IMPROVING THE INTERSAMPLE BEHAVIOR BY USING A MULTIESTIMATION SCHEME WITH MULTIRATE SAMPLING

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Abstract: A multiestimation adaptive control scheme for linear time-invariant (LTI) continuous-time plant with unknown parameters is presented. The set of discrete adaptive models is calculated from a different combination of the correcting gain β in a fractional order hold (FROH) and the set of gains to reconstruct the plant input under multirate sampling with fast input sampling. The reference output is given by a continuous transfer function to evaluate the continuous tracking error of all the possible discrete models. Then the scheme selects online the model with the best continuous tracking performance. The estimated discrete unstable zeros are avoided through an appropriate design of the multirate gains so that the reference model might be freely chosen with no zeros constrains. A least-squares algorithm is used to estimate the plant parameters. However, only the active model is updated by using a least squares algorithm. The remaining possible models are updated by first calculating an estimated continuous-time transfer function, which results to be identical for all the models while their discretized versions are distinct in general.

Keywords: Adaptive control, Fractional Order Hold, Multirate sampling, Multiestimation, Supervised switching

1. INTRODUCTION

It is well-known that the unstable either continuous or discrete plant zeros should be transmitted to the reference model in a model matching problem (Aström and Wittenmark, 1990). In the context of discrete-time controllers acting on continuous-time plants, an appropriate choice of the correcting gain β of a FROH (potentially including zero-order holds, ZOH, for $\beta = 0$ and first-order holds, FOH, for $\beta = 1$) as well as the sampling period can locate some of the discretization zeros in the stable zone (Bilbao-Guillerna et al., 2005; Ishitobi, 1996; Liang and Ishitobi, 2005). However, this is not always possible because of the presence of unstable continuous-time zeros or because of the required range of the sampling period which can instabilize either the discretization or the intrinsic zeros. A solution of general applicability to avoid or circumvent this drawback is the use of multirate sampling techniques. A good selection of the multirate gains may make the estimated discrete zeros stable, (Alonso-Quesada and de la Sen, 2006; De la Sen and Bárcena, 2007; Moore et al., 1993; Morris and Neuman, 1981). However, the use of these techniques introduces a disadvantage that should be taken into account. Although the tracking of the desired reference can be achieved at sampling instants by the control law, the behavior of the output during the inter-sample period may not be suitable enough. This behavior depends on the choice of β, the sampling period and the reconstruction method used to generate the continuous plant input from the computed control sequence at sampling instants.

The main objective of this paper is to improve the inter-sample behavior by an appropriate selection of the gain β and the multirate gains through a fully freely chosen reference model even when the continuous plant possesses unstable zeros. In order to achieve this objective, we introduce a parallel multiestimation scheme, (Bilbao-Guillerna et al., 2005; Narendra and Balakrishnan, 1994 and 1997). The various models of this scheme are obtained via different values of the gain β in the FROH and different multirate gains. Since the plant parameters are unknown they have to be estimated and the models composing the multiestimation scheme are time-varying. *The main novelty of this paper compared with previous background work (Alonso-Quesada and de la Sen 2006, De la Sen and Alonso Quesada, 2007) is that the reference output is supplied by a stable continuous transfer function. Then the scheme is able to partly regulate the continuous-time tracking error while the controller is essentially discrete-time and operated by a FROH in general.* However, since the controller is designed to be discrete, it is necessary to obtain a discrete transfer function from the continuous-time reference one. In this way, each discretized plant model possesses a different discrete model which is obtained via discretization of the continuous reference model under a FROH with its associated gain β. As a result, each estimated model tends asymptotically to a different reference one. In (Alonso-Quesada and de la Sen, 2006), all of them converged asymptotically to the same reference model. The closed-loop performance is evaluated for all the possible discretized plant models by calculating their corresponding plant control signal and testing and monitoring its effect on an estimated continuous plant. Then, a performance index evaluates the continuous tracking performance of the estimated outputs related to the reference ones and the switching scheme selects the one with the lowest value. The active model currently in operation is used to online parameterize a discrete controller for matching the corresponding discrete reference model. A minimum residence time between consecutive switches is required for closed-loop stability purposes, (Aström and Wittenmark, 1990; Narendra and Balakrishnan, 1994 and 1997). Finally, some simulations will be displayed to show the effect of the proposed scheme.

2. DICRETE TRANSFER FUNCTION

Since the controller is discrete-time and the plant is continuous-time, we need to generate a continuous-time signal from the discrete control input, before injecting it to the plant. Two different reconstruction methods are considered in order to generate such an input. One method is governed by the large sampling period while the other one is governed by the small sampling period. The continuous-time plant is defined by the following state-space equations:

$$\dot{x}(t) = Ax(t) + bu(t); \quad y(t) = c^T x(t) \tag{1}$$

where $u(t)$ and $y(t)$ are, respectively, the input and output signals, $x(t) \in \Re^n$ denotes the state vector and A, b and c^T are constant matrix and vectors of appropriate dimensions.

2.1 Input Reconstruction Method I (Ruled by the large sampling period T)

The plant input is generated by the following equation:

$$u(t) = \alpha_j \left\{ u_k + \beta \frac{u_k - u_{k-1}}{T} (t - kT) \right\} \quad (2)$$

for $t \in \left[kT + (j-1)T', kT + jT' \right)$ and $j \in \bar{N} \equiv \{1, 2, ..., N\}$, where u_k is the input signal at $t = kT$ and $\beta \in [-1, 1]$ is the FROH correcting gain. T is the large sampling period associated with slow sampling rate of the output and $T' = T/N$ is the small sampling period associated with the fast sampling rate of the input. In other words, the large sampling period is divided in N equal subperiods in order to generate the multirate input. It is possible to ensure the stability of the zeros of all the discretized plant models which relate the input and output sequences defined over the sampling period T by an appropriate choice of the multirate gains α_j since the discretized plant zeros are parameterized by such gains. The discrete transfer function is

$$H_\beta(z) = B_\beta(z) / A_\beta(z) \quad (3)$$

The denominator of the transfer function does not depend on the choice of the gains α_j and it can be calculated as:

$$A_\beta(z) = \begin{cases} Det\left(zI_n - \psi^N\right) = z^n + \sum_{\ell=1}^{n} a_\ell z^{n-\ell} & if\, \beta = 0 \\ zDet\left(zI_n - \psi^N\right) = z^{n+1} + \sum_{\ell=1}^{n} a_\ell z^{n-\ell+1} & if\, \beta \neq 0 \end{cases} \quad (4)$$

The numerator can be written as:

$$B_\beta(z) = \begin{cases} c^T Adj\left(zI_n - \psi^N\right) C_\Delta g = \sum_{\ell=1}^{n} b_\ell^{(0)} z^{n-\ell} & if\, \beta = 0 \\ c^T Adj\left(zI_n - \psi^N\right)\left(zC_\Delta - \beta C'_\Delta\right) g = \sum_{\ell=1}^{n+1} b_\ell^{(\beta)} z^{n-\ell+1} & if\, \beta \neq 0 \end{cases} \quad (5)$$

with $C_\Delta = \left[\psi^{N-1}\Delta_1, ..., \psi\Delta_{N-1}, \Delta_N \right] \in \Re^{n\times N}$

$$C'_\Delta = \left[\psi^{N-1}\Delta'_1, ..., \psi\Delta'_{N-1}, \Delta'_N \right] \in \Re^{n\times N}$$

$$\Delta_j = \left(1 + \frac{j-1}{N}\beta \right)\Gamma + \frac{\beta}{T}\Gamma' \in \Re^{n\times 1}; \Delta'_j = \frac{j-1}{N}\Gamma + \frac{1}{T}\Gamma' \in \Re^{n\times 1}$$

$$\Gamma = \int_0^{T'} \phi(T' - s) B ds \in \Re^{n\times 1};\ \Gamma' = \int_0^{T'} \phi(T' - s) B s ds \in \Re^{n\times 1},$$

$$\psi^j = \phi\left(\frac{j}{N} T \right) = e^{j\frac{AT}{N}} \text{ and } g^T = [\alpha_1, \alpha_2, ..., \alpha_N]$$

with $Adj(.)$ and $Det(.)$ denoting, respectively, the adjoint matrix and the determinant of the square matrix (.) and I_n denoting the n-th order identity matrix. g is the vector of multirate gains with

$$\deg(A_\beta) = \deg(B_\beta) + 1 = \begin{cases} n & if\, \beta = 0 \\ n+1 & if\, \beta \neq 0 \end{cases} \quad (6)$$

The denominator can be rewritten as:

$$B_\beta(z) = \begin{cases} \sum_{\ell=1}^{n} \left(\sum_{j=1}^{N} b_{j,\ell}^{(0)} \alpha_j \right) z^{n-\ell} = \sum_{\ell=1}^{n} b_\ell^{(0)} z^{n-\ell} & if\, \beta = 0 \\ \sum_{\ell=1}^{n+1} \left(\sum_{j=1}^{N} b_{j,\ell}^{(\beta)} \alpha_j \right) z^{n-\ell+1} = \sum_{\ell=1}^{n+1} b_\ell^{(\beta)} z^{n-\ell+1} & if\, \beta \neq 0 \end{cases} \quad (7)$$

The coefficients $b_{j,\ell}$ depend on the parameters of the continuous time plant, the large sampling period T and the correcting gain β of the FROH considered in the discretization process. The value of the vector of multirate gains is relevant to stabilize the discrete plant zeros by appropriate choice of its components. Note that if we choose $\alpha_j = 1$ for all $j \in \bar{N}$, then this reconstruction method becomes the common one obtained with a FROH without multirate sampling working at the large single sampling period T.

2.2 Input Reconstruction Method II (Ruled by the small sampling period T')

In this method, the plant input is governed with the fast sampling and generated by the following equation:

$$u(t) = u_k^{(j)} + \beta \frac{N}{T}\left(u_k^{(j)} - u_k^{(j-1)} \right)\left(t - \left(k + \frac{j-1}{N} \right) T \right) \quad (8)$$

for $t \in \left[kT + (j-1)T', kT + jT' \right)$ and $j \in \bar{N}$, where

$$u_k^{(j)} := u(kT + (j-1)T') = \alpha_j u_k \text{ and}$$

$$u_k^{(0)} := u(kT - T') = \alpha_N u_{k-1}$$

The denominator of the transfer function is identical to that obtained in (4), while the numerator is

$$B_\beta(z) = \begin{cases} B_0^T(z) g & if\, \beta = 0 \\ z\left[B_0^T(z) g + \frac{\beta N}{T} B_1(z) g' \right] - \frac{\beta N}{T} \alpha_N B_{1,1}(z) & if\, \beta \neq 0 \end{cases} \quad (9)$$

where, $B_0^T(z) = \left[B_{0,1}(z), B_{0,2}(z), ..., B_{0,N}(z) \right]$;

$$B_1^T(z) = \left[B_{1,1}(z), B_{1,2}(z), ..., B_{1,N}(z) \right]$$

with $B_{0,j}(z) = C^T Adj\left(zI_n - \psi^N \right) \psi^{N-j} \Gamma$;

$$B_{1,j}(z) = C^T Adj\left(zI_n - \psi^N \right) \psi^{N-j} \Gamma' \text{ and}$$

$$g'^T = [\alpha_1, \alpha_2 - \alpha_1, ..., \alpha_N - \alpha_{N-1}]$$

Now the multirate is ruled by the fast sampling rate because it is necessary to know the value of the input at the fast sampling instants to generate the continuous-time plant input. Note that if $\beta = 0$ both methods lead to the same transfer function.

2.3 Compact Representation

The discretized plant model can be described in a compact and clear way as

$$y_k = \begin{cases} -\sum_{\ell=1}^{n} a_\ell y_{k-\ell} + \sum_{\ell=1}^{n}\sum_{j=1}^{N} b_{j,\ell}^{(0)} \alpha_j u_{k-\ell} = \theta^{(0)T} \varphi_{k-1}^{(0)} & if\ \beta = 0 \\ -\sum_{\ell=1}^{n} a_\ell y_{k-\ell} + \sum_{\ell=1}^{n+1}\sum_{j=1}^{N} b_{j,\ell}^{(\beta)} \alpha_j u_{k-\ell} = \theta^{(\beta)T} \varphi_{k-1}^{(\beta)} & if\ \beta \neq 0 \end{cases} \quad (10)$$

where $\theta_\beta = \left[\theta_a^T\ \theta_{b,1}^T\ \theta_{b,2}^T \ldots \theta_{b,n+1}^T \right]^T$; $\theta_0 = \left[\theta_a^T\ \theta_{b,1}^T\ \theta_{b,2}^T \ldots \theta_{b,n}^T \right]^T$

$$\varphi_{\beta,k} = [\varphi_y^T\ \varphi_{u,1}^T\ \varphi_{u,2}^T \ldots \varphi_{u,n+1}^T]^T; \varphi_{0,k} = [\varphi_y^T\ \varphi_{u,1}^T\ \varphi_{u,2}^T \ldots \varphi_{u,n}^T]^T;$$

$$\theta_a = \left[-a_1\ -a_2 \ldots -a_n \right]^T; \theta_{b,\ell} = \left[b_{\ell,1}\ b_{\ell,2} \ldots b_{\ell,N} \right]^T$$

$$\varphi_y = \left[y_{k-1}\ y_{k-2} \ldots y_{k-n} \right]^T; \varphi_{u,\ell} = \left[\alpha_1 u_{k-\ell}\ \alpha_2 u_{k-\ell} \ldots \alpha_N u_{k-\ell} \right]^T$$

The above notation will be then useful in order to formulate properly the estimation scheme with the given expanded regressor. The coefficients of the numerator of the discrete transfer function can be rewritten as

$$v_\beta = M_\beta g \quad (11)$$

where

$$M_\beta = \begin{bmatrix} b_{1,1}^{(\beta)}\ b_{2,2}^{(\beta)} \cdots b_{N,1}^{(\beta)} \\ b_{1,2}^{(\beta)}\ b_{2,2}^{(\beta)} \cdots b_{N,2}^{(\beta)} \\ \vdots \quad \ddots \\ b_{1,n+1}^{(\beta)} \quad \cdots \quad b_{N,n+1}^{(\beta)} \end{bmatrix} \text{ and } v_\beta = \left[b_1\ b_2 \ldots b_{n+1} \right]^T \text{ if } \beta \neq 0$$

$$M_0 = \begin{bmatrix} b_{1,1}^{(0)} \; b_{2,1}^{(0)} \cdots b_{N,1}^{(0)} \\ \vdots \quad \ddots \\ b_{1,n}^{(0)} \quad \cdots \quad b_{N,n}^{(0)} \end{bmatrix} \text{ and } v_0 = \left[b_1 \; b_2 \; ... \; b_n\right]^T \text{ if } \beta = 0 \quad (12)$$

- Remarks 1:

a) The value of N is chosen to be the minimum one necessary to fix the polynomial of discrete zeros to prescribed coefficients. It means that

$$\begin{cases} N = n & if \quad \beta = 0 \\ N = n+1 & if \quad \beta \neq 0 \end{cases} \quad (13)$$

b) The elements $b_{j,\ell}$ in the matrix M_β and M_0 of equations (12) are different for both reconstruction methods. As a result of this, a different multirate gains vector g is needed for each method to fix the discretized zeros in the same positions.

3. CONTROL SCHEME

A free-design LTI reference stable model given by

$$G_m(s) = N_m(s)/D_m(s) \quad (14)$$

is used in order to generate the continuous-time reference signal to be tracked by the plant output. A model-matching type discrete controller is synthesized to generate the control sequence, so we need to obtain a discrete transfer function of (14). This discrete transfer function is

$$H_{m,\beta}(z) = Z\left(h_\beta(s) \cdot G_m(s)\right) = \frac{B_{m,\beta}(z) A_{0,\beta}(z)}{A_{m,\beta}(z) A_{0,\beta}(z)} = \frac{b_1^{(\beta)} B'_{m,\beta}(z) A_{0,\beta}(z)}{A_{m,\beta}(z) A_{0,\beta}(z)} \quad (15)$$

where, $h_\beta(s) = \left(1 - \beta e^{-sT} + \beta(1-e^{-sT})/Ts\right)\left(1-e^{-sT}\right)/s$ is the transfer function of a β-FROH and Z the Z-transform. $B'_{m,\beta}(z)$ contains the free-design reference model discrete zeros and $A_{0,\beta}(z)$ is a polynomial including the eventual closed-loop stable pole-zero cancellations. Such a stable polynomial is introduced when necessary to guarantee that the relative degree of the reference model is not less than that of the closed-loop system so that the synthesized controller is causal. In the approach of this paper, the multirate techniques allow to stabilize all the discretized plant zeros so that $B_\beta(z) = b_1^{(\beta)} B_\beta^+(z)$ with $B_\beta^+(z)$ being a monic polynomial. The perfect matching at sampling instants is achieved through the control signal:

$$u_k = \left(T_\beta / R_\beta\right) u_{c,k} - \left(S_\beta / R_\beta\right) y_k \quad (16)$$

where the controller polynomials are obtained from $T_\beta = B'_{m,\beta} A_{0,\beta}$ and R_β (monic) and S_β are the unique solutions of the polynomial diophantine equation

$$A_\beta R_\beta + B_\beta S_\beta = B_\beta^+ A_{m,\beta} A_{0,\beta} \Leftrightarrow A_\beta R_{1,\beta} + b_1^{(\beta)} S_\beta = A_{m,\beta} A_{0,\beta}$$

with $R_\beta = B_\beta^+ R_{1,\beta}$ and degrees fulfilling.

$$\deg\left(R_{1,\beta}\right) = \deg\left(A_{0,\beta}\right) + \deg\left(A_{m,\beta}\right) - \deg\left(A_\beta\right),$$

$$\deg\left(S_\beta\right) = \deg\left(A_\beta\right) - 1,$$

$$\deg\left(A_{0,\beta}\right) = 2\deg\left(A_\beta\right) - \deg\left(A_{m,\beta}\right) - \deg\left(B_\beta^+\right) - 1$$

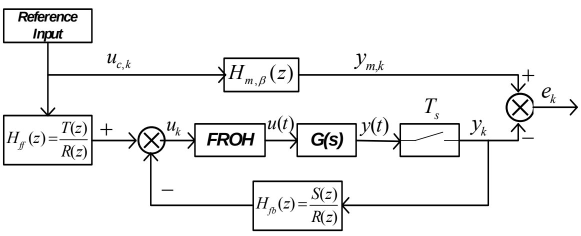

Figure 1: Basic control scheme

4. MULTIESTIMATION SCHEME

Different estimators compose the parallel multiestimation scheme. Each estimator is used to identify a different discretization of the continuous plant under any of both reconstruction methods. The main idea for scheme's implementation is that all the estimator/controller pairs are running in parallel at the same time while calculating each control law, but only one of them actually generates the control law. Each controller parameterization is updated for all time although only one being active is generating the control signal. The closed-loop performance of all the models should be simulated by calculating its corresponding controller and applying the obtained input to an analogic (transfer function) estimated model of the continuous-time plant. Then, the obtained output is compared with the desired one. The strategy is to use the controller obtained from the best estimation model at each time interval. The closed-loop stability is guaranteed if the time interval between consecutive switchings is larger than an appropriate residence time. The estimated output for each i^{th} identifier at sampling instants is calculated as,

$$\hat{y}_k^{(i)} = \hat{\theta}_{\beta^{(i)},k}^{(i)T} \varphi_{\beta^{(i)},k}^{(i)} \text{ for } 1 \le i \le n_e \text{ and all } k \ge 0 \quad (17)$$

where $\hat{\theta}_{\beta^{(i)},k}^{(i)}$ and $\varphi_{\beta^{(i)},k}^{(i)}$ are respectively, the parameter estimation vector and associated regressor defined in section II.C. n_e denotes the number of models in the multiestimation scheme. In order to simplify the notation the subscript $\beta^{(i)}$ is removed from $\hat{\theta}_{\beta^{(i)},k}^{(i)}$ and $\varphi_{\beta^{(i)},k}^{(i)}$. Figure 2 shows a typical multiestimation scheme, where the estimated outputs are compared with the reference output. $\hat{G}_k(D)$ denotes the estimated continuous transfer function, where $D \triangleq d/dt$ is the time-derivative operator, formally equivalent to the Laplace operator s, with $D^0 = 1$ and $D^{i+1} = D^i D; \; i = 0,1,...$

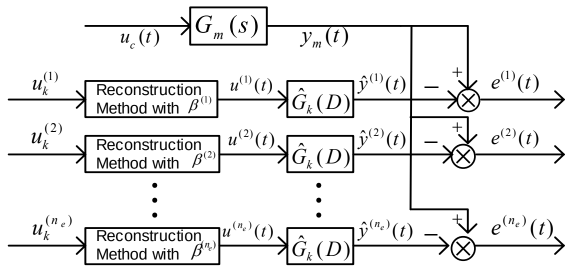

Fig.2: Multiestimation scheme

4.1 Estimation Method

Only the active estimator is updated by using following least squares estimation algorithm

$$\hat{\theta}_{k+1}^{(c_k)} = \hat{\theta}_k^{(c_k)} + \frac{P_k^{(c_k)} \varphi_k^{(c_k)} e_k^{(c_k)}}{1 + \varphi_k^{(c_k)^T} P_k^{(c_k)} \varphi_k^{(c_k)}}; \quad \hat{\theta}_0^{(c_k)} \text{ arbitrary}$$

$$P_{k+1}^{(c_k)} = P_k^{(c_k)} - \frac{P_k^{(c_k)} \varphi_k^{(c_k)} \varphi_k^{(c_k)^T} P_k^{(c_k)}}{1 + \varphi_k^{(c_k)^T} P_k^{(c_k)} \varphi_k^{(c_k)}}; \quad P_0^{(c_k)} = P_0^{(c_k)^T} > 0 \quad (18)$$

where $e_k^{(c_k)} = y_k - \hat{y}_k^{(c_k)}$ is the identification error and c_k denotes the active model at k^{th} sample. To get the estimated values for the rest of estimator we use the active one. Since we know the order of the plant transfer function, it is possible to obtain the estimated parameters of the continuous transfer function from the currently active discretization estimation model. Once the estimated continuous transfer function is updated we can obtain the rest of discretization estimation models via discretization of such an estimated continuous transfer function. The following steps describe this method:

a) Obtain $\hat{\theta}_{k+1}^{(c_k)}$ and $P_{k+1}^{(c_k)}$ from (18).

b) Calculate $\hat{G}_{k+1}(D)$ from $\hat{\theta}_{k+1}^{(c_k)}$, i.e., from $\hat{H}_\beta^{(c_k)}$.

c) Obtain $M_{k+1}^{(i)}$ from $\hat{G}_{k+1}(D)$ for each model.

d) Build $\hat{\theta}_{k+1}^{(i)}$ from $M_{k+1}^{(i)}$ for $i=1,...,c_k-1,c_k+1,...,n_e$.

e) Update the multirate gains vector via (11) so that the estimated discrete numerator remains identical.

Note that in the third step the matrix $M_{k+1}^{(i)}$ can be obtained from $\hat{G}_{k+1}(s)$ for each model, because its elements do not depend on the multirate gains, which are updated later in the last step. The estimated matrixes $\hat{A}_k$, $\hat{b}_k$ and $\hat{c}_k$ in (1) can be directly obtained from $\hat{G}_{k+1}(D)$ by first selecting any state-space like, for instance, controllability or observability forms or a canonical real state-space realization. In this sense, note that any state-space realization leads to the same $\hat{M}_{k+1}^{(i)}$.

4.2 Tracking Performance index

The objective of the supervisor is to evaluate the tracking performance of the possible controllers connected to the plant with the aim of choosing the current controller from the set of parallel parameterized controllers, each of them corresponding to a different value of the gain β. The following estimated tracking performance index is proposed,

$$J_k^{(i)} = \sum_{j=k-N}^{k} \lambda^{k-j} \int_0^T \left(y_m(jT+\tau) - \hat{y}^{(i)}(jT+\tau) \right)^2 d\tau \qquad (19)$$

for $1 \le i \le n_e$ and, where, $\lambda \in (0,1]$, $N>0$, y_m is the reference continuous output and $\hat{y}^{(i)}$ the estimated one corresponding to the i^{th} estimation process. Note that the use of the identification error in the performance index is not sensible in this problem since the current plant output is generated by only one active FROH used for the current plant discretization.

4.3 Switching Rule

Now, the switching rule for the basic adaptive controller reparameterization is obtained from the performance index as follows:

- Let the switching sampling times sequence be denoted by $TS = \left\{ t^{(1)}, t^{(2)}, ..., t^{(\pi)} \right\}$ where π, which may be finite or infinite countable, is the number of consecutive switching instants in increasing order and $\left(t^{(i+1)} - t^{(i)} \right) \ge \tau_r = N_r T$ (a known minimum residence time) for all $t^{(i)}$, $t^{(i+1)} \in TS$. Thus, the c_k-estimation scheme with $1 \le c_k \le n_e$, which parameterizes the basic adaptive controller for all $k \ge 0$ at any switching time in TS is updated as follows. Assume that the last switching time for the controller re-parameterization was $t^{(i)}$. Thus, for each current k sampling time, define the auxiliary integer variable:

$\overline{c}_k = Arg\left[i : J_k^{(i)} = Min\left(J_k^{(j)} \right); i,j \in n_e \right]$, all integer $k \ge 1$

if $kT \ge t^{(i)} + \tau_r$ ***then*** $c_k \leftarrow \overline{c}_k$ ***end_if***

if $c_k \ne c_{k-1}$ ***then*** $t^{(i+1)} \leftarrow kT$ ***and*** $TS \leftarrow \left\{ TS, t^{(i+1)} \right\}$ ***end_if***

It is well-known that there is always a minimum residence time that guarantees the closed-loop stability under switched parameterizations. This value could be obtained from an ‘a priori’ knowledge or from experimental research. If a minimum residence time at each parameterization guaranteeing close-loop stability is not available, then it may be updated online as follows. First, start with a very small value. Then increase it with small positive increments until finding a proper value while the transients are found to heuristically be compatible with stability.

5. TECHNICAL RESULTS OF STABILITY OF THE PLANT ZEROS AND ITS ESTIMATES

- ***Definition 1:*** Let $\rho(z)$ be a polynomial of degree *m*. Then, the class $C_\rho(\delta,\varepsilon)$ is the set

$$C_\rho(\partial,\varepsilon) := \left\{ \delta(z) := \rho(z) + \varepsilon \partial(z) : \deg(\delta(z)) = \deg(\rho(z)) = m \right\} \qquad (20)$$

Note that according to definition 1 any polynomial $\delta(z)$ of degree less or equal to *m* is valid to establish the class $C_\rho(\partial,\varepsilon)$.

- ***Proposition 1:*** Assume that $\rho(z)$ is a stable polynomial (i.e., $\rho(z)=0 \Leftrightarrow |z| \le 1-\gamma$, for some $\gamma \in \mathbb{R}^+$). Then, it exists $\varepsilon^* \in \mathbb{R}^+$ such that the class $C_\rho(\partial,\varepsilon)$ is formed by stable polynomials for all $\varepsilon \in \left[0,\varepsilon^*\right]$ fulfilling $\deg(\partial(z)) \le \deg(\rho(z))$ for each $\partial(z)$.

- ***Proof:*** Consider $\delta(z) = \rho(z) + \varepsilon\, \partial(z)$ for each prefixed $\partial(z)$ of the same (or less) degree as $\rho(z)$. The zeros of $\delta(z)$ satisfy the characteristic equation $1 + \varepsilon \dfrac{\partial(z)}{\rho(z)} = 0$. Then, from the root locus technique, the zeros $z_i\,(i=1,...,m)$ of $\delta(z)$ are those of $\rho(z)$ as $\varepsilon = 0$. These zeros satisfy $|z_i| \le 1-\gamma$ for some $\gamma \in (0,1) \cap \mathbb{R}$. From continuity of the root locus, there exists $\varepsilon^* \in \mathbb{R}^+$ for each given $\mathbb{R}^+ \ni \gamma_0 < \gamma$ such that for all $\varepsilon \in \left[0,\varepsilon^*\right]$ all the zeros of $\delta(z)$ are in $|z_i| \le 1-\gamma_0[\ ]$ with ε^* being dependant on γ and the polynomials $\rho(z)$ and $\partial(z)$. Since all the zeros are in $|z_i| < 1$, all the polynomials in the class $C_\rho(\partial,\varepsilon)$ are stable for all $\varepsilon \in \left[0,\varepsilon^*\right]$ and for any $\partial(z)$ of degree less than or equal to that of $\rho(z)$.

- ***Alternative proof of Proposition 1:*** Consider $0 < \gamma \le 1$ such that any complex *z* satisfying $\rho(z)=0$ is inside the open region $R_\gamma := \left\{ |z| \in \mathbf{C} : |z| < 1-\gamma \right\}$. Assume also that $\varepsilon^* < |\rho(z)/\partial(z)|$ for $|z| < 1-\gamma$. Then from Rouché Theorem for zeros of analytic functions (De la Sen and Bárcena, 2004), all the zeros of $\delta(z)$, $\delta(z) = \rho(z) + \varepsilon\, \partial(z)$, are also in $|z| < 1-\gamma < 1$.

Note that in the alternative proof of Proposition1 there exists $\tilde{\varepsilon} \in \mathbb{R}^+$ such that for $\varepsilon^* + \tilde{\varepsilon} \le |\rho(z)/\partial(z)|$ with $|z| < 1-\gamma$ all the zeros of $\delta(z)$ are in $|z| < 1-\gamma_0 < \gamma$ for some $\gamma_0 \in (0,\gamma)$.

Now consider $H_\beta(z) = B_\beta(z) / A_\beta(z)$ and its estimates $\hat{H}_\beta^{(i)} = \hat{B}_\beta(z) / \hat{A}_\beta(z)$ for $i=1,...,n_e$. Note that $\hat{B}_\beta(z) = \hat{B}_\beta^{(c_k)}(z) = \hat{B}_\beta^{(i)}(z)$ since the zeros of all the estimates are fixed to prescribed time-invariant stable positions and also $\hat{A}_\beta(z) = \hat{A}_\beta^{(c_k)}(z) = \hat{A}_\beta^{(i)}(z)$ since they are obtained from equation similar to (4) by substituting ψ by the corresponding $\hat{\psi}_k$ associated to the continuous-time plant estimated at the sampling instant kT. Now define the time-varying polynomial of parametrical error of zeros as

$$\tilde{B}_{\beta,k}(q)=\sum_{i=0}^{m}\left(b_i-\hat{b}_{i,k}\right)q^i=\sum_{i=0}^{m}\tilde{b}_{i,k}q^i \quad (21)$$

- ***Corollary 1:*** Assume that the discrete plant numerator $B_\beta(z)\in C_{\hat{B}_\beta}\left(\tilde{B}_\beta(z),\varepsilon\right)$. Then $B_\beta(z)$ is stable for any $\varepsilon\in\left[0,\varepsilon^*\right]$ with $\varepsilon^*\in(0,1)$ such that $\left|\frac{\tilde{B}_\beta(z)}{\hat{B}_\beta(z)}\right|\le\varepsilon\le\varepsilon^*<1$ for $|z|=1$.

Note that the scalar ε plays the role of a normalizing gain of the parametrical error which vanishes if ε goes to zero.

- ***Remarks 2:*** a) As a result of Corollary 1, the set of multirate gains, which is calculated to stabilize the active estimated numerator, can locate the discrete plant zeros in the stable zone as well if the estimates are not far from the real parameters of the plant and are located near prescribed stable fixed positions by using algorithm (2) or (8) and equation (11).

b) The H_∞-norm of the stable transfer function $\frac{\tilde{B}_\beta(z)}{\hat{B}_\beta(z)}$ is less than unity.

c) The following degrees condition

$$\deg\left(\tilde{B}_\beta(z)\right)\le\deg\left(\hat{B}_\beta\right)=\begin{cases}m=n-1 & for\ \ \beta=0\\ m=n & for\ \ \beta\ne0\end{cases}$$

guarantees that the number of zeros of $B_\beta(z)$ is the same as that of $\tilde{B}_\beta(z)$ which are stable under corollary 1.

- ***Proposition 2:*** The plant zeros are stable if any of the two conditions below holds

(i) $\varepsilon^*\ge\bar{\tilde{b}}_M$ where

$$\bar{\tilde{b}}_M=\sum_{i=0}^{m}\left|\tilde{b}_{i,k}\right|\ge\underset{|q|=1}{Max}\left(\left|\tilde{B}_\beta(q)\right|\right)=\underset{|q|=1}{Max}\left|\sum_{i=0}^{m}\tilde{b}_{i,k}q^i\right|$$

(ii) $\varepsilon^*\ge\bar{\tilde{b}}$ where

$$\bar{\tilde{b}}:=\underset{\varphi\in[0,2\pi]}{Max}\left(\left|\tilde{b}_{i,k}\left(\cos\varphi+j\sin\varphi\right)\right|\right)\ge\underset{|q|=1}{Max}\left(\left|\tilde{B}_\beta(q)\right|\right)=\underset{|q|=1}{Max}\left|\sum_{i=0}^{m}\tilde{b}_{i,k}q^i\right|$$

$\forall k\ge0$, where $j=\sqrt{-1}$ is the complex unity.

- ***Proof:*** Under the given conditions, the plant numerator belongs to the class defined in Proposition 1 whose members are stable by construction.

Note that condition (i) is more restrictive than condition (ii) in Proposition 2 and that Proposition 2(ii) implies Proposition 2(i). As a particular and usual case, consider that the elements b_i and $\hat{b}_{i,k}$ possess the same sign. This is a reasonable assumption based on 'a priori' knowledge. Then Proposition 2 with condition (i) is achieved under the following conditions,

If $\left|\hat{b}_{i,k}\right|\ge\left|b_i\right|$ then $\left|\hat{b}_{i,k}-b_i\right|=\left|\hat{b}_{i,k}\right|-\left|b_i\right|\le\frac{\varepsilon^*}{m+1}$ for $i=1,...,m+1$

If $\left|\hat{b}_{i,k}\right|<\left|b_i\right|$ then $\left|\hat{b}_{i,k}-b_i\right|=\left|b_i\right|-\left|\hat{b}_{i,k}\right|\le\frac{\varepsilon^*}{m+1}$ for $i=1,...,m+1$

- ***Remark 3:*** The closed-loop stability of the overall parallel multiestimation scheme is guaranteed by respecting a minimum appropriate residence time between two consecutive switches of active parameterized controller. A lower-bound of such a residence time may be evaluated from online measurements checking the relative stability of the experiment or computed analytically from worst-case absolute upper-bounds of the plant parameter (see for instance, [Bilbao-Guillerna et al., 2005]).

6. SIMULATION RESULTS

The first simulation is displayed for the following continuous-time transfer function

$$G(s)=(s-1)\big/\left(s^2+6s+8\right) \quad (22)$$

The continuous reference transfer function is chosen as

$$G_m(s)=(s+2)\big/\left(s^2+2s+1\right) \quad (23)$$

Note that the plant transfer function possesses an unstable zero at $s=1$ which is not transmitted to the reference model. A set of 11 discretization processes, as described in previous sections, composes the multiestimation scheme. Each one uses a different gain for the FROH, being the respective gains obtained from:

$$\beta^{(i)}=1-0.2(i-1)\quad\text{for}\quad 1\le i\le 11 \quad (24)$$

The initial value of the estimated continuous transfer function is

$$\hat{G}_0(D,0)=(D-5)\big/\left(D^2+11D+10\right) \quad (25)$$

In the first simulation Reconstruction Method II with the sampling time being 0.25 seconds is used. The multirate gains are chosen so that the numerator is monic with discretization zeros located in $z=0.5$. Initially, the active discretization process is that with $\beta=0$, i.e., $\beta^{(c_0)}=\beta^{(6)}$. The residence period is 8 samples and the reference input is a square signal. Figure 3 shows the obtained output and the reference one. Figure 4 compares, in the stationary regime, the output obtained with the multiestimation scheme with that obtained in case of maintaining $\beta^{(c_k)}=0$ during the whole simulation. It becomes apparent how the proposed multiestimation scheme reduces the inter-sample tracking error compared to that obtained with a ZOH by selecting the model with the best tracking performance behavior. Figure 5 shows the evolution of the active value of β and Figure 6 displays the evolution of the multirate gains α_j. Note that when the current value of β is 0, there are only two multirate gains. α_3 appears when $\beta\ne0$ and it is removed as $\beta=0$ since the ZOH is able to stabilize the discrete plant zeros with $T'=T/2$ and then only two multirate gains.

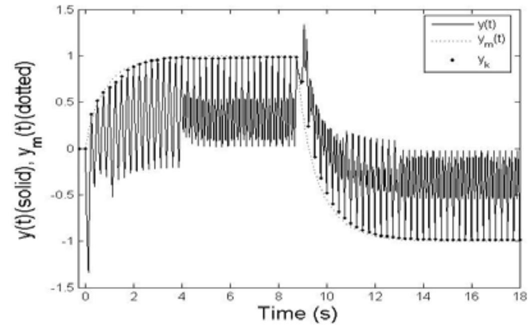

Fig.3: Plant output and reference using Reconstruction Method II

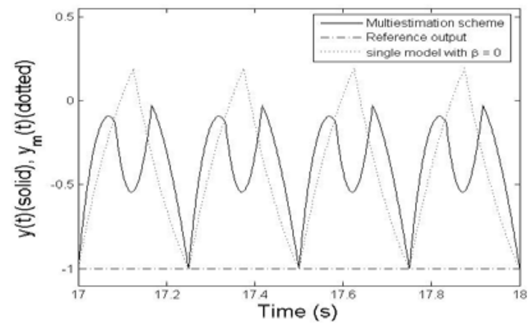

Fig.4: Comparing the multiestimation scheme with the use of a ZOH in steady-state

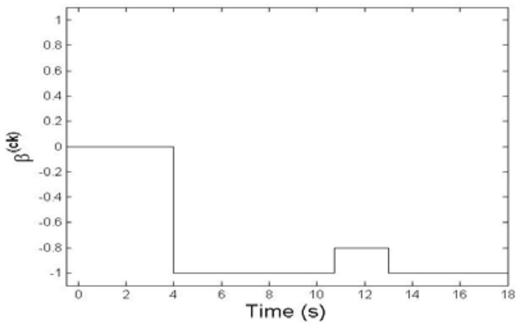

Fig.5: Evolution of the active value of β

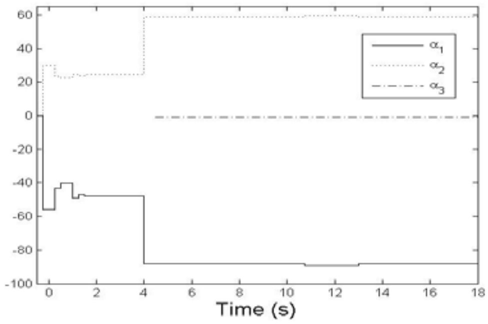

Fig.6: Evolution of the multirate gains

The second simulation is displayed for the following continuous-time unstable transfer function

$$G(s) = (s-1)\Big/\left(s^2 - s - 6\right) \tag{26}$$

for the Reconstruction Method I with T = 0.8s and a step signal as reference input. The remaining parameters are the same as the ones used in previous simulation. Figure 7 shows the plant output, while the evolutions of the active value of β and multirate gains are displayed in Figure 8 and 9, respectively. It has been observed that the tracking error of the switching scheme is lower than the one obtained from the use of a ZOH during the whole simulation.

7. CONCLUSIONS

A multiestimation scheme, consisting of a set of discretization models running in parallel, with a discrete-time model matching controller for an unknown LTI continuous-time plant is presented. Each of these models is calculated from a different combination of the correcting gain β of a FROH and the set of multirate gains to reconstruct the plant input. Unstable zeros of the discretized estimated plant are avoided through an appropriate design of the multirate gains so that the reference model might be freely chosen and perfect matching is achieved at sampling instants without requiring the transmission of the eventual discrete plant zeros to the reference model. The transient response within the inter-sample time period is improved compared to the use of a single model scheme. The tools for such an improvement are the appropriate on-line choice of the correcting gain β and multirate gains corresponding to the discretization process which provides the best behavior.

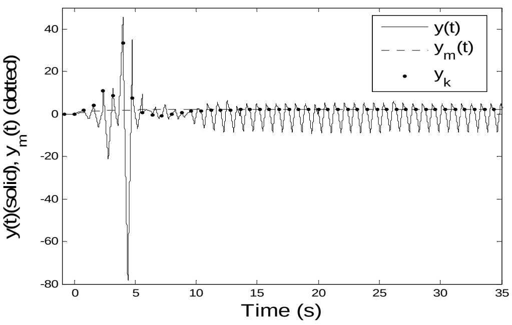

Fig.7: Plant output and reference one with Reconstruction Method I

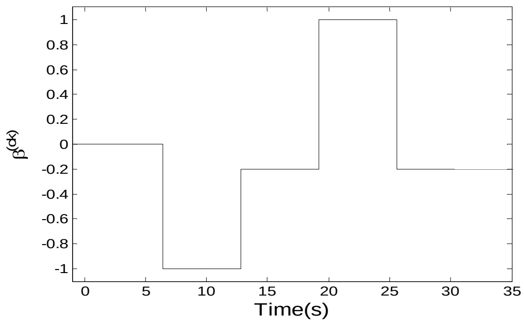

Fig.8: Evolution of the active value of β

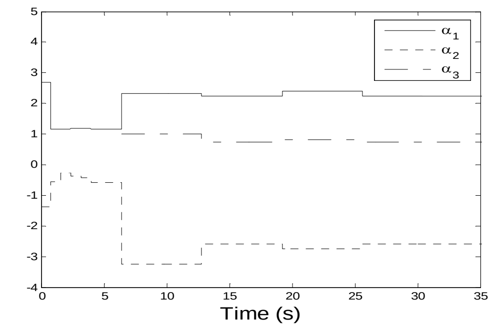

Fig.9: Evolution of the multirate gains

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