

TRAJECTORY ANALYSIS TO VERIFY DESIGN PARAMETERS OF PM-BASED DIPOLES FOR SILA FACILITY

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Abstract

In 2020 Russia launched the Federal scientific and engineering programme aimed to advance development in synchrotron science and techniques to support industry and innovation in a wide range of applications. An important part of these activities is the synchrotron radiation source SILA to be built and commissioned at the Kurchatov Institute by 2033. SILA combines two types of accelerators: a 6 GeV storage ring and a free electron laser. The project is aimed to create the related infrastructure and provide solid basis for innovative synchrotron and accelerator technologies.

This paper presents a numerical method applied to optimize and validate the design parameters of permanent magnet dipoles of the SILA magnet system through simulation of charged particle dynamics in a 3D magnetic field.

Key words

Permanent magnet, dipoles, particle dynamics, SILA facility, 3D magnetic field.

1 Introduction

The study addresses the magnetic design and optical performance of the SILA facility [Ashanin et al., 2018; Gavrish et al., 2024] planned to be built at the NRC “Kurchatov Institute”. This is the fourth-generation synchrotron radiation source designed as a combination of a storage ring and a free electron laser. The storage ring magnetic structure will include a number of dipoles utilizing highly coercive permanent magnets (PM).

The design of precision dipole magnets based on permanent magnets involves a number of engineering challenges. The material consideration addresses a low-carbon steel yoke, FeNi-based thermal shims, iron shims, and possible fluctuations of PM specification to provide desired field tuning. Also, a special attention should be paid to nonlinear magnetic crosstalks of neighboring PMs. Another critical aspect is the need for field tunability. Unlike electromagnets which offer easy field adjustment with coil currents, a PM-based dipole faces mechanical complexity to enable tunability. This requires thorough selection of dimensions and locations of PM blocks and shims to provide the desired field distribution. The resultant field is checked through magnetic measurements and comparison with magnetic simulations for every PM assembly prior to installing dipoles in the storage ring.

This necessitates the use of detailed numerical models for the PM dipoles both at the design and prototyping phases. Experience from state-of-the-art light sources, such as ESRF-EBS in Grenoble, France [ESRF, 2024] and Sirius in Campinas, Brazil [Liu et al., 2013], suggests that parametric simulations with detailed 3D models are inevitable to finalise the engineering design of the fully assembled magnets and verify that the required performance can be achieved.

The main goal of this study is to develop an algorithm for optimization and validation of the magnet design, specifically dipoles, using trajectory simulations. Two alternative methods are applied for modeling particle dynamics. The first is based on a matrix method. It allows a compact description of the linear dynamics and

changes in beam parameters, taking into account the effects of system components. The second approach involves numerical integration of the motion equations to describe particle trajectories in a 3D magnetic field. A comparison of the results allows us to confirm their reliability. Optical properties derived from the trajectory simulations can be used as a technical compliance criterion for the magnet design.

The proposed approach has been elaborated and validated in a set of numerical experiments performed for the short (SBM) and long (DL1) PM-based dipoles of the SILA storage ring using realistic and idealized magnet models.

2 Field distribution

Field distributions for SBM and DL1 were simulated with the use of the codes KOMPOT and KLONDIKE [Arslanova et al., 2024; Arslanova et al., 2023]. Both SBM and DL1 dipoles are shown schematically in Figures 1 and 2. Figures 3–6 illustrate simulated field profiles in the right-handed coordinate system where the radial axis X is directed outward, the vertical axis Y is directed upwards; and the longitudinal axis Z is co-directed with the beam. Note that the field distribution is considered here only for a drift – magnet – drift channel.

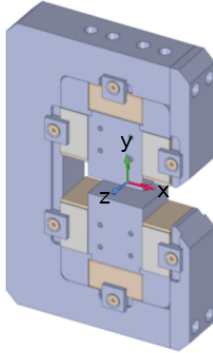


Figure 1. Layout of SBM dipole for SILA facility.

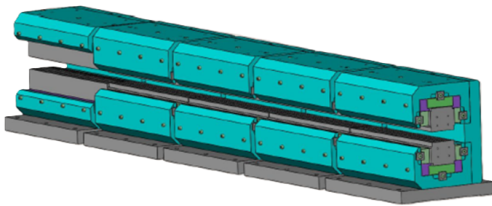


Figure 2. Layout of DL1 dipole for SILA facility.

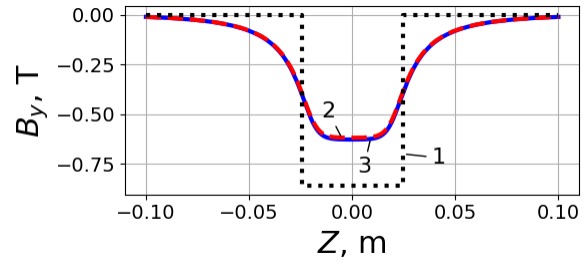


Figure 3. Vertical field profile for SBM. Curve 1 – design profile; curves 2, 3 – simulated profiles.

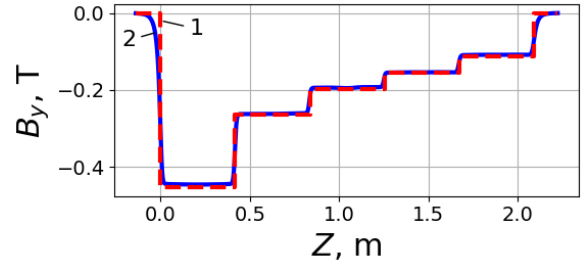


Figure 4. Vertical field profile for DL1. Curve 1 – design profile; curve 2 – simulated profile.

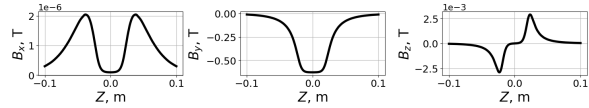


Figure 5. Field profiles near SBM longitudinal axis ($X = 0.1$ mm, $Y = 0.1$ mm).

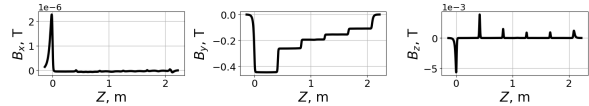


Figure 6. Field profiles near DL1 longitudinal axis ($X = 0.1$ mm, $Y = 0.1$ mm).

In the preliminary magnetic design the key requirement was to provide the specified integrated field, which determines the total bending strength of the magnet. For this reason the magnetic field configurations considered further are assumed to preserve the design value of the field integral, while their longitudinal field profiles may differ from the design distribution. These differences reflect the influence of the realistic 3D magnet geometry, fringe fields, and possible variations in the arrangement of PM blocks and shimming elements. Thus, the following trajectory simulations are intended to verify whether preservation of the field integral is sufficient for the required beam dynamics or whether deviations of the field profile itself lead to noticeable changes in the equilibrium orbit and optical parameters.

The storage ring lattice design (Appendix A, Figure 15) and beam parameters of the SILA facility were specified at the NRC “Kurchatov Institute” on the basis of idealized magnet models [Dyubkov, 2023]. The following definitions will be used herefrom:

- equilibrium orbit is a closed trajectory along which a particle with the nominal (required) energy moves;
- idealized magnet is a model with a magnetic field that corresponds to the design values, in which fringe field effects, alignment errors, amplitude setting errors, etc. are not taken into account.
- reference orbit is an equilibrium orbit formed by idealized magnets in the absence of perturbations;
- magnet axis is a straight line between the entry and exit points of the reference orbit along the magnet bore, see Figure 7.

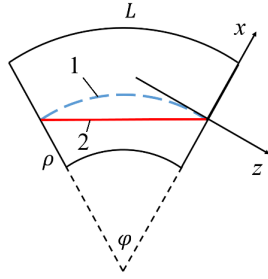


Figure 7. Idealized dipole magnet. 1 – reference orbit, 2 – magnet axis.

Two models were applied to simulate the field for SBM shown in Figure 3. In Model 1 the field integral over the drift – magnet – drift region is 98.67% of the design value, whereas for Model 2 it is 97.12%. Their main difference lies in the different pole profiles, which accounts for the different field distribution and the different values of the field integral. The DL1 model (Figure 4) gives the field integral of 99.72% of the design value.

Simulated field profiles in Figures 5 and 6 demonstrate that the beam dynamics is affected primarily by the vertical field B_y .

3 Beam dynamics simulation

The beam dynamics was simulated using two different approaches:

1. matrix method;
2. numerical integration of the particle motion equations.

3.1 Matrix method

It is assumed that particles have minor deviations from the equilibrium orbit. Then their motion in a field of the idealized magnet can be described with the Hill's equations [Wolski, 2014]:

$$\begin{cases} x'' + \left(\frac{1}{\rho^2} + \frac{G(s)}{B\rho} \right) x = 0 \\ y'' - \frac{G(s)}{B\rho} y = 0 \end{cases} \quad (1)$$

where x, y is the horizontal and vertical deviations from the equilibrium orbit, respectively; ρ is the bending (normalization) radius; $B\rho$ is the magnetic rigidity; $G(s)$ is the field gradient; s is the arc of the equilibrium orbit. The trajectory derivatives are taken with respect to s .

The equations (1) are linear and can be easily recast in matrix form. The field along the equilibrium orbit is approximated piecewise as a series of elementary magnets. Every elementary magnet is described with its own transport matrix. A set of elementary magnets is represented as a matrix product. To simulate beam dynamics using the matrix method we apply the MAD-X software [CERN, 2023], which is widely used for accelerator design and optimization.

To describe the beam dynamics for a realistic magnet, (1) take the form:

$$\begin{cases} x'' + \left(\frac{1}{\rho^2} + \frac{G(s)}{B\rho} \right) x = \frac{\Delta B_y(s)}{B\rho} \\ y'' - \frac{G(s)}{B\rho} y = \frac{\Delta B_x(s)}{B\rho} \end{cases}$$

where $\Delta B_y(s)$, $\Delta B_x(s)$ are deviations of the corresponding field components from the ideal ones.

As the vertical field B_y is dominant, the dynamics of particles would be determined primarily by movement in the horizontal plane. We neglect vertical oscillations due to their small magnitude.

For the matrix method simulation of SBM and DL1 magnets in the MAD-X package, piecewise constant approximation models based on the distribution of B_y along the magnet axis (Figure 8) were used.

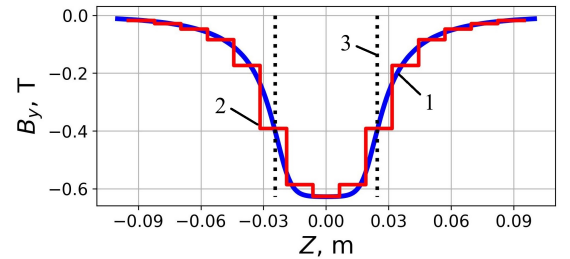


Figure 8. Piecewise constant approximation model for SBM. 1 – estimated distribution of B_y , 2 – B_y approximation, 3 – magnet boundaries.

The described models are based on replacing the magnet and adjacent drift spaces with a large set of idealized magnets whose constant fields match the given distribution. The local coordinate system in MAD-X is associated with the equilibrium orbit. Therefore, to evaluate

variations of the equilibrium orbit we must change to a coordinate system associated with the reference orbit via translation and rotation by a required angle.

This method is described in detail in [Knyazev and Levinsky, 2025; Belyaeva and Knyazev, 2024; Ovsyanikov et al., 2025].

3.2 Integration of motion equations

The matrix method involves simplified 1D models and is used for preliminary simulations of the designed magnets. A more comprehensive study requires consideration of a 3D magnetic field distribution. This will allow us to more accurately determine the optical parameters and effect of various factors, such as:

- fringe fields,
- nonlinear magnetic field distribution,
- magnetic crosstalks,
- manufacturing/assembly tolerances,
- radiation,
- eddy currents,
- external temperature stability etc.

A possible solution is to use the numerical integration of particle trajectories in a 3D field. The particle motion in an electromagnetic field is generally described with the Newton – Lorentz equation [Molokovsky and Sushkov, 2005]. We will reduce consideration to purely magnetic field, i.e. at $\vec{E} = \vec{0}$:

$$\frac{d\vec{p}}{dt} = e[\vec{v} \times \vec{B}] \quad (2)$$

Let us introduce an independent variable for the path length of a specific particle: $s(t) = |\vec{v}|t$. Then (2) can be rewritten as:

$$\frac{d\vec{p}}{dt} \frac{ds}{ds} = \frac{d\vec{p}}{ds} \frac{ds}{dt} = \frac{d\vec{p}}{ds} |\vec{v}| = e[\vec{v} \times \vec{B}] \quad (3)$$

After the simple transformation (3) yields:

$$\frac{d\vec{p}}{ds} = e \left[\frac{m\vec{v}}{m|\vec{v}|} \times \vec{B} \right] = e \left[\frac{\vec{p}}{|\vec{p}|} \times \vec{B} \right] \quad (4)$$

In addition,

$$\frac{\vec{p}}{|\vec{p}|} = \frac{\vec{v}}{|\vec{v}|} = \frac{d\vec{r}/dt}{ds/dt} = \frac{d\vec{r}}{ds} \quad (5)$$

where $\vec{r} = (x, y, z)$ is the position vector.

After substituting (5) in (4), we have differentiation with respect to s :

$$\begin{cases} \frac{dp_x}{ds} = e(y'B_z - z'B_y) \\ \frac{dp_y}{ds} = e(z'B_x - x'B_z) \\ \frac{dp_z}{ds} = e(x'B_y - y'B_x) \end{cases} \quad (6)$$

We may derive that

$$\frac{d\vec{p}}{ds} = |\vec{p}| \frac{d^2\vec{r}}{ds^2}$$

then (6) can be reduced to the usual form:

$$\begin{cases} x'' = \frac{e}{|\vec{p}|} (y'B_z - z'B_y) \\ y'' = \frac{e}{|\vec{p}|} (z'B_x - x'B_z) \\ z'' = \frac{e}{|\vec{p}|} (x'B_y - y'B_x) \end{cases}$$

Then, the particle coordinates and slope angles (x, y, z, x', y', z') define its phase space, where $\frac{d\vec{r}}{ds} = \sin \varphi \approx \varphi$ at small angles.

To integrate the resultant equations, we use the adaptive Runge – Kutta method (RK) in the Dormand – Prince development [Dormand and Prince, 1980]. The solution is calculated iteratively to the fifth-order accuracy. The fourth-order accuracy is used for comparison and error estimation.

To simplify description of a particle motion in the 3D field, we introduce local coordinate systems related to the end points of the reference orbit as shown in Figure 7. Thus, we separate the particle position in global coordinates and its local deviations from the reference orbit at the exit.

This approach was implemented in the Python language and presented at [Knyazev, 2025].

3.3 Field map interpolation

To correctly process simulated 3D field maps for the trajectory analysis, data interpolation is necessary. To select the optimal interpolation, we compared 3 methods: the linear interpolation, cubic spline interpolation, and nearest neighbor interpolation [Burden et al., 2016]. The mean absolute error (7), mean squared error (8), and mean absolute percentage error (9) were used for valuation:

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i^{true} - y_i^{pred}| \quad (7)$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i^{true} - y_i^{pred})^2 \quad (8)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{y_i^{true} - y_i^{pred}}{y_i^{true}} \right| \cdot 100\% \quad (9)$$

The SBM 3D field map was taken as reference. Some data were used to find interpolation coefficients, while the remaining data were used to evaluate the interpolation. The comparative results are presented in Table 1. The linear interpolation was chosen to proceed with the trajectory calculation as the best compromise on the desired accuracy and computational cost.

Table 1. Comparison of interpolation methods

Interpolation	MAE	MSE	MAPE	Time, s
Linear	2e-04	3e-07	8e-04	108.7
Cubic spline	3e-05	1e-08	1e-04	240.29
Nearest neighbor	3e-03	6e-05	2e-02	166.57

4 Numerical experiments

The described approach was validated in comparative simulations of the electron trajectories in the SILVA storage ring. Parameters of the SBM and DL1 dipoles were estimated using MAD-X computations based on the matrix method and trajectory computations with 3D field maps using numerical integration.

The MAD-X results are described in detail in [Knyazev et al., 2025; Kazachenko et al., 2025], in this paper they are presented briefly in Table 2 (Appendix B). The initial data were specified as a set of particles forming a boundary ellipse in the $X-X'$ phase plane. At this stage we simulated the influence of magnetic fields on particle motion only, without radiation or other effects.

The storage ring consists of 40 identical superperiods, each including 16 quadrupole magnets, 4 dipole-quadrupole magnets, one SBM, and two DL1 magnets. Correctors and radio frequency resonators were excluded from consideration.

The following algorithm was adopted to evaluate the beam optics.

1. Find a new equilibrium orbit and calculate the beam deviations from it with a 3D field distribution taken into account.
2. Evaluate the beta function and dispersion as key factors for the beam quality.
3. Calculate the betatron tune.

4.1 Trajectory analysis of SBM

Let us investigate how the beam behaves with allowance for a 3D field distribution in SBM using two models described in Section 2. Deviations of the equilibrium orbit from the reference one are presented in Figure 9. As seen, the 98.67% field model, which is closer to the ideal field, gives almost twice less deviation, than the 97.12% model.

Figure 10 illustrates the beam phase portrait obtained in the plane $X-X'$ with Model 1. As only the field map was taken into account, the boundary ellipse keeps its

shape unchanged according to Liouville's theorem [Wolski, 2014]. Since the equilibrium orbit changes when the realistic 3D field distribution is considered, the boundary ellipse originally defined relative to the reference trajectory becomes shifted with respect to the new closed orbit. The reference trajectory coincides with the equilibrium orbit only for idealized magnetic fields.

Therefore, the beam position and angle at injection should be adjusted in order to reduce the maximum particle deviations. At this stage, such an adjustment is treated as a numerical matching procedure.

In further studies, a more realistic closed orbit correction scheme should also be included. In particular, corrector magnets distributed along the storage ring may compensate for part of the closed orbit distortion caused by deviations of the actual dipole field profile from the design one. This would allow one to separate correctable orbit distortions from magnetic field deviations that require refinement of the dipole design itself.

The beam parameters are estimated in terms of the beta function (10) and dispersion (11), as well as the betatron tune.

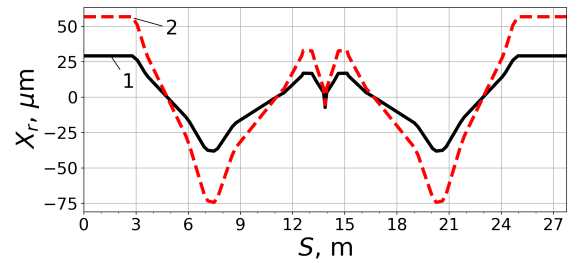


Figure 9. Horizontal coordinate of the equilibrium orbit obtained with the 3D SBM field map in local coordinate systems. Curve 1 is for Model 1, curve 2 is for Model 2.

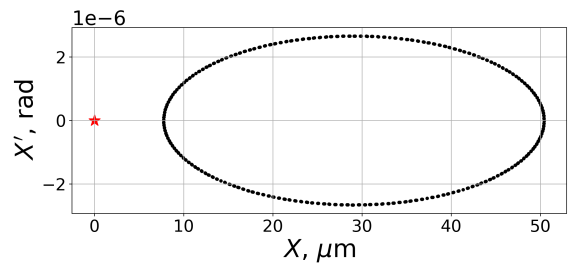


Figure 10. Beam phase portrait in $X-X'$ plane with Model 1 of SBM field.

$$\beta_x(s) = \frac{\Delta x_{max}^2}{\varepsilon_x} \quad (10)$$

$$D_x(s) = \frac{\partial x_r(s)}{\partial p} \quad (11)$$

Here Δx_{max} is the maximal horizontal deviation from the equilibrium orbit; ε_x is the horizontal emittance

treated as the ellipse area; x_r is the coordinate x on the equilibrium orbit.

The horizontal beta function and dispersion are presented in Figure 11 and Figure 12. For both SBM field models they are almost identical to each other and to the idealized model.

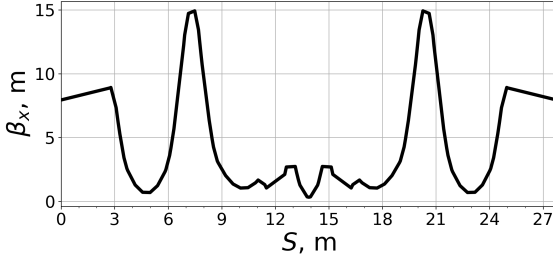


Figure 11. Horizontal β -function with respect to SBM field map.

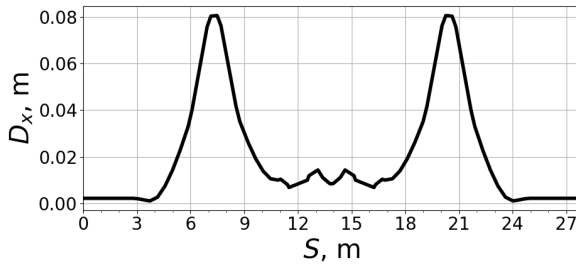


Figure 12. Horizontal dispersion with respect to SBM field map.

Thus, the considered SBM field models do not provide the required parameters. The equilibrium orbit deviates at the boundaries of the superperiod by an amount comparable to the beam size ($\sim 30 \mu\text{m}$ for Model 1). This can significantly affect radiation analysis and particle tracking. A 1.66% increase in the SBM field would reduce the deviations to near zero ($0.07 \mu\text{m}$).

4.2 Trajectory analysis of DL1

A similar numerical experiment was performed for DL1. Again, other components were described with the idealized models. The deviations of the closed orbit were about $170 \mu\text{m}$ at the superperiod boundaries and up to $80 \mu\text{m}$ inside the magnet, as shown in Figure 13. The β -function and dispersion were in qualitative agreement with those for the SBM models.

The horizontal betatron tune was estimated as 2.4075 on the superperiod and 96.3 on the ring.

As seen, the DL1 field model that takes into account the 3D field map demonstrates that required beam parameters are also not achieved. In this case, the deviation of the equilibrium orbit is significantly greater than the beam size. To correct it, the field should be increased by 0.27%. This will reduce the deviations at the superperiod boundaries to near zero ($0.001 \mu\text{m}$).

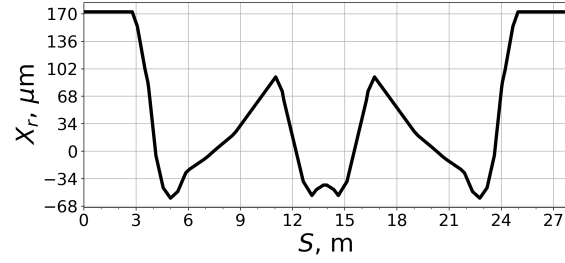


Figure 13. Horizontal coordinate of the equilibrium orbit obtained with the 3D DL1 field map in local coordinate systems.

4.3 Analysis of combined effect of DL1 and SBM

We also studied a combined effect of the 3D field distribution on the beam parameters in both SBM and DL1. For evaluation we took the SBM Model 1. Figures 9, 13, 14 reveal that the equilibrium orbit would deviate by a total $200 \mu\text{m}$ at the boundaries of the superperiod, by $60 \mu\text{m}$ in DL1, and $50 \mu\text{m}$ in SBM. The β -function and dispersion also match the ideal model (Figures 16 and 17, Appendix C). The horizontal betatron tune with respect to the 3D field of both magnets is expected to be 2.4065 on the superperiod and 96.259 on the ring.

These evaluations support the previous recommendations: to bring deviations of the closed orbit down to zero it is necessary to increase the SBM field by 1.66% and the DL1 field by 0.27%.

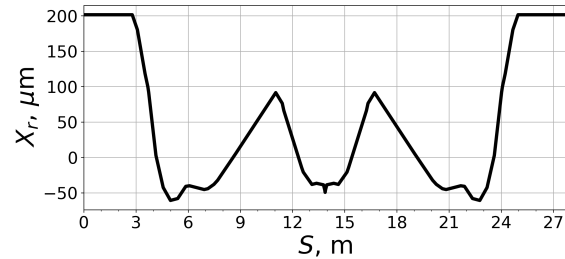


Figure 14. Horizontal coordinate of the equilibrium orbit obtained with the 3D DL1 and SBM field maps in local coordinate systems.

5 Comparison of simulations

The magnet models were validated in comparative computations with the developed software utilizing the RK integration of motion equations and the MAD-X (M) simulation based on the matrix method. As key parameters we selected the relative errors in the β -function and dispersion (12), the horizontal betatron tune in the superperiod, and the deviation of the equilibrium orbit at the boundaries of the superperiod.

$$\begin{aligned}\epsilon(\beta_x) &= \frac{|\beta_x^{ideal} - \beta_x|}{\beta_x^{ideal}} \cdot 100\% \\ \epsilon(D_x) &= \frac{|D_x^{ideal} - D_x|}{D_x^{ideal}} \cdot 100\%\end{aligned}\quad (12)$$

Results of the comparison are presented in Table 2 (Appendix B).

The comparison suggests that the proposed approaches give identical results for idealized models. The estimates of the equilibrium orbit shift and the tune under the influence of SBM and DL1 magnets also agree. The deviations in the β -function are caused by numerical errors in the integration and approximation. In contrast, the differences in the equilibrium orbit position and in the dispersion are caused by the 3D magnetic field model.

6 Conclusion

The performed study has yielded the following results.

- The methodology has been proposed for validation of the dipole magnet design using 3D field maps.
- An in-house software package for simulation of particle dynamics in 3D magnetic fields has been developed and validated using independent computations.
- Electron trajectories in the storage ring of the SILA facility have been simulated with respect to the magnetic field models for the SBM and DL1 dipoles.
- The influence of SBM and DL1 dipoles on the beam parameters has been evaluated, and recommendations for the magnet designs have been refined: specifically, to increase the SBM field by 1.66% and the DL1 field by 0.27%.

7 Acknowledgements

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Appendix A Superperiod structure of the SILA facility ring

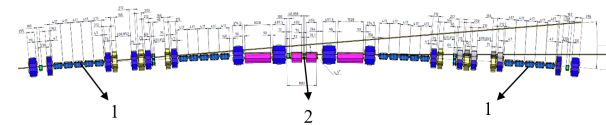


Figure 15. The regular superperiod structure of the storage ring at the SILA facility. 1 – long dipole DL1, 2 – short dipole SBM.

Appendix B Comparison of beam dynamics simulations

Table 2. Comparison of beam dynamics simulations

Field model	Method	$\epsilon(\beta_x)$, %	$\epsilon(D_x)$, %	Q_x	X_r , μm
Idealized model	RK	0.16	0.003	2.4079	0
	M	0	0	2.4079	0
SBM 1	RK	1.19	8.193	2.4069	29.09
	M	0	6.323	2.4079	29
SBM 2	RK	2.23	16.042	2.4059	56.71
	M	0	12.349	2.4079	56.7
DL1	RK	1.29	16.877	2.4075	172.3
	M	0	34.932	2.4079	170.5
DL1 & SBM	RK	1.63	10.332	2.4065	201.4
	M	0	30.61	2.4079	200.7

Appendix C Beta-function and dispersion for combined effect of DL1 and SBM

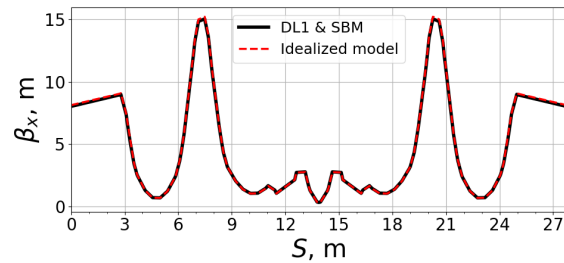


Figure 16. Horizontal β -function with respect to DL1 and SBM field maps.

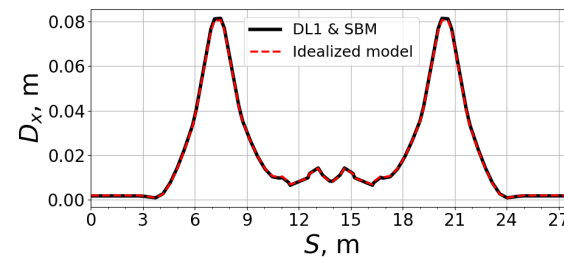


Figure 17. Horizontal dispersion with respect to DL1 and SBM field maps.