# CONTROL AND TRANSFER OF ENTANGLEMENT IN A SYSTEM OF COUPLED MICRO-TOROIDAL RESONATORS

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**Abstract**

Quantum systems that comprising atoms coupled to optical cavities have been normally used for quantum information processing. Due to improvement of the experimental control under such systems, new architectures involving one or two atoms and cavities with two modes have been proposed. The entanglement is here the fundamental quantum feature which plays an important role in quantum processing. In this work, we investigated dynamical entanglement of quantum states transfer of the coupled atoms with micro-toroidal cavities. Each cavity supports two counter-propagating whispering-gallery modes coupled simultaneously an atom through their evanescent fields. We show that, it is possible to transfer with high fidelity a superposition state of one atom which is coupled to a micro-toroidal cavity to the another atom which is coupled to a second micro-toroidal cavity. Normally this kind of propagating medium has high quality factor however they can have some small deformation that can also introduce coupling between the gallery modes and in that situation generating entanglement between them. We also observed that, is possible transfer this entangled states between the cavities. In addition, the influence of the coupling between the modes in dynamics of entanglement between the two atoms is analysed.

**Key words**

Entanglement, quantum processing, micro-toroidal cavity

## 1 Introduction

The quantum networks usually consist of distant nodes connected by quantum communication channels. Each nodes can process, store and distribute the quantum information under the network via reversible and irreversible processes channels [Boozer, Boca, Kimble, 2007]. Numerous proposals have been realized - both experimental and theoretical - for implementation of the atoms coupled to optical cavities to become the nodes that make up a quantum network [Raimond, Brune, Haroche, 2001]. In this case, coupled optical cavities can be implemented as quantum channels [Cirac, Zoller, Kimble, 2009]. Different architectures of cavities have been developed (micro-fabricated) due to need for ultrahigh factors ($Q$) and scalability to large number of devices [Kimble, 2008]. For this propose, micro-toroidal or micro-spherical resonators have present a technical feature to achieve efficient optical communications, such as, small-mode-volume, ultra-high quality factors and strong coupling between an atomic system and the cavity electromagnetic mode [Spillane, Kippenberg, Painter, Vahala, 2003]. Therefore it would be interesting to extend their research to consider the dynamics of quantum state transfer in a system formed by two-level atoms coupled to a micro-toroidal cavities.

Here we investigated the dynamics of the two coupled micro-toroidal cavities via the evanescent field of two intracavity modes and where each of them is coupled to a single two-level atom. Our main result consist in transfer a superposition state of one atom which is coupled to a cavity for the other atom which is coupled to another cavity with high fidelity, taking into account mechanism of the system losses. The possibility of the coupled between the intracavity modes (generated for imperfection in cavity) for the dynamical entanglement between the atoms is also studied.

## 2 Theory

Our system consist of two coupled micro-toroids interacting with two-level atoms shown in Fig. 1. The micro-toroids and atoms are depicted by label $i = 1, 2$. The two degenerate counter-propagating whispering-gallery modes (WGM's) of frequency $\omega_{Ci}$ , with annihilation (creation) operators $\hat{a}_i$ $(\hat{a}_i^\dagger)$ and $\hat{b}_i$ $(\hat{b}_i^\dagger)$ of each one of the cavities, are coupled simultaneously to a single two-level atom with coupling constant $g_i$ and transition frequency $\omega_{eg}^{i}$ . We assume that the interaction of the atoms and toroid with the surrounding environment is described by spontaneous emission rate of the atoms ($\gamma_A$) and cavity decay rate ($\kappa$). We also consider the intermode backscattering between the two WGM's (strength constant $J_i$) induced by small deformation in the toroid [Spillane, Kippenberg, Painter, Vahala, 2003].

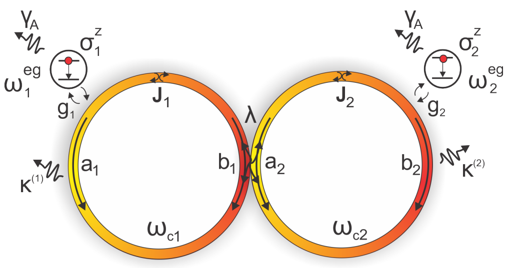


Figure 1. Scheme experimental of the two atoms- two microtoroidal cavity system. Each cavity consists of two modes coupled to a two-level atom.

According to the above scheme, the Hamiltonian of the atom-microtoroid system is given by

$$H = H_{A1C1} + H_{A1C2} + H_{C1C2} \tag{1}$$

where

$$\begin{aligned} H_{A1C1} &= \hbar\omega_1^{eg}\sigma_1^{+}\sigma_1^{-} + \hbar\omega_{C1}(\hat{a}_1^{\dagger}\hat{a}_1 + \hat{b}_1^{\dagger}\hat{b}_1) + \\ &\quad \hbar J_1(\hat{a}_1^{\dagger}\hat{b}_1 + \hat{b}_1^{\dagger}\hat{a}_1) + \hbar(g_1^{*}\hat{a}_1^{\dagger}\sigma_1^{-} + g_1\hat{a}_1\sigma_1^{\dagger}) + \\ &\quad \hbar(g_1\hat{b}_1^{\dagger}\sigma_1^{-} + g_1^{*}\hat{b}_1\sigma_1^{\dagger}), \qquad (2a) \\ H_{A2C2} &= \hbar\omega_2^{eg}\sigma_2^{+}\sigma_2^{-} + \hbar\omega_{C2}(\hat{a}_2^{\dagger}\hat{a}_2 + \hat{b}_2^{\dagger}\hat{b}_2) + \\ &\quad \hbar J_2(\hat{a}_2^{\dagger}\hat{b}_2 + \hat{b}_2^{\dagger}\hat{a}_2) + \hbar(g_2^{*}\hat{a}_2^{\dagger}\sigma_2^{-} + g_2\hat{a}_2\sigma_2^{\dagger}) + \\ &\quad \hbar(g_2\hat{b}_2^{\dagger}\sigma_2^{-} + g_2^{*}\hat{b}_2\sigma_2^{\dagger}), \qquad (2b) \\ H_{C1C2} &= \hbar\lambda(e^{-i\phi}\hat{a}_1^{\dagger}\hat{b}_2 + e^{i\phi}\hat{b}_2^{\dagger}\hat{a}_1 + e^{-i\phi}\hat{b}_1^{\dagger}\hat{a}_2 + \\ &\quad e^{i\phi}\hat{a}_2^{\dagger}\hat{b}_1) \qquad (2c) \end{aligned}$$

with $\hbar\omega_i^{eg}$ denotes the energy required of separation between of the atom $i$ ($i = 1, 2$) for excited and ground states by $|e\rangle_i$ and $|g\rangle_i$, $\sigma_i^{+} = |e\rangle_i\langle g|$ and $\sigma_i^{-} = |g\rangle_i\langle e|$ are the raising and lowering operators of the atom $i$, $\lambda$ is the coupling constant between the two micro-toroid and determine the speed of the energy transfer between them [Zhou, Zheng-Yuan Xue, 2014]. The phase $\phi$ take into account the propagation distance between the micro-toroids. For neglect effects of the retardation at time of flight of the light, a short distance limit between the toroids should be imposed.

In the Eqs.(2) $H_{A1C1}$ and $H_{A2C2}$ describe the first and second atom-toroid interacting systems, respectively, and $H_{C1C2}$ describes the coupling between the toroids.

To get information about the evolution of the system state considering the interaction of system with the surrounding environment, which in practice introduce loss mechanisms, we need help of the master equation (that for the case where the reservoir is in the temperature $T = 0$ and weak coupling) is written as [Carmichael, 1993]

$$\begin{aligned} \frac{d}{dt}\rho(t) &= -\frac{i}{\hbar}[H, \rho(t)] + \\ &\quad \sum_{i=1}^{2}\frac{\kappa_i}{2}(2\hat{a}_i\rho(t)\hat{a}_i^{\dagger} - \hat{a}_i^{\dagger}\hat{a}_i\rho(t) - \rho(t)\hat{a}_i^{\dagger}\hat{a}_i) + \\ &\quad \sum_{i=1}^{2}\frac{\kappa_i}{2}(2\hat{b}_i\rho(t)\hat{b}_i^{\dagger} - \hat{b}_i^{\dagger}\hat{b}_i\rho(t) - \rho(t)\hat{b}_i^{\dagger}\hat{b}_i) + \\ &\quad \sum_{i=1}^{2}\frac{\gamma_A}{2}(2\sigma_{-}^{(i)}\rho(t)\sigma_{+}^{(i)} - \sigma_{+}^{(i)}\sigma_{-}^{(i)}\rho(t) - \\ &\quad \rho(t)\sigma_{+}^{(i)}\sigma_{-}^{(i)}) \end{aligned} \tag{3}$$

where $\rho(t)$ is the density operator of the atom-microtoroid systems and $H$ is given by Eq. (1). Having in mind the quantum state processing our main objective here is the quantum state transfering between the two atoms, following the quantum transmission between two qubit as defined by [Cirac, Zoller, Kimble, 2009], i.e.,

$$(c_a|1\rangle_1 + c_b|0\rangle_1) \otimes |0\rangle_2 \Rightarrow |0\rangle_1 \otimes (c_a|1\rangle_2 + c_b|0\rangle_2)$$

where $c_a$ and $c_b$ are complex numbers. We can understanding the quantum state transmission process matching tomographically the evolved state of the system or subsystem on the initial state, such as, a subsystem having the first qubit in a superposition state (in Fock bases) and the second in vacuum state, and at certain elapsed time $t$ the first qubit of subsystem is projected on the vacuum state and second in a superposition state, performing a perfect and complete transmission. In our case, preparing the initial state $|\psi_i\rangle = (\cos\theta|g\rangle_1 + e^{i\alpha}\sin\theta|e\rangle_1)|g\rangle_2|00\rangle_{c1}|00\rangle_{c2}$ with $|00\rangle_{c1} = |0\rangle_{a1} \otimes |0\rangle_{b1}$ and $|00\rangle_{c2} = |0\rangle_{a2} \otimes |0\rangle_{b2}$ the goal is to obtain the final state $|\psi_f\rangle = |g\rangle_1(\cos\theta|g\rangle_2 + e^{i\alpha'}\sin\theta|e\rangle_2)|00\rangle_{c1}|00\rangle_{c2}$. Following the time evolution of the quantum state it is possible find the exact time when the state of the atom 1 is completely transferred to the atom 2. We have done this using the Fidelity as defined by [Nielsen, Chuang, 2000]

$$F = \langle\psi_f|\hat{\rho}_{A1A2}(t)|\psi_f\rangle \tag{4}$$

where $\hat{\rho}_{A1A2}(t)$ is the reduced density matrix of the two atoms, which takes into account all mechanisms of losses considered. In order, to observe the dynamics of state transfer and degree of entanglement between the atoms, we also have used the Negativity, as proposed by [Vidal, Werner ,2002]

$$N = \sum_i |\mu_i^-| \tag{5}$$

where $\mu^-$ are the negative eigenvalues of $\rho_{A1A2}(t)$. When $N = 0$ indicates that the states of each atoms are separable and for $N = 1$ the two atoms are in a maximum entanglement state. This measure is important to certify that at instant of time of complete quantum state transfer, Fidelity will be equal to one, the Negativity must be zero (indicating that the initial product state reach after transference a final product state, having at this time a maximum Fidelity and Negativity null (product state or separable state).

## 3 Quantum state transfer

In this section we present results about the dynamics of the quantum state transfer in a system of two coupled resonators including the presence of losses. Firstly, we examine the implementation of a swap gate between the two atoms (analogous the swap gate two-qubit) observing time evolution of the Fidelity. Then, we extend our investigation for the case of a transference of superposition state from atom 1 to atom 2, when the modes of the resonators are in vacuum state. The influence of the dissipation effects and the dynamics entanglement between the two atoms are also considered. Besides, we also consider in our scheme the transference of entangled state.

### 3.1 Swap gate

Quantum computation require the successful implementation of the quantum gates. In this way, we describe as a swap gate can be applied in our scheme, making use of the fidelity. For this purpose, we will use in the eq.(4) the initial state of the system, not the final state like there. The new equation is now represented by

$$F_i = \langle\psi_i|\hat{\rho}_{A1A2}(t)|\psi_i\rangle. \tag{6}$$

We can observe in Fig. 2 that the function $F_i$ at instant of time $\tau_n = 2n\frac{\pi}{\sqrt{2}}$ $(n = 0, 1, 2, ...)$ reaches the value unitary for any value of $\theta$. This is interesting because the system periodically returns to the initial state (reversible processes), behaving as a swap gate between the two atoms, with repetition period of $2\pi/\sqrt{2}$. In this case, we assume that there is no coupling of the system with the environment.

### 3.2 Lossless quantum state transfer

Now, we study the possibility of the transferring of quantum state for two coupled micro-toroidal cavities via evanescent field, where each of them is coupled to a single two-level atom. For sake of simplify, we imposed two conditions on that system: (i) $\lambda \gg g, J$ (strong coupling regime) and (ii) the resonant case, i.e., the frequency of the two WGM's is equal the frequency of the atomic transition ($\omega_{Ci} = \omega_{eg}^i = \omega$). For this case, considering the initial state $\psi_i$ and we obtain the following solution for state of the system in the Schödinger picture

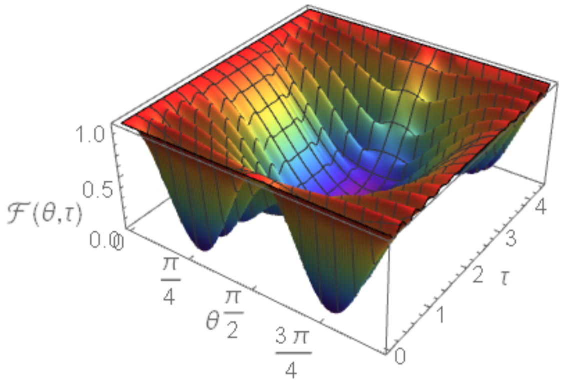


Figure 2. Time evolution of the fidelity related to state $|\psi_i\rangle$ as a function of normalize time ($\tau = gt$) and $\theta$. The results were obtained for $\omega/g = 20$ and $\alpha = 0$.

$$\begin{aligned}|\psi(t)\rangle = {} & [\cos\theta e^{\frac{i\omega}{g}\tau}|g\rangle_1|g\rangle_2 - \\ & e^{i\alpha}\sin^2(\frac{\tau}{\sqrt{2}})\sin\theta|g\rangle_1|g\rangle_2 + e^{i\alpha}[ \\ & \cos^2(\frac{\tau}{\sqrt{2}})\sin\theta|e\rangle_1|g\rangle_2]|00\rangle_{c1}|00\rangle_{c2}] + \frac{i}{\sqrt{2}}[ \\ & e^{i\alpha}\sin(\sqrt{2}\tau)\sin\theta|g\rangle_1|g\rangle_2|00\rangle_{c1}|01\rangle_{c2}]\end{aligned} \tag{7}$$

In the instant of time $\tau_n = \frac{(2n+1)\pi}{\sqrt{2}}$ $(n = 0, 1, 2...)$ the state of the system is given by:

$$|\tilde{\psi}_1\rangle = |g\rangle_1[\cos\theta|g\rangle_2 + e^{-i\alpha'}\sin\theta|e\rangle_2]|00\rangle_{c1}|00\rangle_{c2} \tag{8}$$

where $\alpha' = \alpha - \frac{(2n+1)\pi}{\sqrt{2}}\frac{\omega}{g}$.

Under the condition $\omega/g = 2l$ (l integer), the state of the atom 2 at the instant of time $\tau_n$ is exactly same atom 1 superposition state, i.e., the atomic state was completely transferred from the atom 1 to the atom 2. To observe the temporal evolution of system in the process of transferring this state, without dissipation mechanism, we using eq.(3) from which we have obtained the fidelity as in function of the parameters $\theta$ and $\tau$, that is shown in Fig. 3. As expected, the state is completely transferred at $\tau = \pi/\sqrt{2}$ $(F = 1)$, for any value of angle $\theta$. We also observe that, the Fidelity present some oscillations before reaching the value unit, which depend of the ratio $\omega/g$.

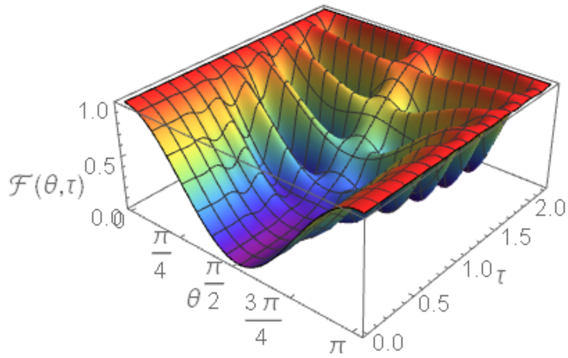


Figure 3. Time evolution of the fidelity related to state $|\psi_f\rangle$ as a function of normalize time ($\tau = gt$) and $\theta$. The results were obtained for $\omega/g = 20$ and $\alpha = 0$.

### 3.3 Transfer in the presence of losses

From an experimental point of view to transfer quantum state efficiently under realistic conditions, one must take into account the presence losses, e.g., spontaneous emission of the atoms ($\gamma_A$) and decays of the two resonators ($\kappa_1$ and $\kappa_2$). In this case, we using the eq. (3) for estimate the performance of our scheme in quantum state transfer. In the Fig. 4 (Top) is plotted the Fidelity (first maximum) as function of normalize time $\tau$ for fixed values of the angle $\theta = \pi/4$ and different process and values of losses. The solid line (color black) represents the case when the system is coupled weakly with the surround environment, $\kappa = \gamma = 2 \times 10^{-3}g$, indicating that the transmission is reliable, in this regime. The dotted line (color red) represent the case of intermediate coupling with environment, when $\kappa = 2 \times 10^{-1}g$ and $\gamma = 1 \times 10^{-2}g$, reaching a maximum transmission of $F = 0.957$ indicating still, an efficient transmission. The dashed line illustrate the case of dissipation more intense when the system is coupled strongly with environment, as we can seen with $\kappa = \gamma = g$ (color blue), the transmission Fidelity reach at $F = 0.652$, indicating that the quantum state transfer is inefficient. In the Fig. 4 (Botton) is plotted the Fidelity (first maximum) only for the central peak to show that the Fidelity is exactly one in the neighborhood of $\tau = 2.20$, indicating maximum transference of the initial state.

In order to confirm the complete transference of the superposition state, we observe the dynamics of entanglement between the atoms using, as witness of perfect transference, the negativity for initial state $|\psi\rangle_i$. As shown in the Fig. 5, when the negativity is zero ($N = 0$ at time $\tau = \pi/\sqrt{2}$), corresponding to the situation where the atoms are in a separate state occur the maximum quantum state transfer (see Fig. 3), meaning that we have complete transference of the atomic state of the atom 1 to the atom 2. At this instant of time the state $|\psi_f\rangle$ is a separate state, in accordance with a negativity $= 0$, as expected. This allows us use projective measurement over one atomic state without disturb the state of the other atom, indicating that this system could be used for processing quantum information.

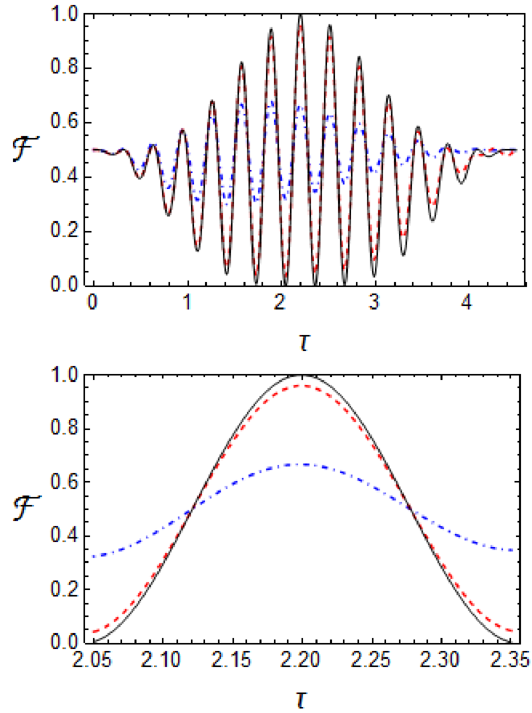


Figure 4. (Upper) Time evolution of transmission fidelity related to state $|\psi_f\rangle$ as a function of normalize time ($\tau = gt$) for $\theta = \pi/4$. The solid line (color black) is for $\kappa = \gamma = 2 \times 10^{-3}g$, doted line (color red) for $\kappa = 2 \times 10^{-1}g$ and $\gamma = 1 \times 10^{-2}g$ and dashed line (color blue) for $\kappa = \gamma = g$ The results present were obtained with $\omega/g = 20$ and $\alpha = 0$. (Lower) Zoom for range $2.05 < \tau < 2.35$ with same parameters above.

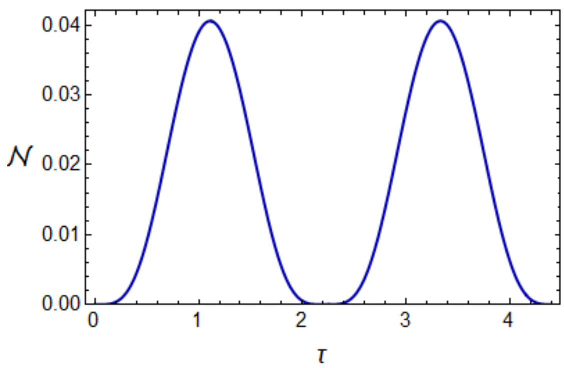


Figure 5. Time evolution of the negativity as a function of normalize time ($\tau = gt$) when the pair of atoms is initially prepared in state $|\psi_i\rangle$ for $\theta = \pi/4$ . The result present were obtained with $\omega/g = 20$ and $\alpha = 0$.

Besides the possibility of transfer an atomic superpostion state we note that our system support also the transference of a quantum entangled state. In such a case, the atom 1 is initially prepared in a superposition state and the modes of resonator 1 is in maximality entangled state and other parts of the system are in their fundamental states, e.g., $|\psi_i^{(2)}\rangle = (\cos\theta|g\rangle_1 + e^{i\alpha'}\sin\theta|e\rangle_1)|g\rangle_2\frac{1}{\sqrt{2}}(|10\rangle_{c1} + |01\rangle_{c1})|00\rangle_{c2}$. The goal here is transferring the superposition state from atom 1 to atom 2 and the entangled state from resonator 1 to resonator 2. Then, again, we observed the evolution of the fidelity as function of normalize time ($\tau$) for $\theta = \pi/4$. This result is shown in the Fig. 6. From this figure we can see that, at time $\tau_n = \frac{(2n+1)\pi}{\sqrt{6}}$ ($n = 0, 1, 2...$) the quantum entangled state is completely transferred, showing more efficiency (decrease in transfer time) for quantum information processing. This result is equivalent to the transfer of the entangled state of two qubits [Nohama, Roversi, 2008], only that in the present case instead of the vibrational-electronic state we are performing the transmission of the entangled state between the modes of the resonator 1 to the resonator 2.

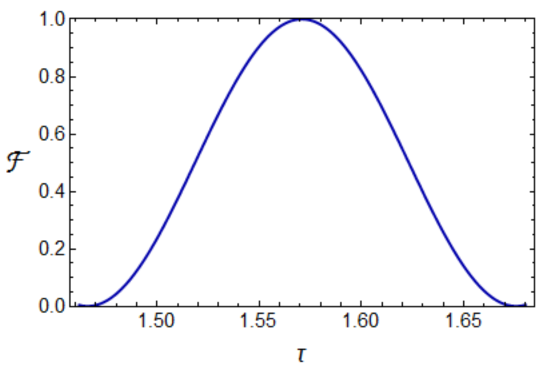


Figure 6. Evolution of the fidelity as a function of normalize time ($\tau = gt$) for $\theta = \pi/4$, related to the state $|\psi_f\rangle = |g\rangle_1(\cos\theta|g\rangle_2 + e^{i\alpha'}\sin\theta|e\rangle_2)|00\rangle_{c1}\frac{1}{\sqrt{2}}(|10\rangle_{c2} + |01\rangle_{c2})$. The results present were obtained with $\omega/g = 20$ and $\alpha = 0$.

## 4 Conclusion

We explored a system formed by two microtoroidal cavities coupled by evanescent field, where each cavity interacts with a single two-level atom. It was observed that the set of quantum state (separate and entangled) is completely transferred from atom 1 coupled with the resonator 1 to atom 2 coupled with resonator 2. Even under the influence of interaction between the system and environment (reservoir at temperature $T = 0$) the transference could be done with high efficiency ($F = 0.957$). The system of two coupled microtoroidal cavities shown that the period of separable state (negativity null) during the transference is relatively large allowing quantum information processing without disturb the states of others subsystems involved in the process. it is also important to say that the transference of the quantum state had shown more efficient when the initial state is an entangled state.


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