STATISTICS AND CONTROL OF CHAOTIC ATOMIC TRANSPORT IN AN OPTICAL STANDING-WAVE FIELD

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Abstract
Centre-of-mass atomic motion in an optical lattice is shown to be near the resonance a chaotic walking due to the interplay between coherent internal atomic dynamics and spontaneous emission. Statistical properties of chaotic atomic motion can be controlled by the single parameter, the detuning between the atomic transition frequency and the laser frequency. We derive a Fokker-Planck equation in the energetic space to describe the atomic transport near the resonance and demonstrate numerically how to manipulate the atomic motion varying the detuning.

Key words
Cold atom, control of statistics, Levy flights, Monte Carlo wavefunction, chaotic walking

1 Introduction

Let us consider a cold two-level atom in a laser standing wave forming a 1D optical lattice. Depending on the values of atomic and lattice parameters, different regimes of the centre-of-mass motion can be identified: oscillations in wells of the optical potential, ballistic flights with acceleration and deceleration, velocity grouping, Brownian motion and random walking with Lévy flights [Kazantsev et al, 1990; Bardou et al, 2002; Chu, Wieman, 1989]. Theoretical study and experimental realization of these effects have been done, mainly, in the context of laser cooling of atoms when the laser is far detuned from the atomic transition frequency and the excited state can be adiabatically eliminated. Near the resonance, coherent interaction of atoms with a laser field may strongly affect the atomic motion. It has been shown in Refs. [Prants, Konkov, 2001; Argonov, Prants, 2007] that, when neglecting spontaneous emission (SE), there should exist *a deterministic chaotic transport* of atoms with exponential sensitivity to small variations in initial conditions and/or lattice parameters. In our recent work [Argonov, Prants, 2007] we developed a semiclassical theory of this phenomenon resembling a random walking but without any random forces and noise.

In this Letter we use a Monte Carlo wavefunction approach to take into account SE and demonstrate a new type of motion in an optical lattice, *a chaotic atomic walking*, with the properties both of the chaotic atomic transport, caused by coherent atomic dynamics, and of a random process due to SE kicks. It is a kind of random walking with specific statistical properties that cannot be classified neither as deterministic chaotic motion nor as normal diffusion nor as sub(super)diffusion and nor as Lévy flights. We derive a Fokker-Planck equation in the energetic space, study the statistical properties of the chaotic walking and demonstrate numerically how to manipulate these properties varying the atom-field detuning. Small changes in this parameter may affect drastically the atomic transport, transforming the atomic motion from a practically regular one to anomalous diffusion.

2 Monte Carlo wavefunction approach

We start with the non-Hermitian Hamiltonian of a two-level atom interacting with a strong standing-wave 1D laser field forming an optical lattice. In the frame rotating with the laser frequency ω_f, it has the form

$$
\begin{aligned}
\hat{H} =& \frac{\hat{P}^2}{2m_a} + \frac{1}{2}\hbar(\omega_a - \omega_f)\hat{\sigma}_z - \\
& - \hbar\Omega\left(\hat{\sigma}_- + \hat{\sigma}_+\right)\cos k_f\hat{X} - i\hbar\frac{\Gamma}{2}\hat{\sigma}_+\hat{\sigma}_-,
\end{aligned} \tag{1}
$$

where $\hat{\sigma}_{\pm,z}$ are the Pauli operators for the internal atomic degrees of freedom, $\hat{X}$ and $\hat{P}$ are the atomic position and momentum operators, ω_a, ω_f and Ω are the atomic transition, laser and Rabi frequencies, respectively, and Γ is the spontaneous decay rate. Internal atomic states are described by the wavefunction $|\Psi(t)\rangle = a(t)|2\rangle + b(t)|1\rangle$, with a and b being the complex-valued probability amplitudes to find an atom in the excited $|2\rangle$ and ground $|1\rangle$ states. Note that the norm of the wavefunction, $|a|^2 + |b|^2$, is not conserved due to non-Hermitian term in the Hamiltonian.

To study the atomic dynamics in the optical lattice we use the standard Monte Carlo wavefunction technique to get the most probabilistic outcome that can be compared directly with corresponding experimental observations with single atoms. The method is based on the evolution of an atomic state $|\Psi(t)\rangle$ while a continuous measurement of radiation-field state is performed by an ideal photodetector. The evolution consists of two parts: (1) jumps to the ground state ($a = 0$, $|b|^2 = 1$) each of which is accompanied by the emission of an observable photon at random time moments with the mean time $(|a|^2\ \Gamma)^{-1}$ and (2) coherent evolution with continuously decaying norm of the atomic state vector without the emission of an observable photon. The decay of the norm of the state vector is equal to the probability of spontaneous emission of the next photon. This coherent decay without emission of a photon is usually interpreted within the context of the quantum measurement theory. Let us introduce the new real-valued variables (normalized all the time) instead of the amplitudes a and b (renormalized only after SE events)

$$u \equiv \frac{2\,\mathrm{Re}\,(ab^*)}{|a|^2+|b|^2}, \quad v \equiv \frac{-2\,\mathrm{Im}\,(ab^*)}{|a|^2+|b|^2}, \quad z \equiv \frac{|a|^2-|b|^2}{|a|^2+|b|^2}, \tag{2}$$

which have the meaning of synphase and quadrature components of the atomic electric dipole moment and the population inversion, respectively. We stress that the length of the Bloch vector, $u^2 + v^2 + z^2 = 1$, is conserved.

The average atomic momentum is supposed to be large as compared with the photon momentum $\hbar k_f$, so the translational atomic motion can be treated classically using the Hamilton equations. The whole atomic dynamics is governed by the following Hamilton-Schrödinger equations [Argonov, Prants, 2006]:

$$\begin{aligned}
&\dot{x} = \omega_r p, \quad \dot{p} = -u\sin x + \sum_{j=1}^{\infty} p_j \delta(\tau - \tau_j),\\
&\dot{u} = \Delta v + \frac{\gamma}{2} uz - u\sum_{j=1}^{\infty} \delta(\tau - \tau_j),\\
&\dot{v} = -\Delta u + 2z\cos x + \frac{\gamma}{2} vz - v\sum_{j=1}^{\infty} \delta(\tau - \tau_j),\\
&\dot{z} = -2v\cos x - \frac{\gamma}{2}(u^2+v^2) - (z+1)\sum_{j=1}^{\infty} \delta(\tau - \tau_j),
\end{aligned} \tag{3}$$

where $x \equiv k_f\langle\hat{X}\rangle$ and $p \equiv \langle\hat{P}\rangle/\hbar k_f$ are the normalized atomic centre-of-mass position and momentum, respectively. The dot denotes differentiation with respect to the normalized time $\tau \equiv \Omega t$. The values of the normalized decay rate $\gamma \equiv \Gamma/\Omega$ and the recoil frequency $\omega_r \equiv \hbar k_f^2/m_a\Omega \ll 1$ are fixed in this paper, and the normalized detuning $\Delta \equiv (\omega_f - \omega_a)/\Omega$ is a variable parameter. In the equations of motion (3) τ_j are random time moments of SE events and p_j are random recoil momenta with the values between ± 1 in a one-dimensional case. In terms of the normalized time τ the mean frequency of SE events is equal to $\gamma(z+1)/2$. At the time moments $\tau = \tau_j$, the atomic variables change as follows: $p \to p + p_j$, $u \to 0,\ v \to 0,\ z \to -1$. The well-known effects of acceleration, deceleration and velocity grouping have been successfully simulated with Eqs. (3) in Ref. [Argonov, Prants, 2006]. In the present numerical simulation we take a cesium atom with the working transition $6S_{1/2} - 6P_{3/2}$ ($\lambda_a = 852.1$ nm and $\Gamma = 3.2 \cdot 10^7$ Hz) interacting with a rather strong field with the resonant Rabi frequency $\Omega = 10^{10}$ Hz. Thus, the corresponding normalized recoil frequency is $\omega_r = 10^{-5}$ and the spontaneous decay rate is $\gamma = 3.3 \cdot 10^{-3}$.

It follows from Eqs. (3) that the centre-of-mass motion is described by the equation for a nonlinear physical pendulum with a frequency modulation caused by coherent internal atomic dynamics and random kicks of the momentum. Besides SE recoils any atomic trajectory is determined by the coherent evolution of the synphase component of the electric dipole moment u between the events of SE and its jumps at random time moments τ_j. These jumps are not small. In our recent paper [Argonov, Prants, 2007] we developed a theory of atomic transport in an optical lattice (in the absence of SE) based on a specific behavior of the variable u which performs shallow and fast oscillations between the nodes of the standing laser wave and changes suddenly its value when atoms cross the nodes. The theory predicts deterministic chaotic transport at small values of the detuning $|\Delta| \ll 1$ whose statistical properties are very well described by a stochastic map for the deterministic variable u [Argonov, Prants, 2007]. In fact, atom moves in a rigid optical lattice just like as in a random optical potential with a complicated alternation of oscillations in potential wells and flights over many wells when it can change its direction of motion many times. SE causes further complication of this motion.

3 Chaotic walking at small detunings, $|\Delta| \ll 1$

Near the resonance ($|\Delta| \ll 1$), the following quantity is almost conserved between any two acts of SE:

$$\begin{aligned}
\tilde{H}_j \equiv \frac{\omega_r}{2}p^2 - u\cos x - \frac{\Delta}{2}z - \frac{\Delta\gamma}{4}\langle 1 - z^2\rangle(\tau - \tau_j) =\\
= H - \frac{\Delta\gamma}{4}\langle 1 - z^2\rangle(\tau - \tau_j),
\end{aligned} \tag{4}$$

where $\tau_j \leqslant \tau < \tau_{j+1}$. H is the total atomic energy which is a constant in the purely Hamiltonian system, i.e. without any relaxation [Argonov, Prants, 2007]. The last term in (4) with the averaging over a time exceeding the period of the Rabi oscillations compensates the relaxation. The energy H changes suddenly at the moments of SE and decays linearly in between (Fig. 1) whereas the pseudoenergy $\tilde{H}$ changes suddenly as well

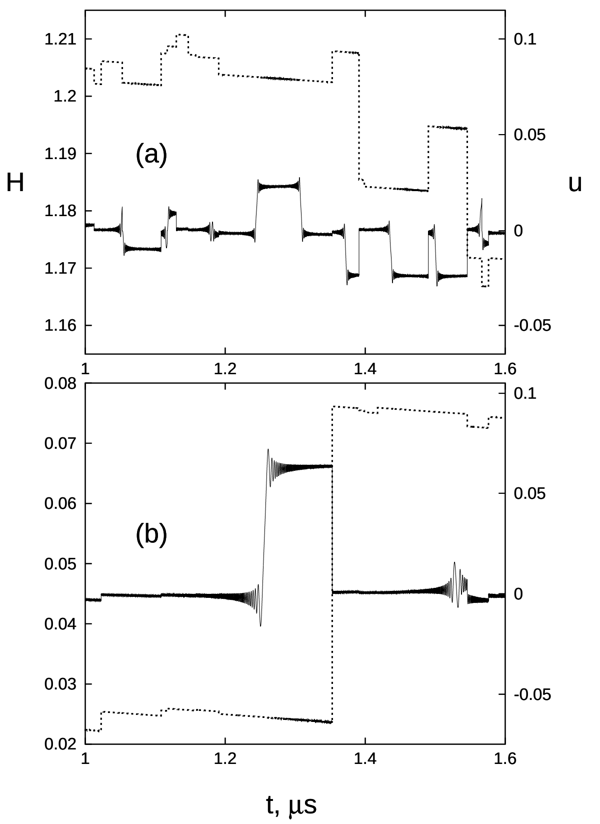

Figure 1. Time evolution of the atomic energy H (dashed line) and the synphase component of the electric dipole moment $u0$ (solid line) at the same small value of the detuning $\Delta = -0.001$ but with different initial conditions.

at the same moments but is approximately a constant in between. At $H \gtrsim |u|$, the atom moves ballistically, and its momentum cannot be zero because at small detunings $|\Delta|$ the kinetic energy is larger than all the other terms in (4). If $H \lesssim 0$, the atom changes surely the sign of the momentum during its motion. Thus, if we could construct a mapping for the pseudoenergy $\tilde{H}_j$, we would know approximately when atoms move ballistically and when they turn and could estimate the duration of the atomic flights which is a time interval between two successive events when atom changes the sign of p.

Just after SE at $\tau = \tau_j$, we have $u \to 0$, $z \to -1$, $p \to p + p_j$ and $H \to \tilde{H}_j$, and a change in the energy H during the time interval between the acts of SE is equal to the difference between the values of the pseudoenergy just before ($\tilde{H}_{j-1}$) and just after the j-th SE ($\tilde{H}_j$)

$$
\begin{aligned}
H_j - H_{j-1} = \tilde{H}_j - \tilde{H}_{j-1} = \omega_r p(\tau_j) p_j + \frac{\omega_r}{2} p_j^2 + \frac{\Delta}{2} + \\
+ u(\tau_j)\cos x(\tau_j) + \frac{\Delta}{2} z(\tau_j) + \frac{\Delta\gamma}{4}\langle 1 - z^2 \rangle (\tau_j - \tau_{j-1}),
\end{aligned} \tag{5}
$$

where H_j is a value of the energy just after $\tau = \tau_j$ and $x(\tau_j)$, $u(\tau_j)$, $z(\tau_j)$, $p(\tau_j)$ are the values of the corresponding variables just before the moments $\tau = \tau_j$ which are determined by coherent evolution between SE events. Changes in H at $\tau = \tau_j$ are conditioned mainly by the corresponding changes in u (Fig. 1). We stress that sudden changes in u occur at the moments of crossing the nodes of the standing wave and SE events. In Fig. 1b the jump just after $t = 1.2$ μs occurs when the atom crosses a node whereas the jump just before $t = 1.4$ μs is caused by a SE event.

Let us estimate the average value of the energy jump's magnitude. Analytical estimate and simulation show that with sufficiently large values of the momentum, $\omega_r |p| \gtrsim \gamma/2$, and at small detunings, we get $\langle u \cos x \rangle \simeq \langle z \rangle \simeq 0$ during the coherent evolution. The component u never goes far away from zero, and z performs frequency-modulated harmonic oscillations in the range $-1 \lesssim z \lesssim 1$. The probability of SE is proportional to $(z+1)/2$. We estimate the average value of the population inversion just before SE events at $\tau = \tau_j$ to be equal to $\langle z(\tau_j) \rangle \simeq 0.5$. In the expression (5) the first and fourth terms are estimated to be zero in average but the second, third, fifth and sixth ones are not. The total average change in H_j due to SE and the relaxation term is

$$
\langle H_j - H_{j-1} \rangle = \frac{\omega_r}{6} + \Delta, \tag{6}
$$

where we estimated the average value of z^2 to be $1/2$ and the average value of the squared recoil momentum p_j^2 is $1/3$. The mean time between SE events is $\langle \tau_j - \tau_{j-1} \rangle = 2/\gamma$.

Thus, the evolution of the energy H is an asymmetric random walking. At positive values of the detuning Δ, the values of H increase in average, whereas at $\Delta < 0$ they may increase or decrease depending on the relations between ω_r and Δ. We take the values of the detuning to be $|\Delta|/2 \gg \omega_r/9$, and if $\Delta < 0$ then H_j decreases in average. The corresponding physical effects — light-induced acceleration and deceleration of atoms — are well known [Kazantsev et al, 1990].

The diffusion coefficient in the energetic space can be estimated with the help of the two largest terms in Eq. (5) as

$$
\begin{aligned}
D \equiv \frac{\langle (H_j - H_{j-1})^2 \rangle - \langle H_j - H_{j-1} \rangle^2}{4 \langle \tau_j - \tau_{j-1} \rangle} \simeq \\
\simeq \frac{\langle \omega_r^2 p^2 p_j^2 \rangle \gamma + \langle u^2(\tau_j) \cos^2 x(\tau_j) \rangle \gamma}{8} \simeq \frac{\omega_r H_j \gamma}{12} + \frac{\Delta^2}{16},
\end{aligned} \tag{7}
$$

where we suppose u to be a random-like process (Fig. 1) described by the Eq. (11) from Ref.[Argonov, Prants, 2007], which is correct only under the condition $|p| \ll 1/\omega_r$. This condition is fulfilled for all the atomic flights found in the numerical simulation. By the other hand, there is a stronger condition, $\omega_r|p| \gtrsim \gamma/2$, that is needed to neglect correlations between u^2 and $\cos^2 x$. Thus, the expression (7) fails to give a result supporting the corresponding numerics for sufficiently slow atoms. The drift velocity of particles in the energetic space at sufficiently large values of p is

$$c \equiv \frac{d\langle H\rangle}{d\tau} = \frac{\langle H_j - H_{j-1}\rangle}{\langle \tau_j - \tau_{j-1}\rangle} = \frac{\omega_r\gamma}{12} + \frac{\Delta\gamma}{2}. \qquad (8)$$

This result is also correct only with sufficiently fast atoms. In Fig. 2 we compare numerical results of computation of the diffusion D and drift c coefficients with those obtained with (7) and (8). Some cases (especially D for $\Delta = -0.0001$) show very good correspondence between numerical and analytical data, some (D for $\Delta = -0.001$) have much less precision. Analytical results for drift coefficient c are not applicatable for most slow atoms. However, in all the cases we have rather good qualitative correspondence. At the same condition as (8) we can estimate the friction force acting on the atom to be the following:

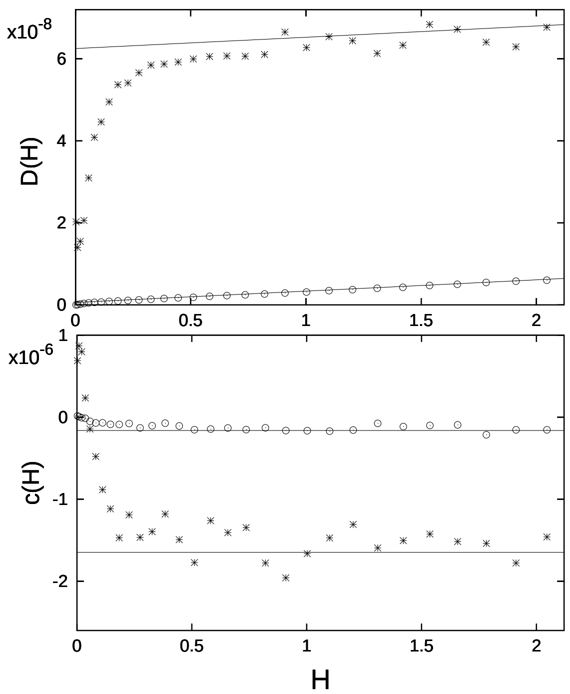

Figure 2. The diffusion D and drift c coefficients in the enegry space vs the energy H: stars — $\Delta = -0.001$, circles — $\Delta = -0.0001$ (numeric data). Solid lines show the corresponding analytic results using formulas (7) and (8).

$$F \equiv \frac{d\langle p\rangle}{d\tau} \simeq \frac{\Delta\gamma}{2\omega_r p}. \qquad (9)$$

This kind of decreasing F with increasing p is a well-known fact [Kazantsev et al, 1990].

Random jumps of the atomic energy just after SE give rise to a random walking of atoms in an optical lattice. In order to find distribution of the durations of atomic flights we consider the problem of the first passage time for the quantity H_j to return to its zero value (to be more correct, a return of H to $|u|$ must be considered, but with $|\Delta| \ll 1$ the variable u cannot reach large values and always returns to zero value for the time $\sim 2/\gamma$, see Fig. 1). In the very beginning of any flight we have $H_j \approx 0$, then it can reach a rather large value and after that it returns to zero. The duration of this process is a flight duration T.

If the random jumps of H_j would be symmetric, the probability to find the flight duration to be equal to T would be proportional to $T^{-1.5}$, where the exponent does not depend on the diffusion coefficient (a classical result in theory of symmetric random walking [Feller, 1964]).

What will happen if we take into account that the random walking of H_j is asymmetric? At $\Delta > 0$, atoms begin to accelerate without any flights. At $\Delta < 0$, the friction force tends to stop atoms, and instead of the power-law decay $T^{-1.5}$ we get an exponential one at large T. At large times exceeding the mean SE time $2/\gamma$, one may treat the evolution of H as a diffusion process with a drift described by the Fokker-Planck equation in the energetic space

$$\dot{P}(H,\tau) = -2c\frac{\partial P}{\partial H} + D\frac{\partial^2 P}{\partial H^2}, \qquad (10)$$

where the diffusion D and drift c coefficients are given by Eqs. (7) and (8), respectively. If the coefficients c and D would not change with changing the energy, the PDF for flight durations would be equal to [Feller, 1964]

$$P_{\text{fl}} \propto e^{-c^2T/D}T^{-1.5}. \qquad (11)$$

This result agrees qualitatively with the results of numerical simulation shown in Fig. 3a for a few values of the detuning Δ. Really, all the PDFs in Fig. 3a have power-law fragments followed by exponential tails at large T in accordance with the formula (11). However, the length of these fragments depends strongly on the value of the detuning. At very small value $\Delta = -10^{-5}$ (when coherent atomic dynamics is practically regular [Argonov, Prants, 2007] and atom performs a random walk due to SE), $P_{\text{fl}} \sim T^{-1.5}$, whereas at larger values of $|\Delta|$ the power-law fragments are much shorter.

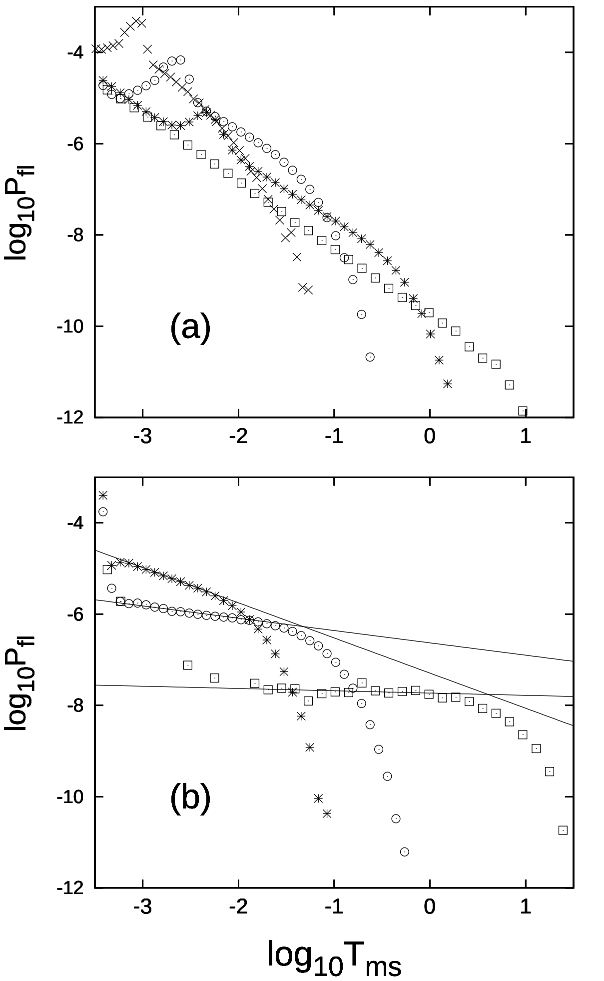

Figure 3. The PDFs $P_{\rm fl}$ for the duration of atomic flights T_{ms} in milliseconds with (a) small detunings (crosses $\Delta = -0.01$, stars $\Delta = -0.001$, circles $\Delta = -0.0001$, squares $\Delta = -0.00001$) and (b) medium detunings (stars $\Delta = -0.09$, $\alpha = -0.77$; circles $\Delta = -0.1$, $\alpha = -0.27$; squares $\Delta = -0.12$, $\alpha = -0.05$). Straight lines show slopes α of the power-law fragments of the PDFs in log-log scale.

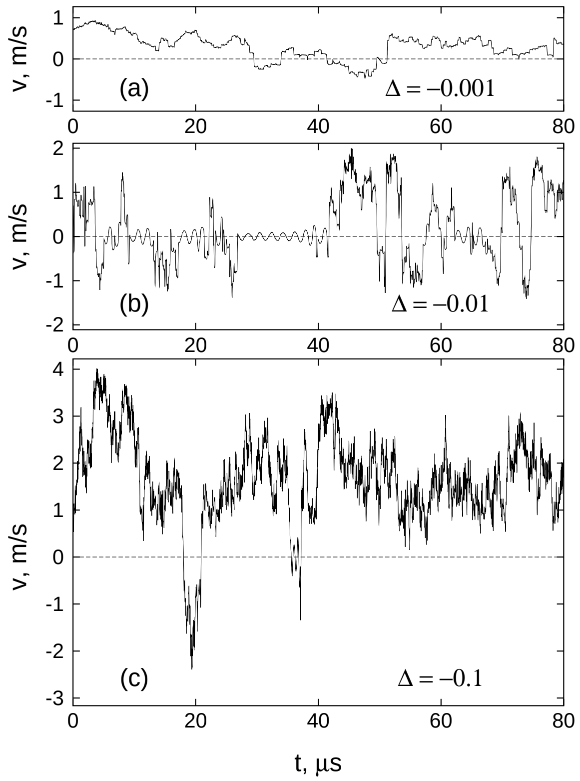

Figure 4. Dependencies of the current atomic velocity v on time: (a) $\Delta = -0.001$, chaotic walking with comparatively long flights, (b) $\Delta = -0.01$, chaotic walking with short flights, (c) $\Delta = -0.1$, velocity grouping effect.

In fact, both c and D depend on the value of H, therefore, Eqs. (7) and (8) are not correct for small values of the momentum p, and more accurate formula for $P_{\rm fl}$ is required.

To illustrate the behavior of the atomic momentum at different values of the detuning we plot in Fig. 4a a typical chaotic walking of an atom with comparatively long flights at $\Delta = -0.001$ and in Fig. 4b chaotic walking with short flights at $\Delta = -0.01$.

Qualitatively, the statistics in Fig. 3 are similar to those computed in Ref. [Argonov, Prants, 2007] with a purely Hamiltonian coherent dynamics. Both of the PDFs contain power-law fragments with exponential decays at their tails, but the origin of these fragments and tails is different. Statistics of the purely Hamiltonian system [Argonov, Prants, 2007] is governed by a deterministic diffusion of the quantity u in a bounded space (between its maximal and minimal values ± 1), *at a constant energy* H. Solution for the first-passage-time problem for the quantity $u = \pm H$ gives a PDF with a power-law fragment with the slope -1.5 and an exponential tail. The quantity u jumps to zero value at the moments of SE (see Fig. 1), and its evolution cannot be treated as a diffusion process when we take into account SE. However, the energy now is not a constant and can walk randomly (with a drift) within a broad range. The statistics of the system with SE is defined by a random walking of the energy H, not u. Thus, in both the systems the condition $H \approx \pm u$ defines the value of the energy which allows atoms to stop and turn back. Transformation of power-law fragments into exponential tails is explained in the purely Hamiltonian system [Argonov, Prants, 2007] by a limitation of the quantity u, whereas in the system with SE it is explained by a drift of the energy H. Both of those factors prevent randomly walking quantities to go far from their critical values and decrease exponentially the probability of long atomic flights.

4 Chaotic walking at moderate detunings, $|\Delta| \lesssim 1$

The effect of velocity grouping, when there are one or a few values of the capture momentum p_g to which current momenta of different atoms tend to, is known to occur at moderate negative values of the detuning [Kazantsev et al, 1990]. This effect has been numerically demonstrated with Eqs. (3) in our recent paper [Argonov, Prants, 2006]. Chaotic walking of atoms may occur in the regime of velocity groping if the values of p_g are sufficiently small and atoms can change the direction of motion due to fluctuations of momentum (caused by chaos in coherent evolution and/or SE, see Fig. 4c).

Computed statistics of atomic flights at medium values of the detuning $|\Delta| \lesssim 1$ (Fig. 3b) are similar to that at small detunings but the slope of the power-law decay may differ considerably. The PDFs $P_{\rm fl}(T)$ shown in Fig. 3b demonstrate decrease of the slope with increasing the values of $|\Delta|$ with corresponding increase of the lengths of the power-law fragments. It should be emphasized that the length of the power-law fragments (and the mean flight length) increases significantly with a rather small increas in $|\Delta|$.

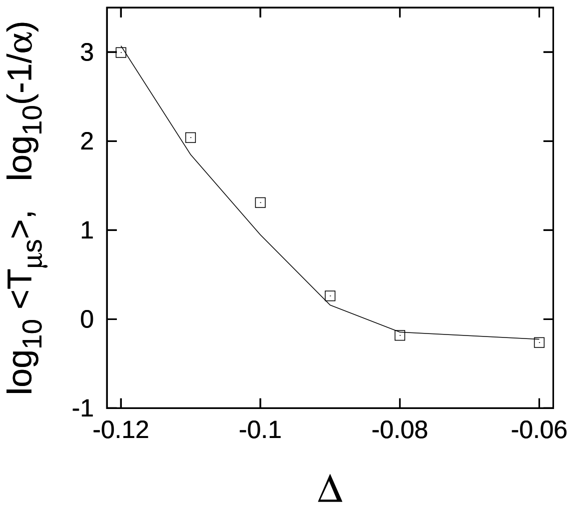

Figure 5. Dependencies of the logarithms of the mean duration of atomic flights $\langle T_{\mu s}\rangle$ (in microseconds) (solid line) and of the slope α of the PDF power-law fragments (squares) on the medium detuning Δ.

Varying the value of the detuning Δ, one can manipulate atomic transport in an optical lattice and its statistical properties. In Fig. 5 we plot the dependencies of the mean duration of atomic flights $\langle T\rangle$ (in microseconds) and the slope of the power-law fragments of $P_{\rm fl} \sim T^\alpha$ on the detuning in the range of medium values. The dependencies correlate well with each other. Figure 3 b demonstrates clearly that the length of the power-law fragments increases with increasing the absolute values of the detuning, whereas the absolute value of the slope α decreases. The mean duration of flights $\langle T\rangle$ increases correspondingly with increasing $|\Delta|$.

The control is nonlinear in the sense that when slightly decreasing Δ, say, from -0.08 to -0.12, the mean time of flights increases in a few orders of magnitude (see Fig. 5). This effect is explained by increas in the value of the capture momentum p_g and decreasing fluctuations of the current atomic momentum with increas in the absolute value of $|\Delta|$. For example, at $\Delta = -0.1$, one gets $p_g \simeq 500$ ($\simeq 1.5$ m/s) with the momentum fluctuations of the same order. Thus, the atom can change its direction of motion (see Fig. 4 c, with $\Delta = -0.1$ the mean time of flights is $T_{\mu s} \sim 10\mu$s) but not so frequently as in the case of smaller detunings (see Fig. 4b). The value of p_g increases with increasing $|\Delta|$ and, say, at $p_g \simeq 1000$ the momentum fluctuations are of the order of 300, and the atom cannot change its direction of motion. Reduction in the momentum fluctuations is caused by increas in the friction force F. With the values of p smaller than p_g, the force F is so large that changes in the energy H with time is not a process of random walking (as it is in the case with smaller detunings) but rather a directed drift in the momentum space to the value of the capture momentum. Thus, it is practically impossible for atoms to decrease their current values of p to zero value, and the process of chaotic walking eventually stops. The slope α of the PDF power-law fragments can go to zero due to existence of an exponential decay at the very tail of $P_{\rm fl}$. With a purely power-law decay the minimal slope would be $\alpha = -1$.

In conclusion, we have shown that near the resonance atomic transport in an optical lattice is a complicated process of chaotic walking caused by an interplay between coherent but deterministically chaotic internal atomic dynamics and spontaneous emission random events. It is possible to manipulate this process and its statistical characteristics by varying the single control parameter, the atom-laser detuning Δ.

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