A NEW APPROACH TO MRAC PROBLEM WITH DISTURBANCE REJECTION[1]

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Abstract: This work represents the development of robust control methods in the tasks of adaptation of SISO linear time-invariant plant with unknown parameters and disturbance. This problem is solved by $((2n+1)\rho-1)$-order regulator.

Keywords: Adaptive control, robustness, high-order tuners, complex systems, model reference adaptive control, disturbance rejection.

1. INTRODUCTION

In work (Morse, 1980) Morse proposed a new adaptation scheme (high-order tuner) to solve a task of control of linear time-invariant plant with unknown parameters. In paper (Nikiforov, 1999) Nikiforov proposed a robustified variant of the high-order tuner. In comparison with Morse's tuner, the scheme by Nikiforov (Nikiforov, 1999) has an essentially simplified structure. This paper deals with the problem of designing an adaptive output-feedback controller for unknown linear time-invariant plants. The earliest decisions were based on the certainty-equivalence principle and augmented error concept (Monopoli, 1974). However, it was soon noticed that the proof of stability of equivalence schemes with adaptation laws, forced by an augmented error, is not trivial and can be accomplished after lengthy signal analysis (Morse, 1980). Recently, such a class of adaptive controllers has been proposed in (Kristic, et al., 1994) with the use of nonlinear design tool: integrator backstepping. Now it is well known that these new controllers possess several useful properties which were unattainable for the traditional adaptive systems (Kristic, et al., 1995).

An alternative approach to the design of adaptive controllers with unnormalized adaptation laws was proposed by Morse in (Morse, 1992) and introduced the notion of a "high-order tuner". Robustified variant of the Morse's high-order tuner was proposed by Nikiforov in (Fradkov, et al., 1999; Nikiforov, 1999). The algorithm proposed in (Fradkov, et al. 1999; Nikiforov, 1999) has essentially simplified structure and smaller dimension in comparison with Morse's tuner. In comparison with approaches by Morse, algorithm by Nikiforov has smaller dimension:

- algorithm Morse's has dimension $2n(2\rho-1)-2$;
- algorithm Nikiforov's has dimension $2n(\rho+1)+\rho-4$.

This work represents the development of methods of adaptive (Morse, 1992) and robust control (Fradkov, et al., 1999; Nikiforov, 1999) in the tasks of adaptation single-input, single-output linear time-invariant plants with unknown parameters. The algorithm proposed in this paper has essentially simplified structure and smaller dimension in comparison with Morse's tuner (Morse, 1992) and scheme by Nikiforov (Nikiforov, 1999).

2. STATEMENT OF THE PROBLEM

This article deals with the problem of adaptive control of linear time-invariant single-input, single-

[1] This work was supported in part by RFBR under Grant № 05-08-33388-a.

output plants described by the following input-output relationship (Fradkov, et al., 1999; Nikiforov, 1999)

$$d(t) = k\frac{b(p)}{a(p)}[u(t) + w(t)], \qquad (1)$$

where $d = d(t)$ and $u = u(t)$ are the plant output and input, respectively; p denotes the differential operator; k is a high-frequency gain; $a(p) = p^n + a_{n-1}p^{n-1} + ... + a_1 p + a_0$ and $b(p) = p^m + b_{m-1}p^{m-1} + ... + b_1 p + b_0$ are monic coprime polynomials with unknown coefficients; $w = w(t)$ is the unknown smooth disturbance and $|w(t)| \le w_0 < \infty$.

The following standard assumptions (Monopoli, 1974; Morse, 1992; Narendra, 1978; Nikiforov, 1999) are made:

(A1) polynomial $b(p)$ is Hurwitz;
(A2) degrees n and m are known and $\rho = n - m > 2$;
(A3) high-frequency gain $k = 1$.

Let the desired behaviour of the closed-loop system be specified by the following reference model

$$d_r(t) = \frac{k_r}{a_r(p)} r(t), \qquad (2)$$

where $d_r = d_r(t)$ is the reference output, $r = r(t)$ is a piece-wise continuous bounded reference input (command signal), $\alpha_r(p)$ is a monic Hurwitz polynomial of the order $\rho = n - m$, $k_r > 0$.

A tracking error is considered

$$y = d - d_r. \qquad (3)$$

Then the purpose of control is

$$|y(t)| \le \varepsilon, \qquad (4)$$

where ε is a small positive number.

3. MODEL PARAMETERIZATION

In order to design an appropriate control law, first it is necessary to derive a suitable error model. For any Hurwitz monic polynomial

$$\gamma(p) = p^{n-1} + \gamma_{n-2}p^{n-2} + ... + \gamma_1 p + \gamma_0 \qquad (5)$$

of an order $n-1$, the tracking error $y = d - d_r$ can be presented in the form

$$y(t) = \frac{1}{a_r(p)}\left[\omega(t)^T \theta + u + w\right] + \varepsilon_d, \qquad (6)$$

where ε_d exponentially decays due to nonzero initial conditions, $\boldsymbol{\theta} \in R^{2n}$ is a vector of unknown constant parameters and $\boldsymbol{\omega} \in R^{2n}$ is a standard regressor vector (Fradkov, et al., 1999; Nikiforov, 1999), i.e.

$$\boldsymbol{\omega}(t) = \left[\frac{1}{\gamma(p)}u(t); \frac{p}{\gamma(p)}u(t); ...; \frac{p^{n-2}}{\gamma(p)}u(t); d(t);\right.$$
$$\left.\frac{1}{\gamma(p)}d(t); \frac{p}{\gamma(p)}d(t); ...; \frac{p^{n-2}}{\gamma(p)}d(t); r(t)\right]^T. \qquad (7)$$

Further a transfer function $W(p)$ with the relative degree $\rho^* = \rho - 1$ is chosen such that

$$W(p) = \frac{(p + \alpha_0)}{a_r(p)}, \qquad (8)$$

where α_0 is a strictly positive constant. Then the error model (6) can be rewritten as (Nikiforov, 1999)

$$y = \frac{1}{p + \alpha_0}\left[\varpi^T \boldsymbol{\theta} + \bar{u} + \bar{w}\right] + \delta \qquad (9)$$

or

$$\dot{y} = -\alpha_0 y + \varpi^T \boldsymbol{\theta} + \bar{u} + \bar{w} + \bar{\delta}, \qquad (10)$$

where $\bar{\delta} = \dot{\delta} + \alpha_0 \delta$ and $\delta(t)$ exponentially decays due to nonzero initial conditions, the functions $\varpi = W(p)\boldsymbol{\omega}$, $\bar{w} = W(p)w$ and a new variable $\bar{u}$ is

$$\bar{u} = W(p)u. \qquad (11)$$

Consider the new function

$$\varphi = \varpi^T \boldsymbol{\theta} + \bar{\delta} + \bar{w}, \qquad (12)$$

then for the model (10) it is obtained

$$\dot{y} = -\alpha_0 y + \varphi + \bar{u}. \qquad (13)$$

Thus equation (13) represents the model (6) as system of the first order.

4. MAIN RESULT

The variable $\bar{u}$ is chosen such that

$$\bar{u} = -\varphi', \qquad (14)$$

where φ' is an estimate of the function φ. Then the control is

$$u = -W(p)^{-1}\varphi' . \quad (15)$$

It is easy to see that for realization of control law (12) it is necessary to differentiate function φ' (i.e. it is necessary to obtain $\rho - 1$ derivation of φ'). Also it is clear that exact enough estimation φ' of the function φ provides small error of $y(t)$. The last follows from equation (13). Let $\tilde{\varphi} = \varphi - \varphi'$ than equation (13) takes the form

$$\dot{y} = -\alpha_0 y + \tilde{\varphi} . \quad (16)$$

From last equation it follows that the smaller is $\tilde{\varphi}$ the smaller is $y(t)$. So problems of the design algorithm (15) are the following:

- operation of differentiation of the function φ';
- estimation of the function φ with established precision.

Step 1:
Let the function φ be measured, than let the following algorithm of estimation is selected

$$\begin{cases} \dot{\xi}_1 = \vartheta\bar{\sigma}\xi_2, \\ \dot{\xi}_2 = \vartheta\bar{\sigma}\xi_3, \\ \dots \\ \dot{\xi}_\rho = \vartheta\bar{\sigma}(-k_1\xi_1 - k_2\xi_2 - \dots - k_\rho\xi_\rho + k_1\varphi), \end{cases} \quad (17)$$

$$\varphi' = \xi_1 , \quad (18)$$

where constant $\vartheta > 0$, the strictly positive function $\bar{\sigma} \ge C_0 > 0$ is identical in growth rate with $|\dot{\varphi}|^2$ (i.e. $0 \le \frac{|\dot{\varphi}|^2}{\bar{\sigma}} \le C_1 < \infty$, where C_1 is some positive constant), and $\rho - 1$ derivative of $\bar{\sigma}$ is known or is measurable, coefficients k_i are such that the model (17) is asymptotically stable for the case $\varphi = 0$.

Theorem 1. The algorithm (17), (18) under condition of magnification of parameter ϑ ensures a diminution of the value $|\varphi - \varphi'|$.

Proof. The model (17), (18) is considered in the form

$$\dot{\xi} = \vartheta\bar{\sigma}(\boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{q}k_1\varphi) , \quad (19)$$

$$\varphi' = \boldsymbol{h}^T \boldsymbol{\xi} , \quad (20)$$

where $\boldsymbol{\Gamma} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_1 & -k_2 & -k_3 & \dots & -k_\rho \end{bmatrix}$, $\boldsymbol{q} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$ and $\boldsymbol{h} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$.

Considering the error vector $\boldsymbol{\eta} = \boldsymbol{h}\varphi - \boldsymbol{\xi}$, it is obtained

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \boldsymbol{h}\dot{\varphi} - \vartheta\bar{\sigma}(\boldsymbol{\Gamma}(\boldsymbol{h}\varphi - \boldsymbol{\eta}) + \boldsymbol{q}k_1\varphi) = \\ &= \boldsymbol{h}\dot{\varphi} + \vartheta\bar{\sigma}\boldsymbol{\Gamma}\boldsymbol{\eta} - \vartheta\,\bar{\sigma}(\boldsymbol{q}k_1 + \boldsymbol{\Gamma}\boldsymbol{h})\varphi . \end{aligned} \quad (21)$$

As $\boldsymbol{q}k_1 = -\boldsymbol{\Gamma}\boldsymbol{h}$ (is checked up by substitution), than

$$\dot{\boldsymbol{\eta}} = \boldsymbol{h}\dot{\varphi} + \vartheta\bar{\sigma}\boldsymbol{\Gamma}\boldsymbol{\eta} , \quad (22)$$

where matrix $\boldsymbol{\Gamma}$ is Hurwitz.
The Lyapunov function is considered

$$V = \boldsymbol{\eta}^T \boldsymbol{P}\boldsymbol{\eta} , \quad (23)$$

where matrix $\boldsymbol{P} = \boldsymbol{P}^T > 0$ is such that

$$\boldsymbol{\Gamma}^T \boldsymbol{P} + \boldsymbol{P}\boldsymbol{\Gamma} \le -\lambda\boldsymbol{P} < 0 . \quad (24)$$

Differentiating (18), it is obtained

$$\begin{aligned} \dot{V} &= \vartheta\bar{\sigma}\boldsymbol{\eta}^T(\boldsymbol{\Gamma}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{\Gamma})\boldsymbol{\eta} + 2\boldsymbol{\eta}^T\boldsymbol{P}\boldsymbol{h}\dot{\varphi} \le -\lambda\vartheta\bar{\sigma}\boldsymbol{\eta}^T P\boldsymbol{\eta} + \\ &+ \mu\bar{\sigma}\boldsymbol{\eta}^T\boldsymbol{P}\boldsymbol{h}\boldsymbol{h}^T\boldsymbol{P}\boldsymbol{\eta} + \mu^{-1}\bar{\sigma}^{-1}|\dot{\varphi}|^2 , \end{aligned} \quad (25)$$

where the number μ is such that

$$\dot{V} \le -\bar{\lambda}\,\vartheta\bar{\sigma}\boldsymbol{\eta}^T\boldsymbol{P}\boldsymbol{\eta} + \mu^{-1}\bar{\sigma}^{-1}|\dot{\varphi}|^2 , \quad (26)$$

and constant $\bar{\lambda} > 0$.
In (25) an easily checked inequality was used

$$(\mu\bar{\sigma})^{-1}(\mu\bar{\sigma}a - b)^2 \ge 0 , \quad (27)$$

whence follows, that

$$2ab \le \mu\bar{\sigma}a^2 + (\mu\bar{\sigma})^{-1}b^2 , \quad (28)$$

where $a = \boldsymbol{\eta}^T\boldsymbol{P}\boldsymbol{h}$ and $b = \dot{\varphi}$.
As the function $\bar{\sigma} \ge C_0 > 0$ is such that $0 \le \frac{|\dot{\varphi}|^2}{\bar{\sigma}} \le C_1 < \infty$, then from (26) it is obtained

$$\dot{V} \le -\bar{\lambda}\,\vartheta C_0 \boldsymbol{\eta}^T \boldsymbol{P}\boldsymbol{\eta} + \mu^{-1} C_1 . \qquad (29)$$

From the last inequality follows, that a vector of deviations $\boldsymbol{\eta} = \boldsymbol{h}\varphi - \boldsymbol{\xi}$ is bounded and all its variables can be converged to any small compact set, with the increase of parameter ϑ.
By force of structure of the matrix $\boldsymbol{h}$ it is obtained

$$\left|\boldsymbol{h}^T \boldsymbol{\eta}\right| = \left|\varphi - \boldsymbol{h}^T \xi\right| = \left|\varphi - \varphi'\right| \qquad (30)$$

and for some ϑ (in general case ϑ is large) convergence to any small compact set.

Theorem 2. The signals $\dot{\varphi}(t)$ and $\left|\dot{\varpi}(t)\right| + C_2$ (where number $C_2 > 0$) are proportional.

Proof. Consider the derivative of the function $\varphi(t)$. From equation (9) it is obtained

$$\dot{\varphi} = \dot{\varpi}^T \boldsymbol{\theta} + \dot{\bar{\delta}} + \dot{\bar{w}} \le \left|\dot{\varpi}\right|\left|\boldsymbol{\theta}\right| + \dot{\bar{\delta}} + \left|\dot{\bar{w}}\right| \le$$
$$\le C_3 \left|\dot{\varpi}\right| + \dot{\bar{\delta}} + C_4 , \qquad (31)$$

where numbers $C_3 > 0$ and $C_4 > 0$, $\dot{\bar{\delta}}$ decays exponentially, $\left|\dot{\varpi}\right| = \sqrt{\dot{\varpi}_1^2 + \dot{\varpi}_2^2 + ... + \dot{\varpi}_{2n}^2}$ and $\left|\theta\right| = \sqrt{\theta_1^2 + \theta_2^2 + ... + \theta_{2n}^2}$.
As the function $\dot{\bar{\delta}}$ decays exponentially, then signals $\dot{\varphi}(t)$ and $\left|\dot{\varpi}(t)\right|$ are proportional.

Remark. It is possible to calculate the function $\bar{\sigma} = C_0 + \left|\dot{\varpi}\right|^2 = C_0 + \dot{\varpi}^T \dot{\varpi}$, such that the signals $\dot{\varphi}(t)$ and $\left|\dot{\varpi}(t)\right| + C_2$ are proportional and in the control (15) only $\rho *$ measured derivatives $\varpi(t)$ will be used.

Step 2:
Now it will be constructed a realizable scheme of the algorithm of estimate (17) in the form

$$\begin{cases} \dot{\xi}_1 = \vartheta\bar{\sigma}\xi_2 , \\ \dot{\xi}_2 = \vartheta\bar{\sigma}\xi_3 , \\ ... \\ \dot{\varsigma} = \vartheta\bar{\sigma}(-k_2\xi_2 - ... - k_\rho\xi_\rho + k_1\alpha_0 y) - \vartheta\dot{\bar{\sigma}}k_1 y, \end{cases} \qquad (32)$$

$$\xi_\rho = \varsigma + \vartheta\bar{\sigma}k_1 y . \qquad (33)$$

The system (32), (32) contains variables, which can be measured or calculated.

Theorem 3. The algorithm of evaluation of the aspect (32), (33) is equivalent to algorithm (17).

Proof. From the equation (10) it has been realized that

$$\varphi = \dot{y} + \alpha_0\, y - \bar{u} . \qquad (34)$$

Substituting the last equation into the system (17), it is obtained

$$\begin{cases} \dot{\xi}_1 = \vartheta\bar{\sigma}\xi_2 , \\ \dot{\xi}_2 = \vartheta\bar{\sigma}\xi_3 , \\ ... \\ \dot{\xi}_\rho = \vartheta\bar{\sigma}(-k_1\xi_1 - ... - k_\rho\xi_\rho + k_1(\dot{y} + \alpha_0 y - \bar{u})). \end{cases} \qquad (35)$$

Taking into account that

$$\bar{u} = -\varphi' = -\xi_1 , \qquad (36)$$

it is obtained

$$\begin{cases} \dot{\xi}_1 = \vartheta\bar{\sigma}\xi_2 , \\ \dot{\xi}_2 = \vartheta\bar{\sigma}\xi_3 , \\ ... \\ \dot{\xi}_\rho = \vartheta\bar{\sigma}(-k_2\xi_2 - ... - k_\rho\xi_\rho + k_1(\dot{y} + \alpha_0 y)). \end{cases} \qquad (37)$$

It is necessary to introduce a new variable

$$\varsigma = \xi_\rho - \vartheta\bar{\sigma}k_1 y . \qquad (38)$$

Then differentiating (38) for the system (37), it is obtained

$$\begin{cases} \dot{\xi}_1 = \vartheta\bar{\sigma}\xi_2 , \\ \dot{\xi}_2 = \vartheta\bar{\sigma}\xi_3 , \\ ... \\ \dot{\varsigma} = \vartheta\bar{\sigma}(-k_2\xi_2 - ... - k_\rho\xi_\rho + k_1\alpha_0 y) - \vartheta\dot{\bar{\sigma}}k_1 y, \end{cases} \qquad (39)$$

$$\xi_\rho = \varsigma + \vartheta\bar{\sigma}k_1 y . \qquad (40)$$

So this algorithm has dimension $(2n+1)\rho - 1$.

5. EXAMPLE

It is considered the following linear time-invariant single-input, single-output plant:

$$d(t) = \frac{b_0}{p(p^2 + a_1 p + a_0)}[u(t) + w(t)] , \qquad (41)$$

where a_0, a_1 and b_0 are unknown parameters and uncertain disturbance $w(t)$.

Let the desired behaviour of the closed-loop system be specified by the following reference model

$$d_r(t) = \frac{1}{(p+1)(p^2+p+1)} r(t), \qquad (42)$$

where $d_r = d_r(t)$ is the reference output, $r = 2\sin t$ is a piece-wise continuous bounded reference signal.

Choose Hurwitz monic polynomial

$$\gamma(p) = p^{n-1} + \gamma_{n-2}p^{n-2} + \ldots + \gamma_1 p + \gamma_0 = = (p+1)^2. \qquad (43)$$

The tracking error $y = d - d_r$ can be presented in the form

$$y(t) = \frac{1}{(p+1)^3}\left[\omega(t)^T \theta + u\right] + \varepsilon_d, \qquad (44)$$

where ε_d exponentially decays due to nonzero initial conditions, $\theta \in R^6$ is the vector of unknown constant parameters and $\omega \in R^6$ is the regressor vector

$$\omega(t) = \left[\frac{1}{(p+1)^2}u(t); \frac{p}{(p+1)^2}u(t); d(t); \frac{1}{(p+1)^2}d(t); \frac{p}{(p+1)^2}d(t); r(t)\right]^T. \qquad (45)$$

Further a transfer function $W(p)$ with the relative degree $\rho^* = \rho - 1 = 2$ is chosen such that

$$W(p) = \frac{(p+\alpha_0)}{a_r(p)} = \frac{(p+1)}{(p+1)^3} = \frac{1}{(p+1)^2}, \qquad (46)$$

where $\alpha_0 = 1$.

Choose the control according to equation (12)

$$u = -\frac{\alpha_r(p)}{(p+\alpha_0)}\varphi' = -(p+1)^2\varphi' = -\varphi' - 2\frac{d\varphi'}{dt} - \frac{d^2\varphi'}{dt^2} = = -\xi_1 - 2\vartheta\bar{\sigma}\xi_2 - \vartheta\dot{\bar{\sigma}}\xi_2 - (\vartheta\bar{\sigma})^2\xi_3, \qquad (47)$$

where

$$\varphi' = \xi_1, \qquad (48)$$

$$\begin{cases} \dot{\xi}_1 = \vartheta\bar{\sigma}\xi_2, \\ \dot{\xi}_2 = \vartheta\bar{\sigma}\xi_3, \\ \dot{\varsigma} = \vartheta\bar{\sigma}(-3\xi_2 - 3\xi_3 + \alpha_0 y) - \vartheta\dot{\bar{\sigma}}\, y, \end{cases} \qquad (49)$$

$$\xi_3 = \varsigma + \vartheta\bar{\sigma}\, y. \qquad (50)$$

It is necessary to choose the function $\bar{\sigma}$ according to outcomes of the *remark*. The results of a computer simulation for variable $y(t)$ for the case $a_1 = -1$, $a_0 = -1$, $b_0 = 1$, $C_0 = 1$, $d(t_0) = 0.1$ and disturbance $w(t) = 1 + 2\sin 5t$ are presented in Fig. 1 – 3. Presented control is seen to provide for the error $y(t)$ decrease under extension of parameter ϑ.

6. CONCLUSION

In this work the control schemes permitting to receive less complicated and bulky algorithms were offered. The structure of controller is nonlinear and contains a non-stationary filter (18), (40) and (41), parameters of which are selected from the requirements to the behaviour of an output plant variable. Thus the main contribution of the paper consists in design of an easy approach and reaching the simplified structure of the control law in comparison with Morse's tuner and scheme by Nikiforov. In comparison with approaches by Morse and Nikiforov this algorithm has smaller dimension:

- algorithm by Morse has dimension $2n(2\rho - 1) - 2$;
- algorithm by Nikiforov has dimension $2n(\rho + 1) + \rho - 4$;
- this algorithm has dimension $(2n+1)\rho - 1$.

Regions of the parameters ρ and n are presented in Fig. 4 and 5. From Fig. 4 and 5 can see where the proposed controller is more simple than existing ones.

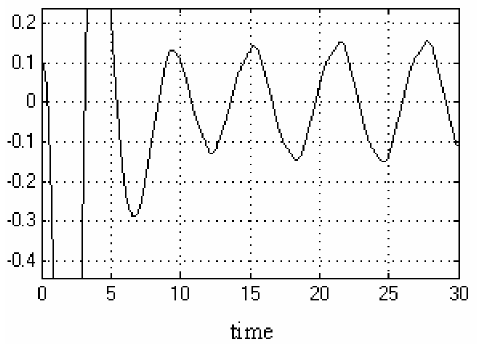

Fig. 1 Transients in control system (41) – (50) for variable $y(t)$ (the case $\vartheta = 5$).

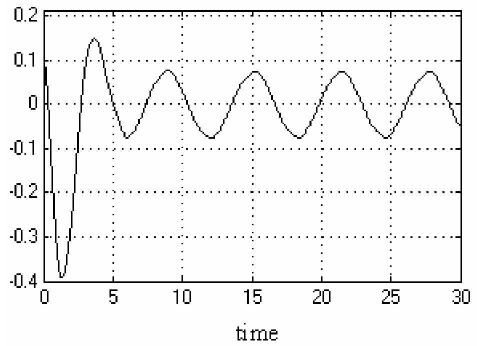

Fig. 2. Transients in control system (41) – (50) for variable $y(t)$ (the case $\vartheta = 10$).

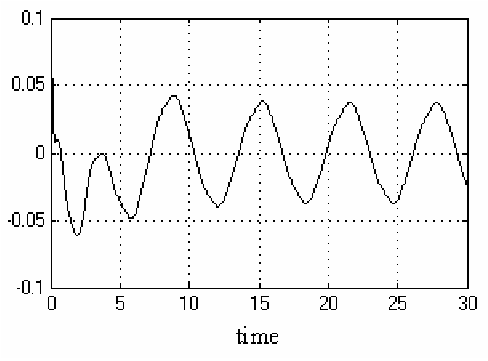

Fig. 3. Transients in control system (41) – (50) for variable $y(t)$ (the case $\vartheta = 20$).

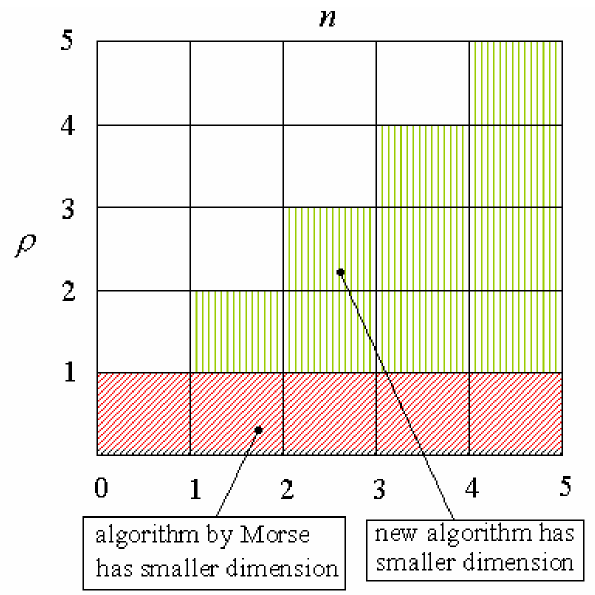

Fig. 4. Comparison the new algorithm and Morse's algorithm by dimension

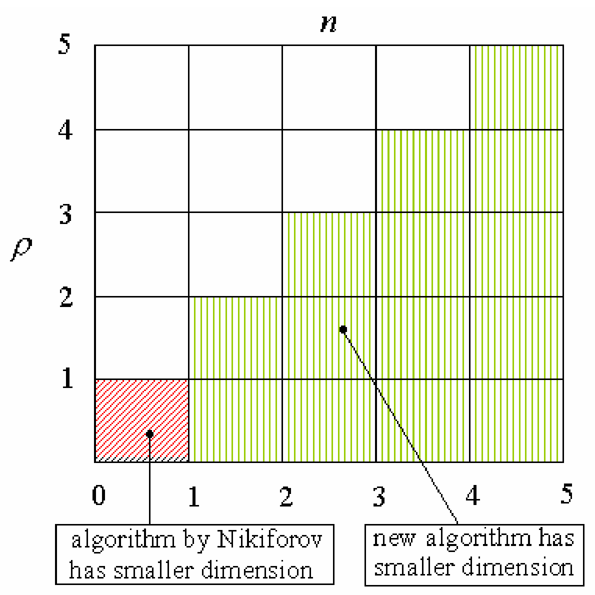

Fig. 5. Comparison the new algorithm and Nikiforov's algorithm by dimension

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