ROBUST CONTROL FOR MULTIPLY CONNECTED SYSTEMS WITH STATE DELAY

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Abstract
The problem of a robust control system design for interconnected systems with structural and parametrical uncertainty was solved for the case where derivatives of input and output parameters cannot be measured. The order of the mathematical model may change over time. Operability of the designed control systems in the case of non-measurable and bounded disturbances acting on the controlled plant was demonstrated. Only the measurable variables of the local subsystems are used to generate the control actions, that is, control is completely decentralized.

1 Introduction

The problem of control with scalar input and output has become one of the classical problems of modern control theory and plenty of methods for robust control design have been developed.. The two key methods of their compensation for the disturbances are given. Under the first approach, the structure and parameters of the controlling systems are chosen in such a way that they would provide insensitivity of the system to disturbance (invariant systems). The second approach is based on a dynamic compensation of internal and external disturbances, when the control of adjusting a device suppresses the influence of disturbances on the parameters of the system.

In [Nikiforov, 2004] an internal model of disturbances is used to solve the problem whereas [Miroshnik, Nikiforov and Fradkov, 1999] use the methods of the theory of robust and adaptive systems. Robust systems with compensation of disturbances that use these methods are studied in [Bobtsov, 2003]

A simple robust control algorithm that remains the same for various types of plants is proposed in [Tsykunov, 2007]. It is shown that the algorithm compensates for parametric and external disturbances with a given accuracy. A closed-loop system works here as an implicitly given nominal model whose parameters are used in control.

It is important to note that almost all the suggested methods are based on an assumption that the structure of a plant is known i.e. the order of a system of differential equations is known and parametric and external disturbances are unknown. At the same time, there are practically no works that are various studies devoted to the problems of control systems with an unknown order pants. Sources [Tao and Ioannou, 1993], [Hoang and Bernstein, 2007] consider control problems of LTI systems with an unknown and constant order of numerator and denominator for their transfer functions. In monograph [Furtat, Parsheva and Tsykunov, 2011] considers a wider class of systems with disturbances that are able to influence both the parameters of the system as well as its order.

Nowadays the problems of developing control systems for various dynamic plants with state delay are still remaining relevant and in-demand for modern control theory [Tsykunov, 2014]. This circumstance is due to the fact that similar automatic systems are very increasingly used on various practical solutions. For example, control of models with delay may be occurring in: systems for manipulation robots [De Oliveira and Lages, 2016], some aircraft tracking systems [He, Guo, and Leang, 2017], systems for voltage converters [Li and Ye, 2018] and others.

It should be noted that often control plants are complex multiply connected systems, the development of regulators for which requires special approaches. One such approach is the decentralized control method, in which the original plant is decomposed into several interconnected subsystems. And then for each local subsystem synthesis of control algorithms is carried out [Zhu and Pagilla, 2007], [Dragicevic, Wu, Shafiee, and Meng, 2017], [Shukla and Mili, 2017].

Decentralized control can be used for a wide rage of large-scale complex systems including satellite networks, group flights [Tavakol and Binazadeh, 2017], electric power systems [Cho, Kato, Spilman, 1993], robots [Pshikhopov and Medvedev, 2011] etc. Decentralized control is also very efficient when there is a need to design the control algorithms relying on local information. Modern computer networks provide an efficient infrastructure for a real implementation of such algorithms.

Current paper considers robust control for interconnected systems with unknown parameters which are subject to be influenced by external and parametrical uncontrolled disturbances. The article deals with the synthesis problem of control system regulator for multivariable dynamic plant with state delay, with the help of results obtained in [Tsykunov, 2014], [Parsheva, 2009]. Mentioned disturbances influence on plant order in unpredictable way. It means that order of plant is unknown and only scalar input and output signals can be measured. To solve concerned problem it is suggested to use simple robust control algorithm that allows to compensate this class of uncertainties with given exactitude for appropriate time. Only measured variables of the local subsystems are used for control action formation and in adjustment algorithms, i.e. completely decentralized control is exercised.

2 Problem Statement

Let us consider an interconnected system whose local subsystems' dynamic processes are described by the following equations

$$Q_i(P,t)y_i(t)+G_i(P,t)y_i(t-\tau_i)=k_i(t)R_i(P,t)u_i(t)+ + f_i(t)+\sum_{j=1}^{M}S_{ij}(P,t)y_j(t),\quad i\neq j,\quad i=\overline{1,M}, \quad (1)$$

where $P=d/dt$ – differential operator;

$$Q_i(P,t)=q_{n_i i}(t)P^{n_i}+q_{(n_i-1)i}(t)P^{n_i-1}+\ldots+q_{0i}(t),$$

$$G_i(P,t)=g_{(n_i-1)i}(t)P^{n_i-1}+\ldots+g_{0i}(t),$$

$$k_i(t)R_i(P,t)=r_{m_i i}(t)P^{m_i}+r_{(m_i-1)i}(t)P^{m_i-1}+\ldots+r_{0i}(t),$$

$S_{ij}(P,t)=s_{n_{ij}ij}(t)P^{n_{ij}}+s_{(n_{ij}-1)ij}(t)P^{n_{ij}-1}+\ldots+s_{0ij}(t)$ – linear differential operators with unknown parameters; $f_i(t)$ – an uncontrolled disturbance; $u_i(t)$ – a scalar control action; $y_i(t)$ – a scalar controlled variable in the i-subsystem which can be measured.

Decentralized control for such a system is defined as the problem of finding M local control blocks, each of which only can access current information about a system [Mirkin, Gandelman and Tsoi, 1989]. Required quality of transition processes in a subsystem is defined by equations of the local nominal models

$$Q_{mi}(P)y_{mi}(t)=k_{mi}r_i(t),\qquad i=\overline{1,M}. \quad (2)$$

Here $Q_{mi}(P)$ are linear differential operators; $k_{mi}>0$; $r_i(t)$ are the scalar bounded control actions.

It is necessary to design a control system for which the following condition will be satisfied:

$$\overline{\lim_{t\to\infty}}|e_i(t)|=\overline{\lim_{t\to\infty}}|y_i(t)-y_{mi}(t)|<\delta \text{ if } t\geq T. \quad (3)$$

Here δ is the accuracy of the dynamic error $e_i(t)$; T is the time beyond of which the dynamic error should not exceed the value δ. It is forbidden to use measurable parameters of one subsystem in other local subsystems.

Assumptions:

i) $Q_{mi}(\lambda)$ are Hurwitz polynomials (λ – complex variable in Laplace transformation);

ii) the operator $R_i(P,t)$ is stable, i.e. trivial solution of equation $R_i(P,t)u_i(t)=0$ is asymptotically stable. For the fixed value t_1 polynomial $R_i(\lambda,t_1)$ is Hurwitz;

iii) the orders of polynomials $\deg Q_i=n_i$, $\deg R_i=m_i$, $\deg S_{ij}=n_{ij}$, $n_{ij}<n_i-1$ are unknown and relative degree of a local subsystem $\gamma_i=n_i-m_i>1$;

iv) the upper bound $\gamma_{ui}\geq\gamma_i$ of relative degree γ_i is known as well as the upper bound of the degree of the polynomial Q_i, i.e. $n_i\leq\bar{n}_i$;

v) the order of the polynomials Q_{mi} is equal to γ_{ui};

vi) the coefficients' signs $k_i(t)$ are known and $k_i(t)>0$;

vii) the operators coefficients $k_i(t)R_i(P,t), Q_i(P,t)$, $G_i(P,t), S_{ij}(P,t)$ are bounded functions; the non-zero coefficients of high orders of operators $R_i(P,t)$ and $Q_i(P,t)$ are positive functions;

viii) the coefficients of differential operators $k_i(t)R_i(P,t)$, $Q_i(P,t)$ depend on vector of unknown parameters $\xi\in\Xi$, where Ξ is a known bounded set;

ix) the actions $r_i(t)$ are bounded functions;

x) the signal of local nominal model $y_{mi}(t)$ and its derivatives γ_{ui} are bounded functions;

xi) the external disturbance $f_i(t)$ is a bounded function of time with an unknown changes range;

xii) it is prohibited to use the derivatives of signals $y_i(t),\ u_i(t),\ r_i(t)$.

Based on the assumptions it is possible to conclude that the dynamic order of the system (1) is unknown and subject to change as the result of parametric disturbances. For instance if $q_{n_i}(t)=0$ and $q_{n_i-1}(t)\neq 0$ then $\deg Q_i(P,t)=n_i-1$; if $q_{n_i}(t)=q_{n_i-1}(t)=0$ and $q_{n_i-2}(t)\neq 0$ then $\deg Q_i(P,t)=n_i-2$ etc. The requirement to know the signs of the non-zero coefficients of high orders of operators $R_i(P,t), Q_i(P,t)$ (assumption vii)) is related to knowing the sign of a high-frequency gain of the system (1).

3 Method of Solution

Let us write the operators $Q_i(P,t), R_i(P,t)$ as

$$Q_i(P,t)=Q_{0i}(P)+\Delta Q_i(P,t),$$

$$k_i(t)R_i(P,t)=k_iR_{0i}(P)+\Delta R_i(P,t),$$

where $Q_{0i}(P)$ is an arbitrary linear differential operator, such as that polynomial $Q_{0i}(\lambda)$ is Hurwitz polynomial, $\deg Q_{0i}=\bar{n}_i$. Then $\Delta Q_i(P,t)$ is the difference $Q_i(P,t)-Q_{0i}(P)$ and $\deg\Delta Q_i\leq\bar{n}_i$, i.e. if $\deg Q_i<\deg Q_{0i}$, then $\deg\Delta Q_i=\deg Q_{0i}$, and if $\deg Q_i=\deg Q_{0i}$, then $\deg\Delta Q_i\leq\bar{n}_i-1$. $R_{0i}(P)$ is an arbitrary linear differential operator $\deg R_{0i}=\bar{n}_i-\gamma_{ui}$ such as that polynomial $R_{0i}(\lambda)$ is Hurwitz. Regarding structure $\Delta R(P,t)$ it's possible to say that if $m_i<\bar{n}_i-\gamma_{ui}$, then $\deg\Delta R_i=\bar{n}_i-\gamma_{ui}$, and if $m_i>\bar{n}_i-\gamma_{ui}$, then $\deg\Delta R_i=m_i$. Thus it is always possible to guarantee correctness of the mentioned decomposition of the operators $Q_i(P,t), R_i(P,t)$, as in one case operators $\Delta Q_i(P,t)$ and $\Delta R_i(P,t)$ have all coefficients non-zero, in other case the correspondent number of components are nonzero. The decomposition [Furtat, Parsheva and Tsykunov, 2011], that allows to solve the problem, differs from known methods of parameterization equations of control plants

Let us transform the equation of a system (1):

$$y_i(t)=\frac{k_iR_{0i}(P)}{Q_{0i}(P)}\left(u_i(t)+\frac{\Delta R_i(P,t)}{k_iR_{0i}(P)}u_i(t)-\right. \quad (4)$$

$$-\frac{G_i(P,t)}{k_iR_{0i}(P)}y_i(t-\tau_i)+\frac{1}{k_iR_{0i}(P)}f_i(t)-$$

$$\left.-\frac{\Delta Q_i(P,t)}{k_iR_{0i}(P)}y_i(t)+\sum_{j=1,i\neq j}^{M}\frac{S_{ij}(P,t)}{k_iR_{0i}(P)}y_j(t)\right),$$

since operators $Q_{0i}(P)$ and $R_{0i}(P)$ are arbitrary, we can choose them in order that the following condition is obeyed

$$\frac{R_{0i}(\lambda)}{Q_{0i}(\lambda)}=\frac{1}{Q_{mi}(\lambda)}. \quad (5)$$

Let us write the equation for error $e_i(t)=y_i(t)-y_{mi}(t)$, subtracting (2) from (4), and taking into consideration (5),

$$Q_{mi}(P)e_i(t)=k_iu_i(t)+\left(\frac{\Delta R_i(P,t)}{R_{0i}(P)}u_i(t)+\right.$$

$$+\frac{1}{R_{0i}(P)}f_i(t)-k_{mi}r_i(t)-\frac{\Delta Q_i(P,t)}{R_{0i}(P)}y_i(t)-$$

$$\left.-\frac{G_i(P,t)}{R_{0i}(P)}y_i(t-\tau_i)+\sum_{j=1,i\neq j}^{M}\frac{S_{ij}(P,t)}{R_{0i}(P)}y_j(t)\right). \quad (6)$$

To obtain the main result, let's use the approach [Tsykunov, 2007], which allows to compensate disturbance. Let choose a local control law in the following form

$$u_i(t)=\alpha_i\vartheta_i(t). \quad (7)$$

where $\alpha_i>0$; $\vartheta_i(t)$ is an additional control action. Then the following equation of error can be derived from (6)

$$Q_{mi}(P)e_i(t)=\vartheta_i(t)+\varphi_i(t), \quad (8)$$

$$\varphi_i(t)=\frac{1}{R_{0i}(P)}\left(\Delta R_i(P,t)\ u_i(t)\right)-$$

$$-\frac{1}{R_{0i}(P)}\left(G_i(P,t)\,y_i(t-\tau_i)\right)-$$

$$-\frac{1}{R_{0i}(P)}\left(\Delta Q_i(P,t)y_i(t)-\sum_{j=1,i\neq j}^{M}S_{ij}(P,t)y_j(t)\right)+ \quad (9)$$

$$+\frac{1}{R_{0i}(P)}f_i(t)-k_{mi}r_i(t)+(k_i\alpha_i-1)\vartheta_i(t).$$

Signal $\varphi_i(t)$ contains all components action of which in the error needs to be compensated. It is necessary to extract the signal.

Let's define the additional loop

$$Q_{mi}(P)\tilde{e}_i(t)=\vartheta_i(t) \quad (10)$$

and write the equation with the error signal $\zeta_i(t)=e_i(t)-\tilde{e}_i(t)$:

$$Q_{mi}(P)\zeta_i(t)=\varphi_i(t).$$

If the derivatives γ_{ui} of the output signal $y_i(t)$ can be measured then defining the variation law of the additional control action in the following form

$$\vartheta_i(t)=-Q_{mi}(P)\zeta_i(t)=-\varphi_i(t), \quad (11)$$

we will get the following equation of the closed loop system using the error equation (8)

$$Q_{mi}(P)e_i(t)=0. \quad (12)$$

Let us show that all the signals in the closed loop system are bounded. It is necessary for the efficiency of the algorithm which will be described later. Equation (12) shows that the signal $y_i(t)$ and its derivatives γ_{ui} are bounded due to assumption x). Then from conditions of the assumptions $\deg\Delta Q_i(P)=\bar{n}_i$ and because $R_{0i}(\lambda)$ is Hurwitz polynomial of $\bar{n}_i-\gamma_{ui}$ degree we can conclude that

$$\varphi_{1i}(t)=\frac{1}{R_{0i}(P)}f_i(t)-k_{mi}r_i(t)-\frac{1}{R_{0i}(P)}\left(\Delta Q_i(P,t)y_i(t)\right)-$$

$$-\frac{1}{R_{0i}(P)}\left(G_i(P,t)\,y_i(t-\tau_i)-\sum_{j=1,i\neq j}^{M}S_{ij}(P,t)y_j(t)\right)$$

is a bounded value. It is necessary to show that the chosen control action is bounded. For that purpose let's substitute $\varphi_i(t)$ in (11) with the statement above and resolve derived equation for $\vartheta_i(t)$:

$$\vartheta_i(t)=-\frac{1}{k_i\alpha_i}\left(\varphi_{1i}(t)+\frac{1}{R_{0i}(P)}\left(\Delta R_i(P,t)u_i(t)\right)\right). \quad (13)$$

Let us substitute $\vartheta_i(t)$ in equation (9) and resolve it for $u_i(t)$, taking into consideration following parameterization $k_i(t)R_i(P)=k_iR_{0i}(P)+\Delta R_i(P,t)$:

$$k_i(t)R_i(P,t)u_i(t)=-\varphi_{1i}(t).$$

From condition of assumption ii) and boundedness of $\varphi_{1i}(t)$ boundedness of local control action $u_i(t)$ is followed.

Because we cannot measure the derivatives, let's formulate the local law of additional control action $\vartheta_i(t)$ in the following form

$$\vartheta_i(t)=-g_{mi}^T\bar{\zeta}_i(t), \quad (14)$$

where $g_{mi}^T=\left[q_{m\gamma_{ui}},\ldots,q_{m1},1\right]$ – vector composed with polynomial coefficients $Q_{mi}(\lambda)=\lambda^{\gamma_{ui}}+q_{m1}\lambda^{\gamma_{ui}-1}+\ldots+q_{m\gamma_{ui}}$; $\bar{\zeta}_i(t)=col\left(\zeta_i,\bar{\zeta}_{i1},\bar{\zeta}_{i2},\ldots,\bar{\zeta}_{i\gamma_{ui}}\right)$; $\bar{\zeta}_{ik}(t)$ is estimation of derivatives $P^k\zeta_i(t)$, obtained from filters

$$\dot{z}_{ik}(t)=\frac{1}{\mu}F_iz_{ik}(t)+\frac{1}{\mu}b_iP^k\zeta_i(t),$$
$$\bar{\zeta}_{ik}=L_{0i}z_{ik},\qquad i=\overline{1,M},\quad k=\overline{1,\gamma_{ui}}. \quad (15)$$

Where $z_{ik}\in R^{\gamma_{ui}}$; $L_{0i}=[1,0,\ldots,0]$; $b_i^T=[0,\ldots,0,1]$;

$$F_i=\begin{bmatrix}-1 & 1 & 0 & \cdots & 0\\ 0 & -1 & 1 & \ddots & \vdots\\ \vdots & \ddots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & -1 & 1\\ 0 & \cdots & 0 & 0 & -1\end{bmatrix};$$

$\mu>0$ is small number.

If we use (14) and (15) in Laplace transformation we'll get the following

$$\vartheta_i(\lambda)=-\frac{Q_{mi}(\lambda)}{(\mu\lambda+1)^{\gamma_{ui}}}\zeta_i(\lambda).$$

Taking into consideration (10) and statement for error signal $\zeta_i(t)=e_i(t)-\tilde{e}_i(t)$ we have

$$\vartheta_i(\lambda) = -\frac{Q_{mi}(\lambda)}{(\mu\lambda+1)^{\gamma_{ui}}-1} e_i(\lambda).$$

Substituting $\vartheta_i(t)$ in equation (7) with the obtained statement and using the original of Laplace transformation we'll get control algorithm. Obviously that control law now is technically feasible since it contains only known or measurable variables.

Proposition. *If assumptions i) - xii) are obeyed then there are numbers* $\mu_0 > 0$, $T_0 > 0$ *such that under conditions* $\mu \le \mu_0$, $T \ge T_0$ *control algorithm*

$$\left((\mu P+1)^{\gamma_{ui}}-1\right)u_i(t) = -\alpha_i Q_{mi}(P) e_i(t) \qquad (16)$$

guarantees that target condition (3) is obeyed, where $\alpha_i > 0$.

It is necessary to note that the described algorithm remains invariant if there is state delay in a system as well as in the case when a system is in a steady state with unknown parameters with known boundaries.

Proposition proof. Let's consider vectors of the estimation error of derivatives $P^k\zeta_i(t)$

$$\eta_{ik}(t) = z_{ik}(t) + F_i^{-1}b_i P^k \zeta_i(t), \quad k=\overline{1,\gamma_{ui}}, \quad i=\overline{1,M}.$$

Here the vector $F_i^{-1}b_i = h_i$ has first component equal to -1. If to prove that the value $|\eta_{ik}(t)|$ is small, then from condition $\left|\bar{\zeta}_{ik}(t) - P^k\zeta_i(t)\right| < |\eta_{ik}(t)|$ it follows that estimation $\bar{\zeta}_{ik}(t)$ is near to $P^k\zeta_i(t)$. From (15) we'll get the equation of dynamic for vectors $\eta_{ik}(t)$:

$$\dot{\eta}_{ik}(t) = \frac{1}{\mu}F_i z_{ik}(t) + \frac{1}{\mu}b_i P^k\zeta_i(t) + F_i^{-1}b_i P^{k+1}\zeta_i(t) =$$
$$= \frac{1}{\mu}F_i\eta_{ik}(t) + h_i P^{k+1}\zeta_i(t),$$
$$\Delta_{ik}(t) = L_i\eta_{ik}(t), \qquad i=\overline{1,M}, \quad k=\overline{1,\gamma_{ui}}.$$

Taking into account that the additional control action is formulated as (14), we can transform the equation of error into the following form

$$Q_{mi}(P)e_i(t) = -q_{mi}^T\Delta_i(t), \qquad (17)$$

where $q_{mi}^T = \left[q_{m\gamma_{ui}-1}, \ldots, q_{m1}, 1\right]$;

$\Delta_i(t) = col\left(\Delta_{i1}(t), \Delta_{i2}(t), \ldots, \Delta_{i\gamma_{ui}}(t)\right)$;

$\Delta_{ik}(t) = \bar{\zeta}_{ik}(t) - P^k\zeta_i(t)$. Let's transform equation (17) into vector-matrix form. As a result we'll get the following equations set of the closed loop system:

$$\begin{cases} \dot{\varepsilon}_i(t) = A_{mi}\varepsilon_i(t) + b_i q_{mi}^T\Delta_i(t), \quad e_i(t) = L_i\varepsilon_i(t), \\ \mu_1\dot{\eta}_{ik}(t) = F_i\eta_{ik}(t) + \mu_2 h_i P^{k+1}\zeta_i(t), \\ \Delta_{ik}(t) = L_i\eta_{ik}(t), \qquad i=\overline{1,M}, \quad k=\overline{1,\gamma_{ui}}, \end{cases} \qquad (18)$$

where $\mu_1 = \mu_2 = \mu$. We've got singularly perturbanced system as μ – small enough number. Let us use Lemma [Brusin, 1995].

Lemma. [Brusin, 1995]. *If a system is defined by the equation* $\dot{x} = f(x, \mu_1, \mu_2)$, $x \in R^{m_1}$, *where* $f(t)$ *is a continuous function that is Lipshits function with respect to x and in the case when* $\mu_2 = 0$ *it has a bounded closed region of dissipation* $\Omega_1 = \{x \mid F(x) < \tilde{C}\}$, *where* $F(x)$ *– positive defined continuous piecewise smooth function, then there is* $\mu_0 > 0$ *such that under* $\mu_2 \le \mu_0$ *the initial system has the same dissipative region* Ω_1, *if for some numbers* $\tilde{C}_1$ *and* $\bar{\mu}_1$ *for* $\mu_2 = 0$ *following condition is obeyed*

$$\sup_{|\mu_1|\le\bar{\mu}_1}\left(\left(\frac{\partial F(x)}{\partial x}\right)^T f(x,\mu_1,0)\right) \le -\tilde{C}_1, \ \text{if} \ F(x) = \tilde{C}. \qquad (19)$$

In the case of $\mu_2 = 0$ in (18) we have asymptotically stable system for variables $\varepsilon_i(t)$ and $\eta_{ik}(t)$, since A_{mi}, F_i are Hurwitz matrixes. It is the same situation which we had for measuring the derivatives i.e. $\lim_{t\to\infty} e_i(t) = 0$. It was proved that if this condition is obeyed all the signals in the system are bounded. It means that there is a certain region

$$\Omega = \{\varepsilon_i(t), \eta_{ik}(t), \zeta_i(t):$$
$$\left|P^{k+1}\zeta_i(t)\right| \le \delta_{1k}, \left|\varepsilon_i(t)\right| < \delta_{2k},$$
$$\left|\eta_{ik}(t)\right| < \delta_{3k}, F(\varepsilon_i, \eta_{ik}) < C_1\}, \quad k=\overline{1,\gamma_{ui}},$$

where signals $e_i(t), \eta_{ik}(t), \zeta_i(t)$ are within their boundaries for some initial conditions from Ω_0.

Let us consider two vectors

$$\theta_i^T(t) = \left[\ddot{\zeta}_i(t), \ldots, \overset{(\gamma_{ui}+1)}{\zeta_i}(t)\right],$$
$$\eta_i^T(t) = \left[\eta_{i1}(t), \eta_{i2}(t), \ldots, \eta_{i\gamma_{ui}}(t)\right],$$

and block-diagonal matrixes with γ_{ui} diagonal blocks

$$F_{0i} = diag\{F_i, F_i, \ldots, F_i\}, \ B_i = diag\{h_i, h_i, \ldots, h_i\},$$
$$C_i = diag\{L_i, L_i, \ldots, L_i\},$$

then equations (18) will take the following form

$$\begin{cases} \dot{\varepsilon}_i(t) = A_{mi}\varepsilon_i(t) + b_i q_{mi}^T\Delta_i(t), \quad e_i(t) = L_i\varepsilon_i(t), \\ \mu_1\dot{\eta}_i(t) = F_{0i}\eta_i(t) + \mu_2 B_i\theta_i(t), \\ \Delta_i(t) = C_i\eta_i(t), \qquad i=\overline{1,M}. \end{cases} \qquad (20)$$

Evidently that condition (19) was obeyed if to take Lyapunov function for F_i

$$V(\varepsilon_i(t), \eta_i(t)) = \sum_{i=1}^{M}\left(\varepsilon_i^T(t)H_{1i}\varepsilon_i(t) + \eta_i^T(t)H_{2i}\eta_i(t)\right), \qquad (21)$$

where the positive defined symmetric matrixes H_{1i}, H_{2i} are determined from equations solution

$$\begin{aligned} H_{1i}A_{mi} + A_{mi}^T H_{1i} &= -\rho_{1i}I_{\gamma_{ui}} - Q_{1i}, \\ H_{2i}F_i + F_i^T H_{2i} &= -\rho_{2i}I_{\gamma_{ui}} - Q_{2i}, \end{aligned} \qquad (22)$$

where $\rho_{1i} > 0$, $\rho_{2i} > 0$, $Q_{1i} = Q_{1i}^T > 0$, $Q_{2i} = Q_{2i}^T > 0$. Thus in accordance with Lemma [Brusin, 1995], there is $\mu_0 > 0$ such that if $\mu < \mu_0$ then Ω remains dissipative region of system (18).

However it is necessary to note that keeping the dissipative region doesn't guarantee that the set of attraction Ω_1 remains the same in a singularly perturbed system.

Let us calculate the full derivative of function (21) on system's trajectories (20), taking into account equation (22) and assigning $\mu_1 = \mu_2 = \mu_0$:

$$\dot V(\varepsilon_i(t),\eta_i(t)) = \sum_{i=1}^{M} \Big(-\rho_{1i}\|\varepsilon_i(t)\|^2 - \varepsilon_i^T(t)Q_{1i}\varepsilon_i(t) + 2\varepsilon_i^T(t)H_{1i}b_i q_{mi}^T \Delta_i(t) - \frac{\rho_{2i}}{\mu_0}\|\eta_i(t)\|^2 - \frac{1}{\mu_0}\eta_i^T(t)Q_{2i}\eta_i(t) + 2\eta_i^T(t)H_{2i}B_i\theta_i(t)\Big). \quad (23)$$

Let us use estimations

$$2\varepsilon_i^T(t)H_{1i}b_i q_{mi}^T \Delta_i(t) \le \|\varepsilon_i(t)\|^2 + \rho_{3i}\|\eta_i(t)\|^2,$$

$$2\eta_i^T(t)H_{2i}B_i\theta_i(t) \le \frac{1}{\mu_0}\|\eta_i(t)\|^2 + \mu_0\rho_{4i},$$

$$-\varepsilon_i^T(t)Q_{1i}\varepsilon_i(t) \le -\lambda_{\min}(Q_{1i})\|\varepsilon_i(t)\|^2 \le \le -\frac{\lambda_{\min}(Q_{1i})}{\lambda_{\max}(H_{1i})}\varepsilon_i^T(t)H_{1i}\varepsilon_i(t),$$

$$-\eta_i^T(t)Q_{2i}\eta_i(t) \le -\lambda_{\min}(Q_{2i})\|\eta_i(t)\|^2 \le \le -\frac{\lambda_{\min}(Q_{2i})}{\lambda_{\max}(H_{2i})}\eta_i^T(t)H_{2i}\eta_i(t),$$

where $\rho_{3i} = \left\|H_{1i}b_i q_{mi}^T C_i\right\|^2$, $\rho_{4i} = \|H_{2i}B_i\| \sum_{i=1}^{2\gamma_{ui}} \delta_{1i}^2$; $\lambda_{\min}$, $\lambda_{\max}$ are the minimal and maximal characteristic numbers of the mentioned matrixes. Using those estimations into (23) we'll get

$$\dot V(\varepsilon_i(t),\eta_i(t)) \le -\sigma_0 V + \sum_{i=1}^{M}\Big(-(\rho_{1i}-1)\|\varepsilon_i(t)\|^2 - \left(\frac{\rho_{2i}}{\mu_0} - \frac{1}{\mu_0} - \rho_{3i}\right)\|\eta_i(t)\|^2 + \mu_0\rho_{4i}\Big),$$

where $\sigma_0 = \min\left\{\frac{\lambda_{\min}(Q_{1i})}{\lambda_{\max}(H_{1i})}, \frac{\lambda_{\min}(Q_{2i})}{\lambda_{\max}(H_{2i})}\right\}$. If to choose ρ_{1i}, ρ_{2i} from conditions

$$\rho_{1i} - 1 > 0, \quad \frac{\rho_{2i}}{\mu_0} - \frac{1}{\mu_0} - \rho_{3i} > 0, \quad (24)$$

the following inequality is correct:

$$\dot V(\varepsilon_i(t),\eta_i(t)) \le -\sigma_0 V(\varepsilon_i(t),\eta_i(t)) + \sum_{i=1}^{M} \mu_0\rho_{4i}.$$

If we solve the inequality

$$V(\varepsilon_i(t),\eta_i(t)) \le V(0)e^{-\sigma_0 t} + \sum_{i=1}^{M}\frac{\mu_0\rho_{4i}}{\sigma_0},$$

we can see that if to choose μ_0 small enough we get the following region of attraction:

$$\Omega_2 = \left\{\varepsilon_i(t), \eta_i(t): \ V(\varepsilon_i(t),\eta_i(t)) \le \sum_{i=1}^{M}\frac{\mu_0\rho_{4i}}{\sigma_0}\right\}.$$

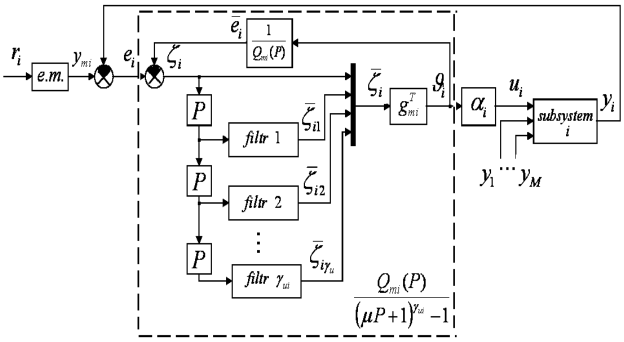

Fig.1. Structural scheme of local robus control system

Inserting the required value T_0 from the target condition (3) into the right part and taking into consideration the inequalities

$$|e_i(t)|^2 \le \|\varepsilon_i(t)\|^2 \le \frac{V(0)e^{-\sigma_0 t}}{\lambda_{\min}(H_{1i})} + \sum_{i=1}^{M}\frac{\mu_0\rho_{4i}}{\sigma_0\lambda_{\min}(H_{1i})}$$

we get the estimation of the value δ in the target condition (3)

$$\delta \le \sqrt{\frac{1}{\lambda_{\min}(H_{1i})}\left(V(0)e^{-\sigma_0 t} + \sum_{i=1}^{M}\frac{\mu_0\rho_{4i}}{\sigma_0}\right)},$$

that shows that there are numbers μ_0 and T_0 guaranteeing that target condition will be obeyed. Thus for $\mu \le \mu_0$ varying ρ_{1i} in (24) and μ, we can get the required value δ in the target condition (3).

Structural scheme of the designed control system is shown in Figure 1.

The drawback of the proposed algorithm is a lack of analytically proved choice of parameters μ and α_i. However they can be easily matched during the modeling phase. For a system (1) minimally possible coefficients of operators $k_i(t)R_i(P,t), Q_i(P,t), G_i(P,t), S_{ij}(P,t)$ are used and maximally possible values of $f_i(t), r_i(t)$ are used for the input. Constant components don't matter. Numbers μ and α_i are selected in order to guarantee a given dynamic error. Number μ is usually varying within 0.005 to 0.05. Error will not exceed a given value for other parameters values and values of external actions from given class of uncertainty.

4 Example

As an example, the system can be used to solve the problem of decentralized control of the trajectory of the group of the pilotless aircrafts of different types in the horizontal plane [Bukov, Bronnikov and Selvesuyk, 2009], [Parsheva, 2009]. The aircrafts do not exchange data with each other. Trajectory control for each aircraft is performed using radio commands from a ground-based control station.

First, the robust local etalon models (2) are selected. Then, we generate the local regulators for each aircraft using (16). Using numerical analysis the group flight under wind disturbance is considered.. The obtained results demonstrate the efficiency of the suggested approach to decentralized control.

Unlike the work [Parsheva, 2009] a broader class of the systems is considered here because of taking into account the ability of the systems to adapt to external, parametrical, and structural disturbance.

To illustrate the quality of synthesized system, let us consider a dynamic system of sixth order represented as two subsystems

$$\dot{x}_1(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5+6\cos 3t & 5+5\cos t & 5+\sin 2t \end{bmatrix} x_1(t) +$$

$$+\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} y_1(t-2) + \begin{bmatrix} 0 \\ 0 \\ 5+\sin 5t \end{bmatrix} u_1(t) + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} f_1(t) + \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} y_2(t),$$

$$\dot{x}_2(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4+3\cos 2t & 4+4\cos t & 4+2\sin t \end{bmatrix} x_2(t) +$$

$$+\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} y_2(t-3) + \begin{bmatrix} 0 \\ 0 \\ 3+\cos t \end{bmatrix} u_2(t) + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} f_2(t) + \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} y_1(t),$$

where x_1 and x_2 are the state vectors of the subsystems, y_1 and y_2 are the measurable scalar outputs of the subsystems, u_1 and u_2 are the scalar control actions whose law of variation is generated according to Eq. (16), and

$$f_1(t) = \sin 2t\ , \quad f_2(t) = 2\sin 3t$$

are the disturbances. The parameters of the local reference models (2) are taken as $Q_{mi}(P) = (P+1)^3$ and $k_{mi} = 1$, $i = 1, 2$, the reference signals r_1 and r_2 are as follows:

$$r_1(t) = 1 + 2\sin t, \quad r_2(t) = 1 + 2\sin 0.5t\ .$$

We represent the considered plant using (1), where

$$k_i(t)R_i(P,t) = k_i + \Delta k_i(t)\ ,$$

$$Q_i(P,t) = P^3 + (q_{1i} + \Delta q_{1i}(t))P^2 + (q_{2i} + \Delta q_{2i}(t))P + q_{3i} + \Delta q_{3i}(t)$$

The class of uncertainty is defined by the inequalities

$$1 \le k_i \le 10\ , \ |\Delta k_i| \le 10\ ; \ |q_{li}| \le 15\ , \ |\Delta q_{li}| \le 10\ , \ l = 1, 2, 3\ .$$

The controller (16) consists of two cascaded blocks with the following transfer functions

$$W_{1i}(\lambda) = \frac{1}{\mu} + \frac{1}{\mu\,\lambda}\ , \quad W_{2i}(\lambda) = \frac{(\lambda+1)^2}{\mu^2\lambda^2 + 3\mu\lambda + 3}\ ,$$

and amplifier with gain of α_i.

Given that $\delta = 0.05$ in the target condition (3), the values $\mu = 0.01$, $\alpha_1 = 1.5$, $\alpha_2 = 2$ allow to achieve the required accuracy.

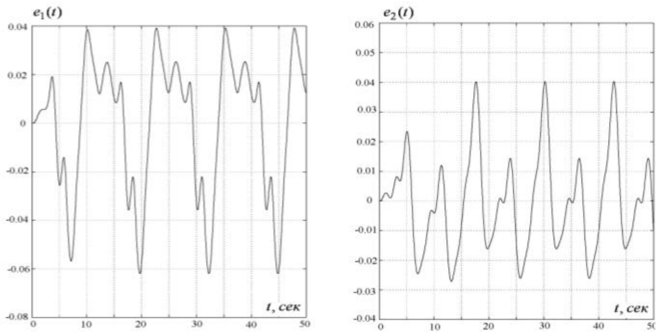

Fig.2. Error trajectories $e_i(t)=y_i(t)-y_{mi}(t)$

Computer-aided modeling demonstrated good operability of the designed systems.

5 Conclusion

Paper considers the problem of decentralized control with an nominal model for interconnected system with unknown parameters and an unknown order when derivatives of input and output signals of the local subsystems cannot be measured. Considered robust control system allows compensating parametric and external disturbances with given accuracy δ for the period of time T. Values δ and T can be small enough using the appropriate parameters of the closed loop system. It is necessary to note that the closed loop system is functioning as an implicitly defined nominal model and parameters of the model are used in control algorithm.

It is important to note that considered algorithm remains the same if there is state delay in a system as well as in the case when a plant is stationary with unknown parameters which values are limited by a certain bounded set. Besides, the advantage of the suggested algorithm consists in the fact that the structure of a local controller is coincided with the structure of a local controller of single-connected system. This gives an advantage for the control of spatially distributed systems. The drawback of the algorithm is a lack of an analytically proved method of selection of the parameters of the controller.

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