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ADAPTIVE DECOUPLING CONTROL OF THREE – TANK – SYSTEM

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Abstract

Adaptive control of a three - tank - system laboratory model is presented. The objective laboratory model is a two input – two output (TITO) nonlinear system with internal interactions between input and output variables. It is based on experience with authentic industrial control applications. Two control algorithms based on polynomial theory and pole – placement are proposed. Decoupling compensators are used to suppress interactions between control loops. The algorithms implemented as self – tuning controllers are then used for control of the model. Results of real-time experiments are also included.

Key words

Multivariable control, self – tuning control, polynomial methods, pole assignment

1 Introduction

Typical technological processes require the simultaneous control of several variables related to one system. Each input may influence all system outputs. The three – tank – system (Fig. 1) is a typical multivariable non-linear system with significant cross-coupling. The design of a controller for such a system must be quite sophisticated if the system is to be controlled adequately. There are many different advanced methods of controlling MIMO (multi input – multi output) systems. The problem of selecting an appropriate control technique often arises. Several of these methods use decentralised controllers. In this case, the system is considered as a set of interconnected subsystems and the output of each subsystem is influenced not only by the input to this subsystem, but also by inputs to the other subsystems. Each subsystem is controlled by a stand-alone controller. Thus, decentralized control is based on decomposition of the MIMO system to subsystems, and the design of a controller for each subsystem [Luyben, 1986; Cui and Jacobsen, 2002; Zhang *et al.*, 2000]. The classical approach to the control of multi-input–multi-output (MIMO) systems is based on the design of a matrix controller to control all system outputs at one time [Albertos and Sala, 2004]. Computation for the matrix controller is realized by a central computer. The basic advantage of this approach is its ability to achieve optimal control performance because the controller can use all the available information about the controlled system. Its disadvantage is its demands on computer resources, because the number of operations and required memory depend on the square of the number of controlled signals. This increases the price of the control system.

In this paper two matrix controllers which utilize decoupling compensators are presented. A serial insertion of a compensator ahead of the system [Krishnawamy *et al.*, 1991; Peng, 1990; Tade *et al.*, 1986; Wittenmark *et al.*, 1987] is a very popular way of controlling MIMO processes. The objective, in this case, is to suppress undesirable interactions between the input and output variables so that each input affects only one controlled variable.

The applied controllers are based on polynomial methods [Kučera 1980; Kučera 1991] which are a standard technique for MIMO control systems. Applications of the polynomial methods yield a suitable controller type and expressions for computation of its parameters. The type of the controller depends on the required properties of the controlled system. This is valid both for SISO systems (with scalar transfer functions for the controllers) and for MIMO systems (where the transfer functions of the controllers are represented by polynomial matrices). Controller synthesis is reduced to the solution of linear Diophantine equations [Kučera 1993].

Dynamic behaviour of the system was described in the neighbourhood of a steady state by a discrete linear model in the form of matrix fraction. It is an input – output model ("black box model") which does not take into consideration an internal structure of the system. It is a model of the system behaviour and its parameters do not have any particular physical denotation.

The model of interconnected tanks is a nonlinear system with variable parameters. A suitable approach to the control of nonlinear systems is application of self – tuning controllers [Landau *et al.*, 1998; Bobál *et al.*, 2005]. Then both controllers were realized as self – tuning controllers with recursive identification of the model of the process. The recursive least squares method with the directional forgetting was used for the identification part.

This paper is organised as follows: Section 2 contains description of the three - tank - system; Section 3 presents a mathematical model of the system which was used for the controllers design; Section 4 describes designs of the controllers; Section 5 describes the system identification method; Section 6 contains the experimental results; Section 7 concludes the paper.

2 Three – tank – system

The three – tank – system laboratory model can be viewed as a prototype of many industrial applications in process industry, such as chemical and petrochemical plants, oil and gas systems [AMIRA-DTS2000, 1996]. The typical control issue involved in the system is how to keep the desired liquid level in each tank. The principle scheme of the model is shown in Fig 1. The basic apparatus consists of three plexiglass tanks numbered from left to right as T1, T2 and T3. These are connected serially with each other by cylindrical pipes. Liquid, which is collected in a reservoir, is pumped into the first and the third tanks to maintain their levels. The level in the tank T2 is a response which is uncontrollable. It affects the level in the two end tanks. Each tank is equipped with a static pressure sensor, which gives a voltage output proportional to the level of liquid in the tank.

Q_1 and Q_2 are the flow rates of the pumps 1 and 2. Two variable speed pumps driven by DC motor are used in this apparatus. These pumps are designed to give an accurate well defined flow per rotation. Thus, the flow rate provided by each pump is proportional to the voltage applied to its DC motor.

There are six manual valves V1, V2...V6 that can be used to vary the configuration of the process or to introduce disturbances or faults. In our case the apparatus was configured so that the valves V3 and V5 were closed and the remaining valves were open.

In our case, the model was controlled as a two input – two output (TITO) system. The outputs are controllable liquid levels of tanks T1 and T2 and the inputs are the pump flow rates Q_1 and Q_2. Each pump flow rate affects both liquid levels. This is the coupling. The systems inputs and outputs interact and the whole system is a multivariable system.

The three – tank – system is a nonlinear system with variable parameters. The nonlinear behaviour is caused by characteristics of the valves, pipes and pumps. Additional nonlinearities are due to air bubbles which are present in the pipes and valves. The bubbles deflate from the pipe system in certain moments.

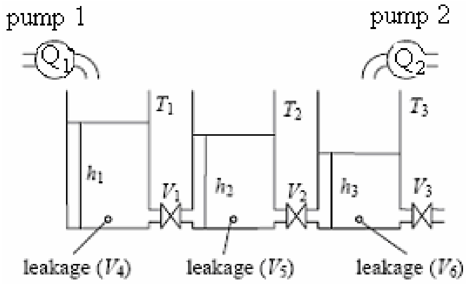

Figure 1. Principle scheme of three – tank – system

3 Mathematical Model of the Controlled Process

A simplified analytical model of the three – tank – system, based on physics and the equipment construction where all the parameters have physical interpretations, is presented in [AMIRA-DTS2000, 1996]. Some simplifications were required during its derivation and some assumptions with limited accuracy were used. The laboratory model is a nonlinear system, as it was mentioned above. Self-tuning controllers are a possible approach to the control of this kind of system. The nonlinear dynamics are described by a linear model in the neighbourhood of a steady state. A suitable model of the real object for control with self-tuning controllers is an input–output model. This is a standard approach for self-tuning controllers. Instead of the often tedious construction of a model from first principles and then calculating its parameters from plant dimensions and physical constants, a general model is chosen and its parameters are identified from data. It is a model of the system behaviour and its parameters do not necessarily have physical interpretations. Of course, not all properties of the plant can be extracted from the data in this way, but when the dominant properties are modelled, the result is sufficient for controller design. The advantages of this kind of model are its simplicity and accuracy in the operational range in which the input–output dependence is measured.

It was necessary to determine a structure for the model in advance. The aim here was to find experimentally the simplest possible structure of the model. The parameters are identified during the process of recursive identification from the measured input and output signals.

A general transfer matrix of a two-input–two-output system with significant cross-coupling between the control loops is expressed as:

$$\boldsymbol{G}(z)=\begin{bmatrix} G_{11}(z) & G_{12}(z) \\ G_{21}(z) & G_{22}(z) \end{bmatrix} \tag{1}$$

$$\boldsymbol{Y}(z)=\boldsymbol{G}(z)\boldsymbol{U}(z) \tag{2}$$

where $\boldsymbol{U}(z)$ and $\boldsymbol{Y}(z)$ are vectors of the manipulated variables (the pump flow rates) and the controlled variables (liquid levels), respectively.

$$\boldsymbol{U}(z)=[u_1(z),u_2(z)]^T \qquad \boldsymbol{Y}(z)=[y_1(z),y_2(z)]^T \tag{3}$$

It may be assumed that the transfer matrix can be transcribed to the following form of the matrix fraction:

$$\boldsymbol{G}(z)=\boldsymbol{A}^{-1}(z^{-1})\boldsymbol{B}(z^{-1})=\boldsymbol{B}_1(z^{-1})\boldsymbol{A}_1^{-1}(z^{-1}) \tag{4}$$

where the polynomial matrices $\boldsymbol{A}\in R_{22}[z^{-1}]$, $\boldsymbol{B}\in R_{22}[z^{-1}]$ are the left coprime factorizations of matrix $\boldsymbol{G}(z)$ and the matrices $\boldsymbol{A}_1\in R_{22}[z^{-1}]$, $\boldsymbol{B}_1\in R_{22}[z^{-1}]$ are the right coprime factorizations of $\boldsymbol{G}(z)$. The model can be also written in the form

$$\boldsymbol{A}(z^{-1})\boldsymbol{Y}(z)=\boldsymbol{B}(z^{-1})\boldsymbol{U}(z) \tag{5}$$

In case of decoupling control using a compensator it is useful to consider matrix $\boldsymbol{A}(z^{-1})$ as diagonal. If the matrix $\boldsymbol{A}(z^{-1})$ was assumed to be non – diagonal, it would have to be included into the compensator. Then, the order of the controller and sophistication of the closed loop system would be increased. This is comprehensively explained in the section V. The control algorithm was first designed for a model with polynomials of the first order. This model proved to be unsuitable for the process description and satisfactory control results were not achieved. Consequently, the algorithm was designed for a model with second-order polynomials. The polynomial orders in this model do not correspond with the orders of transfer functions among the inputs and outputs which were derived in [AMIRA-DTS2000, 1996]. These particular transfer functions have various orders. However, from the point of view of the control system design it is useful when the matrix $\boldsymbol{A}(z^{-1})$ has on the main diagonal polynomials of the same order. This simplified model proved to be effective and sufficiently complex to describe the process, while enabling quite simple computation of the controller. The controller described below is based on this model. The model has 12 parameters:

$$\boldsymbol{A}(z^{-1})=\begin{bmatrix} 1+a_1z^{-1}+a_2z^{-2} & 0 \\ 0 & 1+a_3z^{-1}+a_4z^{-2} \end{bmatrix} \tag{6}$$

$$\boldsymbol{B}(z^{-1})=\begin{bmatrix} b_1z^{-1}+b_2z^{-2} & b_3z^{-1}+b_4z^{-2} \\ b_5z^{-1}+b_6z^{-2} & b_7z^{-1}+b_8z^{-2} \end{bmatrix} \tag{7}$$

4 Design of Decoupling Controllers

There are several ways to control multivariable systems with internal interactions. One possibility is a serial insertion of a compensator ahead of the system [Krishnawamy *et al.*, 1991; Peng, 1990; Tade *et al.*, 1986; Wittenmark *et al.*, 1987]. The objective, in this case, is to suppress undesirable interactions between the input and output variables so that each input affects only one controlled variable. The block diagram for this kind of system is shown in Fig. 2 (***R*** is a transfer matrix of a controller and ***C*** is a decoupling compensator).

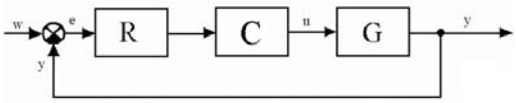

Figure 2. General scheme of closed loop with compensator

The resulting transfer matrix ***H*** is then determined by (operator z^{-1} will be omitted from some operations for the purpose of simplification)

$$\boldsymbol{H}=\boldsymbol{GC} \tag{8}$$

The decoupling conditions are fulfilled when the matrix ***H*** is diagonal.

The matrix ***B*** can be written as

$$\begin{aligned}\boldsymbol{B}&=z^{-1}\begin{bmatrix} b_1+b_2z^{-1} & b_3+b_4z^{-1} \\ b_5+b_6z^{-1} & b_7+b_8z^{-1} \end{bmatrix}= \\ &=z^{-1}\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}=z^{-1}\boldsymbol{B}_\mathrm{x}\end{aligned} \tag{9}$$

and then the matrix ***H*** as

$$\boldsymbol{H}=\boldsymbol{A}^{-1}z^{-1}\boldsymbol{B}_x\boldsymbol{C}=\boldsymbol{A}^{-1}\boldsymbol{H}_x \tag{10}$$

As it was mentioned above, the matrix ***A*** was chosen to be diagonal. The objective of this simplification is apparent from the equation (10). If the matrix ***A*** was assumed to be non – diagonal, it would have to be included into the compensator ($\boldsymbol{AA}^{-1}=\boldsymbol{I}$) to obtain a diagonal matrix ***H***. Then, the order of the controller and sophistication of the closed loop system would be increased. According to this assumption, the compensator ***C*** must be chosen so that multiplication of the matrix $\boldsymbol{B}_\mathrm{x}$ and the compensator leads to a diagonal matrix $\boldsymbol{H}_x$.

A detailed block diagram of the closed loop system with the compensator is shown in Fig. 3.

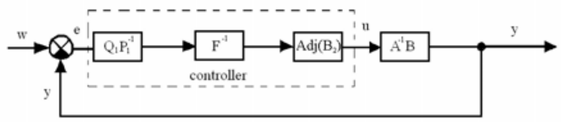

Figure 3. Detailed scheme of closed loop with compensator

The two compensators which are presented in this paper will be further referred to as $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$. The corresponding resulting transfer matrices will be referred to as $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$.

A. Design of controller with compensator C_1

The compensator $\boldsymbol{C}_1$ is defined by the following expression

$$\boldsymbol{C}_1 = \frac{1}{\det(\boldsymbol{B}_x)}\begin{bmatrix} B_{22} & -B_{12} \\ -B_{21} & B_{11} \end{bmatrix} \tag{11}$$

The multiplication of the matrix $\boldsymbol{B}_\mathbf{x}$ and the compensator results in a diagonal matrix $\boldsymbol{H}_1$.

$$\boldsymbol{H}_1 = \boldsymbol{B}_x \boldsymbol{C}_1 z^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} z^{-1} \tag{12}$$

The matrix of the controller can be defined by the following expression

$$\boldsymbol{G}_R = \boldsymbol{C}_1 \boldsymbol{R} = \boldsymbol{C}_1 \boldsymbol{F}^{-1} \boldsymbol{P}^{-1} \boldsymbol{Q} = \boldsymbol{C}_1 \boldsymbol{F}^{-1} \boldsymbol{Q}_1 \boldsymbol{P}_1^{-1} \tag{13}$$

Generally, the vector $\boldsymbol{W}(z^{-1})$ of input reference signals is specified as

$$\boldsymbol{W}(z) = \boldsymbol{F}_w^{-1}(z^{-1})\boldsymbol{h}(z^{-1}) \tag{14}$$

In case of control of the three – tank - system, the reference signals were considered as step functions. In this case $\boldsymbol{h}(z^{-1})$ is a vector of constants and $\boldsymbol{F}_w(z^{-1})$ is expressed as

$$\boldsymbol{F}_w(z^{-1}) = \begin{bmatrix} 1 - z^{-1} & 0 \\ 0 & 1 - z^{-1} \end{bmatrix} \tag{15}$$

The controller was designed so that two basic requirements on control were fulfilled: stability of the closed loop system and asymptotic tracking of the reference signals.

A condition of stability was obtained from the transfer function of the closed loop system. It is possible to derive the following equation for the system output

$$\boldsymbol{Y} = \boldsymbol{A}^{-1}\boldsymbol{H}_1\boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{E} = \boldsymbol{A}^{-1}\boldsymbol{H}_1\boldsymbol{F}^{-1}\boldsymbol{P}^{-1}\boldsymbol{Q}(\boldsymbol{W} - \boldsymbol{Y}) \tag{16}$$

This equation can be modified using the right matrix fraction of the controller into the expression defining the transfer function of the closed loop

$$\boldsymbol{Y} = \boldsymbol{P}_1(\boldsymbol{A}\boldsymbol{F}\boldsymbol{P}_1 + \boldsymbol{H}_1\boldsymbol{Q}_1)^{-1}\boldsymbol{H}_1\boldsymbol{Q}_1\boldsymbol{P}_1^{-1}\boldsymbol{W} \tag{17}$$

The determinant of the matrix in the denominator ($\boldsymbol{A}\boldsymbol{F}\boldsymbol{P}_1 + \boldsymbol{H}_1\boldsymbol{Q}_1$) is the characteristic polynomial of a MIMO system. The roots of this polynomial matrix are the ruling factors for the behaviour of a closed loop system. The roots must be inside the unit circle (of the Gauss complex plain), in order for the system to be stable. Conditions of BIBO (bounded input bounded output) stability can be defined by the following diophantine equation

$$\boldsymbol{A}\boldsymbol{F}\boldsymbol{P}_1 + \boldsymbol{H}_1\boldsymbol{Q}_1 = \boldsymbol{M} \tag{18}$$

Where $\boldsymbol{M} \in R_{22}[z^{-1}]$ is a stable diagonal polynomial matrix. A right pole – placement of the matrix $\boldsymbol{M}$ is very important in order to achieve good control performance.

$$\boldsymbol{M}(z^{-1}) = \begin{bmatrix} 1 + m_1 z^{-1} + m_2 z^{-2} + m_3 z^{-3} + m_4 z^{-4} & 0 \\ 0 & 1 + m_1 z^{-1} + m_2 z^{-2} + m_3 z^{-3} + m_4 z^{-4} \end{bmatrix} \tag{19}$$

Further problem to be solved is the asymptotic tracking of the reference signals. It is possible to derive the following equation for a vector of control errors

$$\boldsymbol{E} = \boldsymbol{W} - \boldsymbol{Y} = \boldsymbol{F}\boldsymbol{P}_1(\boldsymbol{A}\boldsymbol{F}\boldsymbol{P}_1 + \boldsymbol{H}_1\boldsymbol{Q}_1)^{-1}\boldsymbol{A}\boldsymbol{F}_w^{-1}\boldsymbol{h} \tag{20}$$

To fulfill the requirement on the asymptotic tracking, the denominator of the reference signals (matrix Fw) must be eliminated from the expression for the permanent control error. For this purpose, the compensator F was included into the controller. Asymptotic tracking of the reference signals is then obtained if FP1 is divisible by Fw. The compensator F is a component formally separated from the controller (Fig. 3). If the reference signals are step functions, then F is an integrator.

$$\boldsymbol{F}(z^{-1}) = \begin{bmatrix} 1 - z^{-1} & 0 \\ 0 & 1 - z^{-1} \end{bmatrix} \tag{21}$$

The degree of the controller polynomial matrices depends on the internal properness of the closed loop. The structure of matrices $\boldsymbol{P}_1$ and $\boldsymbol{Q}_1$ was chosen so that the number of unknown controller parameters equals the number of algebraic equations resulting from the solution of the diophantine equation using the method of the uncertain coefficients. The matrices of the controller take the following forms:

$$\boldsymbol{P}_1(z^{-1}) = \begin{bmatrix} 1 + p_1 z^{-1} & 0 \\ 0 & 1 + p_2 z^{-1} \end{bmatrix} \tag{22}$$

$$\boldsymbol{Q}_1(z^{-1}) = \begin{bmatrix} q_1 + q_2 z^{-1} + q_3 z^{-2} & 0 \\ 0 & q_4 + q_5 z^{-1} + q_6 z^{-2} \end{bmatrix} \tag{23}$$

The solution of the diophantine equation (18) results in a set of eight algebraic equations with unknown controller parameters. Using matrix notation the algebraic equations can be expressed in the following form

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ a_1 - 1 & 0 & 1 & 0 \\ a_2 - a_1 & 0 & 0 & 1 \\ -a_2 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} p_1 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} m_1 - a_1 + 1 \\ m_2 + a_1 - a_2 \\ m_3 + a_2 \\ m_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ a_3-1 & 0 & 1 & 0 \\ a_4-a_3 & 0 & 0 & 1 \\ -a_4 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} p_2 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} m_5-a_3+1 \\ m_6+a_3-a_4 \\ m_7+a_4 \\ m_8 \end{bmatrix} \quad (24)$$

The controller parameters are obtained by solving these equations. The parameters are then used for computation of a control law

The control law emerges from the block diagram as

$$\boldsymbol{F}U = \boldsymbol{C}_1\boldsymbol{Q}_1\boldsymbol{P}_1^{-1}\boldsymbol{E} \quad (25)$$

B. Design of controller with compensator C_2

The compensator $\boldsymbol{C}_2$ was chosen as

$$\boldsymbol{C}_2 = \frac{1}{\det(\boldsymbol{B}_x)}\begin{bmatrix} B_{11}B_{22} & -B_{12}B_{22} \\ -B_{11}B_{21} & B_{11}B_{22} \end{bmatrix} \quad (26)$$

The resulting diagonal transfer matrix $\boldsymbol{H}_2$ then takes the form

$$\boldsymbol{H}_2 = \boldsymbol{B}_x\boldsymbol{C}_2 z^{-1} = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix} z^{-1} \quad (27)$$

Further procedure of the controller design is analogical to the procedure presented in the subsection *A*. Analogical expressions to the expressions (13), (16), (17), (18), (20) and (25) are valid. Only the compensator $\boldsymbol{C}_1$ is replaced by the compensator $\boldsymbol{C}_2$ and the matrix $\boldsymbol{H}_1$ is replaced by the matrix $\boldsymbol{H}_2$. The matrices of the controller $\boldsymbol{P}_1$ and $\boldsymbol{Q}_1$ as well as the matrix $\boldsymbol{M}$ are defined by the expressions (22), (23) and (19).

Algebraic equations with unknown controller parameters results from the solution of the Diophantine equation

$$\begin{gathered}\begin{bmatrix} 1 & b_1 & 0 & 0 \\ a_1-1 & b_2 & b_1 & 0 \\ a_2-a_1 & 0 & b_2 & b_1 \\ -a_2 & 0 & 0 & b_2 \end{bmatrix}\begin{bmatrix} p_1 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} m_1-a_1+1 \\ m_2+a_1-a_2 \\ m_3+a_2 \\ m_4 \end{bmatrix} \\ \begin{bmatrix} 1 & b_7 & 0 & 0 \\ a_3-1 & b_8 & b_7 & 0 \\ a_4-a_3 & 0 & b_8 & b_7 \\ -a_4 & 0 & 0 & b_8 \end{bmatrix}\begin{bmatrix} p_2 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} m_5-a_3+1 \\ m_6+a_3-a_4 \\ m_7+a_4 \\ m_8 \end{bmatrix}\end{gathered} \quad (28)$$

5 System Identification

The control algorithms were applied as self tuning controllers (as it was introduced in sections 2 and 3). Self-tuning control is based on on-line identification of a model of a controlled process. Each self – tuning controller consists of an on – line identification part and a control part.

Various discrete linear models are used to describe dynamic behaviour of controlled systems. Overview of these models is given for example in [Nelles, 2001]. The most widely applied linear dynamic model is the ARX model. Usually the ARX model is tested first and only if it does not perform satisfactory more complex model structures are examined. But this is not the case because the ARX model matches the structure of many real processes. The parameters can be easily estimated by a linear least squares technique. The ARX model describing our TITO process is defined as

$$\begin{aligned} y_1(k) &= \boldsymbol{\Theta}_1^T(k)\boldsymbol{\Phi}_1(k-1) + n_1(k) \\ y_2(k) &= \boldsymbol{\Theta}_2^T(k)\boldsymbol{\Phi}_2(k-1) + n_2(k) \end{aligned} \quad (29)$$

where $n_1(k)$, $n_2(k)$ are non-measurable disturbances

The parameter vectors are specified as shown below:

$$\boldsymbol{\Theta}_1^T(k) = [a_1 \; a_2 \; b_1 \; b_2 \; b_3 \; b_4] \quad (30)$$

$$\boldsymbol{\Theta}_2^T(k) = [a_3 \; a_4 \; b_5 \; b_6 \; b_7 \; b_8]$$

and the data vectors are

$$\begin{aligned} \boldsymbol{\Phi}_1^T(k-1) = [&-y_1(k-1) \;\; -y_1(k-2) \\ &u_1(k-1) \;\; u_1(k-2) \;\; u_2(k-1) \;\; u_2(k-2)] \\ \boldsymbol{\Phi}_2^T(k-1) = [&-y_2(k-1) \;\; -y_2(k-2) \\ &u_1(k-1) \;\; u_1(k-2) \;\; u_2(k-1) \;\; u_2(k-2)] \end{aligned} \quad (31)$$

When using the least squares method, the influence of all measured input and output samples to the parameter estimates is the same. This is inconvenient for identification of nonlinear systems, where changes of the identified parameters are expected. Tracking of changes of the parameters can be achieved by application of exponential forgetting. This technique ensues from the assumption that new data describes the dynamics of an object better than older data, which are multiplied by smaller weighting coefficients. In case that the identified plant is insufficiently activated – it means that the input and output signals are steady (this situation is typical for closed control systems), the exponential forgetting factor can cause numerical instability of the identification algorithm. A possible solution of this problem is application of the adaptive directional forgetting [Bittanti *et al.*, 1990; Kulhavý, 1987]. This technique changes the forgetting factor according to the level of information in the data. Considering parameters changes of the nonlinear three – tank – system and the expected insufficient activation of the controlled system, the recursive least squares method with adaptive directional forgetting was applied. The parameter estimates, the covariance matrix and the directional forgetting factor are then actualised according to recursive expressions.

6 Experimental Examples

The model was connected with a PC equipped with a control and measurement PC card. Matlab and Real Time Toolbox were used to control the system.

For the experiments presented in this paper, the three – tank – system was configured in such a way that the valves v3 and v5 were closed and the remaining valves were open.
An approximate sampling period was found on the basis of measured step responses so that ten samples cover important part of the step response. The best sampling period T_0=5 s was then tuned according to experiments.
Another problem was finding of suitable poles of the characteristic polynomial. The pole - assignment is a natural part of the polynomial method. Quality of control performance is given by pole – placement of the characteristic polynomial. In comparison with controllers for SISO control loops, pole – placement of multivariable systems is much more complicated. Process of searching suitable poles was following: at first a multiple pole on the real axis was chosen. This pole was moved along the real axis. Control results obtained for particular experiments were compared and a suitable multiple pole was found. Then, a suitable conjunction of various poles in the neighbourhood of the multiple pole was experimentally searched. The suitable right side matrix, obtained from a number of experiments, is as follows:

$$\boldsymbol{M}\left(z^{-1}\right)=\begin{bmatrix} \begin{matrix}1-0{,}9z^{-1}+0{,}19z^{-2}-\\ -0{,}009z^{-3}-0{,}002z^{-4}\end{matrix} & 0 \\ 0 & \begin{matrix}1-0{,}9z^{-1}+0{,}19z^{-2}-\\ -0{,}009z^{-3}-0{,}002z^{-4}\end{matrix}\end{bmatrix} \qquad (32)$$

Figures 4 and 5 show time responses of the control when the initial parameter estimates were chosen without any a priori information. The reference trajectories contain frequent step changes in the beginning of experiments to activate input and output signals and improve the identification.
The controlled variables y_1 and y_2 are liquid levels of the tanks T1 and T2. The manipulated variables u_1 and u_2 are flow rates into the tanks. As w_1 and w_2 are denoted desired liquid levels in the particular tanks (reference signals).

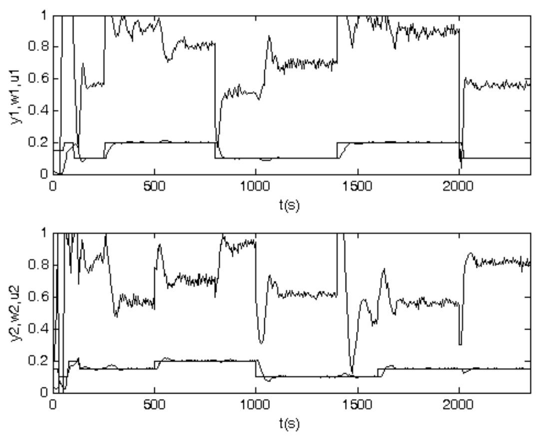

Figure. 4. Control of the three – tank – system with compensator C_1

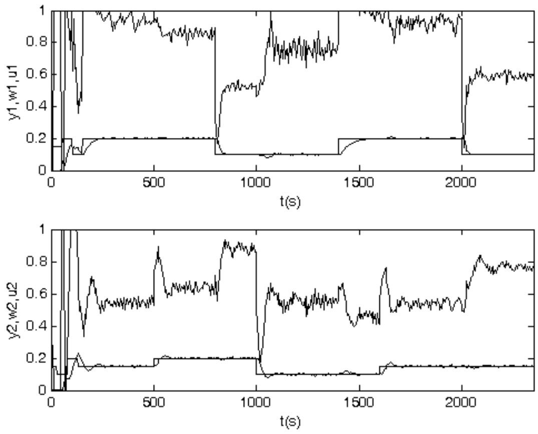

Figure. 5. Control of the three – tank – system with compensator C_2

7 Conclusions

Decoupling multivariable self - tuning controllers were proposed and verified by control of nonlinear time varying system. The adaptive control strategy was applied especially due to nonlinear behaviour of the controlled system.
It is necessary to recognize that self-tuning controllers do not work satisfactorily in the initial adaptation phase if the initial parameter estimates are chosen without a priori information. However, the most important property for practical use of self-tuning controllers is their performance after the adaptation phase.
With regards to decoupling, interactions between control loops were not fully eliminated. Overshoots of one controlled variable caused by step changes of the reference signal of the other one are obvious from Fig. 4. and Fig. 5. This is caused by fact that the decoupling controllers are based on inversion of the controlled plant. Such controllers are sensitive to differences between the model and the plant. But size of the overshoots is negligible in comparison to size of the step changes. This indicates that interactions between the control loops were reduced. The controller with the compensator $\boldsymbol{C}_2$ performed slightly better. This is evident from the control responses in Fig. 4 and Fig. 5.
General principles were elaborated on a specific system with two inputs and two outputs that is often applicable in industrial practice. Control laws based on specific model were derived in the form of self-contained expressions that is especially useful for practical applications of control on common industrial devices. An advantage of the proposed strategy lies in its simplicity and applicability. The control tests executed on the laboratory model provided very satisfactory results, even though its nonlinear dynamics were described by a linear model. The laboratory model simulates technological

processes that frequently occur in industry, and the tests proved that the proposed method could be implemented and used successfully to control such processes.

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