p}_{k+1} = 4g_2^{\rm o}\boldsymbol{\sigma}^q_{k+1},$$

where both the quaternion and vector relations are applied, moreover the MRP vector $\boldsymbol{\sigma}^q_{k+1}$ is defined analytically on the quaternion value $\mathbf{Q}_{k+1}$, and the observer scalar coefficients $g_1^{\rm o}, g_2^{\rm o}$ are calculated by analytic relations (Somov and Butyrin, 2016).

In final stage the MRP vector values $\boldsymbol{\sigma}_s$ are processed by recurrent discrete filter with period T_p. The filtering technology is illustrated by scheme in Fig. 4. Here $\delta\omega(t), \delta\phi(t)$ are the continuous mismatches and their filtered digital values $\delta\omega_l^{\rm f}, \delta\phi_l^{\rm f}$ are presented by black lines when period $T_p = 16T_q$. As a result, one can obtain the MRP vector values $\hat{\boldsymbol{\sigma}}_l$ which are applied for a forming of the quaternion estimate $\hat{\boldsymbol{\Lambda}}_l$, $l \in \mathbb{N}_0$ with given period T_p using transformation $\hat{\boldsymbol{\sigma}}_l \Rightarrow \hat{\boldsymbol{\Lambda}}_l$.

The IMU astronomical correction is temporarily disabled when module $\omega(t)$ of the SC angular rate vector satisfies the inequality $\omega(t) \geq 1$ deg/s during a time interval of the SMR rotational maneuver, but estimation of the SMR angular position continues using the forecast of the $\hat{\mathbf{b}}^{\rm g}_k$ variation.

4 The ADS Alignment

The ADS alignment (calculation of matrix $\hat{\mathbf{S}}^{\Delta}$) and determination of estimate $\hat{m}$ for the error of the scale factor m are carried out off-line by comparing the angular rate vector values, which are recovered autonomously from the IMU signals (vectors $\hat{\boldsymbol{\omega}}^{\rm g}$) and from the AS correction signals (vectors $\hat{\boldsymbol{\omega}}^{\rm a}$) at the same time moments.

Here sets of vectors $\hat{\boldsymbol{\omega}}^{\rm g}_l$ and $\hat{\boldsymbol{\omega}}^{\rm a}_l$ values are formed, fixed to the time moments t_l with a period T_p. For these vectors the values of modules $\hat{\omega}^{\rm g}_l = |\hat{\boldsymbol{\omega}}^{\rm g}_l|$, $\hat{\omega}^{\rm a}_l = |\hat{\boldsymbol{\omega}}^{\rm a}_l|$, units $\hat{\mathbf{e}}^{\rm g}_{\omega l} = \hat{\boldsymbol{\omega}}^{\rm g}_l/\hat{\omega}^{\rm g}_l$, $\hat{\mathbf{e}}^{\rm a}_{\omega l} = \hat{\boldsymbol{\omega}}^{\rm a}_l/\hat{\omega}^{\rm a}_l$ are calculated and the alignment problem is solved on the values of units $\hat{\mathbf{e}}^{\rm a}_{\omega l}$ in basis $\mathbf{A}$ and the units $\hat{\mathbf{e}}^{\rm g}_{\omega l}$ in basis $\mathbf{G}$ using QUEST algorithm.

For calibration of the scale factor error m, the set of values $m_l = 1 - \hat{\omega}^{\rm m}_l/\hat{\omega}^{\rm a}_l$ is calculated, the estimate $\hat{m}$ is obtained by processing this set by the method of least squares. This estimation is applied in the form of $\hat{m}_k$ until the next calibration is completed.

5 Results of Computer Simulation

In Fig. 5 we present typical the SMR attitude guidance law, values of its parameters were computed with the use of developed analytic relations. Here components of vectors $\boldsymbol{\sigma}(t)$, $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ are marked by different colors – blue color on roll, green on yaw and red color on pitch, and modules of vectors $\boldsymbol{\omega}(t)$, $\boldsymbol{\varepsilon}(t)$ are marked by black color. At the computer simulation of the ADS operation we have applied the RMS deviations $\sigma^{\mathrm{b}} = 0.001$ arc sec$\sqrt{\mathrm{Hz}}$ for frequency 128 Hz and $\sigma^{\mathrm{a}} = 0.33$ arc sec$\sqrt{\mathrm{Hz}}$ for frequency 1 Hz in the IMU and AS output signals, accordingly.

The IMU drift vector $\mathbf{b}^{\mathrm{g}} = \{b_i^{\mathrm{g}}\}$ was adopted with the components $b_i^{\mathrm{g}}(t) \in [-1, 1]$ arc sec/s which slowly change. In Fig. 6 errors in the IMU drift estimating are presented for components of vector $\delta\hat{\mathbf{b}}_k^{\mathrm{g}} = \mathbf{b}_k^{\mathrm{g}} - \hat{\mathbf{b}}_k^{\mathrm{g}}$ in the basis $\mathbf{G}$. Hereinafter the errors are marked by the same different colors on channels.

Digital filtering of the ADS output signals on the SC orientation with a frequency of 8 Hz yields vector $\boldsymbol{\delta}^{\mathrm{f}}$, its components are presented in Fig. 7. We represent the ADS errors in determining the angular rate vector and digital filtering of its values with a frequency of 8 Hz in Figs. 8 and 9 for components $\delta\omega_i^{\mathrm{f}}$ of vector $\delta\boldsymbol{\omega}^{\mathrm{f}}$ using the same colors.

For the IMU drift estimation without astronomical correction the forecast variations performed on the set of estimated vectors $\hat{\boldsymbol{\omega}}^{\mathrm{g}}$ and $\hat{\boldsymbol{\omega}}^{\mathrm{a}}$ generated in the last "sliding window" with duration of 20 seconds.

By applying the developed procedures on the ADS alignment and calibration we have obtained the estimation on components of vector $\hat{\boldsymbol{\Delta}}$ with accuracy ≈ 3 arc sec and the estimation $\hat{m}$ on the measure scale coefficient by the angular rate vector $\boldsymbol{\omega}$ with accuracy ≈ 0.025 %.

6 Conclusions

We briefly have presented some principal problems and developed methods for precise attitude determination of free-flying SMRs and the maneuvering land-survey satellites with a variable direction of their angular rate vector. We have developed onboard discrete algorithms for signal processing, calibration and alignment of the astroinertial attitude determination system and presented numerical results on the efficiency of the developed algorithms.

Acknowledgments

This work was supported by RFBR (Russian Foundation for Basic Research, Grant nos. 17-08-01708, 17-48-630637) and also Division on EMMCP of Russian Academy of Sciences (Program no. 13 for basic research).

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