

# Mixed Extended Backstepping- Sliding Modes Control with off line Parameter Optimization used for a class of Solenoid Actuators

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**Abstract:** In this paper, it is described a control-design procedure for a specific class of nonlinear systems that can partially be put in the so called *strict feedback form*. It is used a mixed technique: backstepping and sliding modes, in order to systematically construct the command for a system in more general-form. To illustrate the procedure, an application to a propulsion system micro-pump for small satellites is given. Some practical aspects are then discussed as for instance: how to simplify the analytical form of the command (using decoupling hypothesis) and also how to choose the adjustment parameters involved in the command by using an appropriate optimization technique, in order to preserve a good robustness/performance ratio in case of perturbations.

*Keywords:* nonlinear system, backstepping, sliding modes, propulsion system micro pump.

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## 1. INTRODUCTION

The aim of this paper is to show the possibility to design a robust controller through a simple procedure, similar to the one used in backstepping technique and which is being standard today (Krstic *et al.*, 1995). It is known that this technique is quite nice when dealing with strict feedback form systems, namely systems where the new state variable appears in an affine way. There exists backstepping based approach for systems which are not described in this aforementioned form or which cannot be put neither in *pure-feedback forms* and nor into a lower triangular form (Fontaine and Kokotovic, 1998). The global stabilization of nonlinear systems in the *strict feedforward form*, i.e. refering to upper triangular form is presented in (Sepulchre *et al.*, 1997).

So the procedure described herein consists of using a mixed backstepping technique and sliding modes and may be seen as *a kind of an extension* due to the fact that, in the one hand discontinuous terms are involved in the application under study as well as in the time derivative of the controlled Lyapunov function and in the other hand the new state variable is not necessarily involved in affine way. In doing so, it is shown that the level of robustness with regards to perturbations is improved, at least through the performed application for which one is paying a particular attention.

The first part of the paper is dealing with some theoretical aspects for systematically implementing the proposed command (derived through the combination of both known control techniques: backstepping and sliding modes). It is also emphasized the real interest to this type of command, and it is specified some examples of actuators that might take

profit from this procedure as well. An application to a micro-pump system is shown. This one has been studied by the authors in previous work (Teodorescu *et al.*, 2008a-b) but more investigations are needed in order to increase the performance and especially to enlarge the operating domain while still reaching a good level of robustness. Moreover, the aim is also to reduce as much as possible the complexity of the control law in the perspective of an implementation.

The paper is organized as follows: Section 2 deals with theoretical aspects concerning the design of the command in case of a *strict feedback form* system. Section 3 briefly presents the micro-pump's discontinuous system. In Section 4, the proposed design-scheme is applied to this system. Section 5 shows the simulation results in the nominal case as well as in the perturbed one, which turn out to be quite satisfactory in terms of the required performance. Finally a conclusion is drawn in Section 6.

## 2. PROPOSED CONTROL-METHOD SCHEME

Consider *strict feedback form* system (1) as it has been widely defined in the literature concerning nonlinear control systems (Krstic *et al.*, 1995, Khalil, 2002).

$$(S_1) \begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)x_3 \\ \dots \\ \dot{x}_{n-1} = f_{n-1}(x_1, \dots, x_{n-1}) + g_{n-1}(x_1, \dots, x_{n-1})x_n \\ \dot{x}_n = f_n(x_1, \dots, x_n) + g_n(x_1, \dots, x_n)u \end{cases} \quad (1)$$

where  $f_1, f_2, \dots, f_n$  and  $g_1, g_2, \dots, g_n$  are nonlinear continuous functions depending on the state variables  $x_i$ .

According to the theory presented in (Krstic *et al.*, 1995), one may conceive the control for system (1) using backstepping whose key idea is to let some state variables as *virtual controls* of others. System (1) may be considered as being a particular form of a more general nonlinear system, and according to the sliding modes technique, one may implement it or any of its variants (see for example Utkin, 1996). Nevertheless, backstepping approach is based on Lyapunov theory and therefore can offer robust controller especially needed during the *transient regime*, by using an increased number of adjustment parameters. It is also used continuous-form command, unlike the case of sliding modes which might cause fatigue on actuators, others than electrical ones, due to its discontinuous analytical form. Moreover, for high order systems, the choice of an appropriate sliding surface might be difficult to infer, causing instability and (sometimes) poor performance during *steady state regime*.

Since the goal herein is to control a special type of systems that can partially be put in form (1), meaning that the overall system concerned has a more general form, it is then proposed, in this paper, to combine both control techniques, namely backstepping and sliding modes, in order to take benefit as much as possible from the advantages offered by the first one, while maintaining a simple analytical form for the command  $u$ . Interesting results may be found in (Swaroop *et al.*, 2000), where a synthetic input technique similar to backstepping using multiple surface control methods with filters is described for systems in *strict feedback form*.

As a remark, it is worth mentioning that since the order of complexity for the command  $u$  (if purely designed by backstepping and then by sliding modes) is the same when independently using any of both techniques.

Now, let us consider a class of nonlinear systems having the following form:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_2 = f_2(x_1, x_2) + g_2(x_1, x_2)x_3 \\ \dots \\ \dot{x}_k = f_k(x_1, \dots, x_k) + g_k(x_1, \dots, x_k)x_{k+1} \\ \dot{x}_{k+1} = h_{k+1}(x_1, \dots, x_{k+2}) \\ \dot{x}_{k+2} = h_{k+2}(x_1, \dots, x_{k+2}, x_{k+3}) \\ \dots \\ \dot{x}_{n-1} = h_{n-1}(x_1, \dots, x_n) \\ \dot{x}_n = f_n(x_1, \dots, x_n) + g_n(x_1, \dots, x_n)u \end{cases} \quad (2)$$

where  $f_i$ ,  $h_j$  and  $g_l$  are nonlinear smooth functions.

One may notice that overall system (2) can be divided into two subsystems: the first one corresponds to the first  $k$  equations concerning the state variables  $x_1, x_2, \dots, x_k$  and belongs to the class of *strict feedback form* systems, while the second one (corresponding to the equations of the state variables  $x_{k+1}, x_{k+2}, \dots, x_n$ ) has a lower triangular form. In the

next sections, we may refer to simply as first part (or *first subsystem*) and second part (or *second subsystem*) of (2).

Even though, the proposed design procedure combines backstepping and sliding modes based approaches, there are some essential differences with respect to the literature, systematically presented in  $k+1$  steps below. As known, the main idea in backstepping is to let certain state variables as “*virtual controls*” for others and which along with the controlled Lyapunov function can be used to find a globally stabilizing controller.

In a similar manner, the main idea of this proposed scheme is to systematically design a controlled Lyapunov function in order to ensure global stability of overall system (2) and as results an asymptotic convergence of  $x_1$  towards its reference signal  $x_{1ref}$ . So, let's describe the procedure step by step.

**First step:** Using the reference of the first state variable which is denoted as  $x_{1ref}$ , and first equation of (2), let the tracking error:

$$\varepsilon_1 = x_1 - x_{1ref} \quad (3)$$

to be forced to converge asymptotically towards zero. For this, let us choose the Lyapunov function

$$V_1(\varepsilon_1) = \frac{1}{2} \varepsilon_1^2; V_1(0) = 0. \quad (4)$$

and by imposing the negative definite condition for the time derivative of  $V_1$ , one gets the virtual command (i.e. the desired value) for  $x_2$  – namely  $\phi_{x_2}$ , so that the asymptotic stability is guaranteed for the first equation of (2).

$\phi_{x_2}$  will thus become a reference signal for the second step. In other words,  $x_2$  is used to impose the desired dynamics on  $x_1$ , i.e.  $x_1$  tracks  $x_{1ref}$ .

### 2.1 Simplified scheme

**Second step:** Let's now define the tracking error:

$$\varepsilon_2 = x_2 - \phi_{x_2} \quad (5)$$

and the controlled Lyapunov function

$$V_2(\varepsilon_1, \varepsilon_2) = V_1(\varepsilon_1) + \frac{1}{2} \varepsilon_2^2, V_2(0) = 0 \quad (6)$$

in conjunction with the first two equations of (2) in order to calculate the desired evolution for  $x_3$ , referenced as  $\phi_{x_3}$ . By iterating this procedure for a total of  $k$  steps, one obtains the virtual command corresponding to  $x_{k+1}$  – namely  $\phi_{x_{k+1}}$ .

**Step k+1:** Using  $\phi_{x_{k+1}}$ , a sliding surface can be designed:

$$\mathfrak{Z} = \varepsilon_{k+1}^{(n-(k+1)-1)} + \sum_{j=n-(k+1)-2}^0 \alpha_j \varepsilon_{k+1}^{(j)} \quad (7)$$

$$\text{with } \varepsilon_{k+1} = x_{k+1} - \phi_{x_{k+1}} \quad (8)$$

and  $\alpha_j$  are constant values. The time derivative of  $\mathfrak{Z}$ , namely  $\dot{\mathfrak{Z}} = \dot{\mathfrak{Z}}(\varepsilon_{k+1}^{n-(k+1)})$  reveals  $u$  through its analytical form. Using the Lyapunov function

$$V_{k+1} = V_k + \frac{1}{2} \mathfrak{Z}^2 \quad (9)$$

$$\text{with } V_k = \frac{1}{2} \sum_{i=1}^k \varepsilon_i^2; \quad \varepsilon_i = x_i - \phi_{x_i}, \quad i \in \{1, \dots, k\}$$

and imposing the negative-definite condition on the time derivative  $\dot{V}_{k+1}$ , after some relatively tedious calculus, one gets  $u$ .

Thus we have proved the global stability of overall system (2) and consequently the asymptotic convergence towards a stable point while ensuring that  $x_1$  tracks  $x_{1ref}$ .

As matter of fact, one can significantly simplify the controller expression. Indeed, from dynamics (2), one might decouple the system into two subsystems, making a simplification hypothesis, which is: if the dynamics of the *strict feedback form* subsystem of (2) are (relatively) *slow* compared to the second subsystem one, one might treat the two subsystems independently and may obtain a more simple analytical form for  $u$ . As commonly used in the literature (see for instance Khalil, 2002), the *slow* part of (2) will be used to derive the reference signals for the *fast* one. Also, the simplification hypothesis guarantees that *large* variations of the state variables within the *non strict feedback form* part of (2) cannot not render unstable the first subsystem. Intuitively, for hypothetic system (2) where the first part is *very slow* compared to the other one, the state variables belonging to the first subsystem act like quasi-constant parameters with regards to the second subsystem. Many efficient examples of this kind of system decomposition may be found in several papers (e.g. Rontani *et al.*, 2007).

## 2.2 Extension to discontinuous functions

Consider the system

$$\dot{x}_i = f_i(x_j, u); \quad i, j \in \{1..n\}, u \in \mathfrak{R} \quad (10)$$

with  $u(t)$  and  $f_i(x_j, u)$  being discontinuous functions. It is possible to also systematically construct a *continuous* Lyapunov function  $V = V(x_i)$  in the case of a *discontinuous* system written down in form (2).

## 2.3 Some practical remarks

One has briefly introduced the theoretical aspects, and let's now make the connection with an industrial need.

The practical interest of the proposed control scheme might concern for example the class of *solenoid valve actuators* (Atmann, 2005) like pinch valves and globe valves where fast response is needed (usually 8-12 ms) and where robust nonlinear control might be preferred or suitable in the detriment of complexity (and thus cost). The basic principle of these actuators consists of a coil wrapped around a free metallic piston (sometimes referred to as plunger in the known literature), which moves forward and backward determining the opening ratio of the valve. When modelling each of these systems, the electromagnetic force into the piston depends on the square value of the current according to the relationship:

$$F_{elm} = \frac{1}{2} \frac{dL(x)}{dx} I^2$$

where  $L(x)$  is the coil inductance depending on the position of the inner mobile metallic part, that is the plunger.

By letting the current  $I$  as a state variable when modelling, and by properly arranging the overall system, one might come upon nonlinear SISO system as it can be seen a little later on, through an example.

More precise processes examples may include systems like fuel injectors found in most of nowadays-produced spark-ignition and combustion-ignition engines; EGR (Exhaust Gas Recirculation) valve (e.g. Ribbens, 2003); EVR (Electronic Vacuum Regulator) and magnetic suspensions. Further on, a new concept of micro pump future generation for small satellite propulsion systems will be treated.

## 3. MICRO PUMP SYSTEM

Roughly speaking, the pump consists of a piston that is actuated by an inducing current into a coil wrapped around the piston. By using classical Newton laws for modelling the micro-pump, one can derive the following state space representation of the system:

$$(S) \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \beta_{x_i} - \frac{L_1 x_3^2}{m x_1^3} \\ \dot{x}_3 = \frac{1}{L} \left( u + \frac{2L_1}{x_1^3} x_2 x_3 - R x_3 \right) \end{cases} \quad (11)$$

where

$$\beta_{x_i} = \frac{1}{m} \left[ \frac{1 - \text{sign}(x_2)}{2} P_{res} A + \frac{1 + \text{sign}(x_2)}{2} P_{acc} A - x_2 f \right] - (a_{h0} x_1^3 + a_{h1} x_1^2 + a_{h2} x_1 + a_{h3}) \sigma_2 + a_b x_1 \sigma_1 \quad (12)$$

$$\text{and } \sigma_1 = \begin{cases} 1, & \text{if } x_1 < x_{1l} \\ 0, & \text{if } x_1 \geq x_{1l} \end{cases}; \quad \sigma_2 = \begin{cases} 1, & \text{if } x_1 > x_{1h} \\ 0, & \text{if } x_1 \leq x_{1h} \end{cases} \quad (13)$$

$x_1$  is the piston position;  $x_2$  is the piston speed;  $x_3$  is the current intensity;  $u$  denotes the voltage across the coil;  $L = L(L_1, x)$  is the coil inductance;  $R$  is the windings resistance;  $m$ ,  $P_{res}$ ,  $a_{hi}$ ,  $a_b$ ,  $P_{acc}$ ,  $A$ ,  $x_{1i}$  and  $f$  are constants. The goal is to find the command  $u$  so that  $x_1$  tracks a reference sine-wave signal of frequency preferably over 100Hz.

The discontinuous part of system (11) is due to the presence of two bumpers (an upper and a lower one) thus limiting the position's movement. The controller-design constraint is aimed to avoid any contact with the bumpers. For more information regarding modelling of (11), one may refer to (Teodorescu *et al.*, 2008), where the physical aspects are described in details.

One of the physical constraint concerns the current which is always positive, i.e.  $I > 0$ , and which might be justified due to the type of the power converter realized on the prototype system (Teodorescu *et al.*, 2008), which is reverse current protected. Since solenoids have high inductance, when actuating, one would expect a short lived but very high power voltage spike to occur. After this short lived (~50ms) voltage spike, the voltage will balance at the initial voltage. This is just like driving a dc motor in one direction, then suddenly reversing the direction (see e.g. Krishnan, 2001).

#### 4. CONTROLLER DESIGN

Since the term  $x_3^2$  appears in the second equation of (11), the overall system of the micro-pump is not in a *strict feedback form*. Thus, one can follow the controller design-procedure described in Section 2, where the *strict feedback form* subsystem of (2) consisting of the first equation in (11), and the other two equations will be used in conjunction with a sliding surface in order to get the command  $u$ .

So then, let

$$\varepsilon_1 = x_1 - x_{1ref} \quad (14)$$

$\varepsilon_1$  is being the first tracking error. Using the first equation of (11) in conjunction with the controlled Lyapunov function

$$V_1(\varepsilon_1) = \frac{1}{2} \varepsilon_1^2 \quad (15)$$

one may calculate the desired value for  $x_2$  – namely  $\phi_{x_2}$ , in order to ensure asymptotic convergence of  $V_1 \rightarrow 0$ . Thereby, the time derivative  $\dot{V}_1(\varepsilon_1)$  can be made negative definite, thus yielding  $\phi_{x_2}$ :

$$\dot{V}_1 = \varepsilon_1 \dot{\varepsilon}_1 = \varepsilon_1 (\phi_{x_2} - \dot{x}_{1ref}) < 0 \quad (16)$$

by choosing  $\phi_{x_2} = \dot{x}_{1ref} - k_1 \varepsilon_1$ , in other words

$$\phi_{x_2} = \dot{x}_{1ref} - k_1 (x_1 - x_{1ref}) \quad (17)$$

Next, let us define the sliding surface  $\mathfrak{S}$  and derive the command  $u$ :

$$\mathfrak{S} = \dot{\varepsilon}_2 + \alpha \varepsilon_2, \quad \alpha > 0 \quad (18)$$

with  $\varepsilon_2 = x_2 - \phi_{x_2}$ ;  $\varepsilon_2$  is the second tracking error. Once all calculus are done, it thus yields:

$$\begin{aligned} \mathfrak{S} = & \beta_{x_i} - \frac{L_1 x_3^2}{m x_1^3} - \ddot{x}_{1ref} + k_1 (x_2 - \dot{x}_{1ref}) + \\ & + \alpha [x_2 - \dot{x}_{1ref} + k_1 (x_1 - x_{1ref})] \end{aligned} \quad (19)$$

Notice that  $\dot{\mathfrak{S}} = \dot{\mathfrak{S}}(\dot{x}_3)$ , so the command  $u$  will be extracted from  $\dot{\mathfrak{S}}$ . The controlled Lyapunov function is:

$$V_2 = V_1 + \frac{1}{2} \mathfrak{S}^2 = \frac{1}{2} (\varepsilon_1^2 + \mathfrak{S}^2) \quad (20)$$

By imposing the condition  $\dot{V}_2 < 0$ , it comes out the expression of the command  $u$ . For simplicity reasons, at this point it may be made the *decoupling* simplification hypothesis i.e. the first state variable  $x_1$  of (11) has *slow* dynamics contrary to the two other states variables  $x_2$  and  $x_3$ , which have *fast* dynamics. Using the simplified scheme presented in the above section, one has:

$$V_2^* = \frac{1}{2} \mathfrak{S}^2 \quad (21)$$

Hence, the time derivative of (21) is:

$$\dot{V}_2^* = \mathfrak{S} \dot{\mathfrak{S}} \quad (22)$$

By choosing

$$\dot{\mathfrak{S}} = -k_{21} \mathfrak{S} - k_{22} \text{sign}(\mathfrak{S}) \quad (23)$$

one obtains  $\dot{V}_2^* = -k_{21} \mathfrak{S}^2 - k_{22} |\mathfrak{S}| < 0$ . Therefore, the asymptotic stability of decoupled system (11) is guaranteed.

Some tedious calculus leads to the command expression:

$$u = \frac{m L x_1^3}{2 L_1 x_3} \left( \begin{aligned} & -\frac{1}{m} \left[ \left( \beta_{x_i} - \frac{L_1 x_3^2}{m x_1^3} \right) f - a_b x_2 \sigma_1 \right. \\ & \left. + (3 a_{h0} x_1^2 + 2 a_{h1} x_1 + a_{h2}) x_2 \sigma_2 \right] \\ & + \frac{3 L_1 x_2 x_3^2}{m x_1^4} - \ddot{x}_{1ref} + k_1 \left( \beta_{x_i} - \frac{L_1 x_3^2}{m x_1^3} - \ddot{x}_{1ref} \right) \\ & + \alpha \left[ \beta_{x_i} - \frac{L_1 x_3^2}{m x_1^3} - \ddot{x}_{1ref} + k_1 (x_2 - \dot{x}_{1ref}) \right] \\ & \left. + k_{21} \mathfrak{S} + k_{22} \text{sign}(\mathfrak{S}) \right) - \frac{2 L_1}{x_1^3} x_2 x_3 + R x_3 \end{aligned} \right) \quad (24)$$

*Remark:* in order to simplify a little bit more expression of command (24), according to classical theory one might stick

with  $k_{21} = 0$  and  $k_{22} \neq 0$ . Nevertheless, from numerical point of view, it is recommended to use the continuous part of the command too, i.e.  $k_{21} \neq 0$ . In case of  $k_{21} = 0$  and  $k_{22} \neq 0$  one might then have to use a high value for  $k_{22}$  in order to ensure efficient tracking of the reference signal (one of the requirements is the high frequency imposed to the reference sine wave signal). This might causes problems (due to the *chattering effect*) once the sliding surface has been reached (i.e. in *steady state regime*).

#### 4.1 Reference trajectory

The main objective is to control the system such that the piston position is capable to track a given desired trajectory without chocks with the bumpers. The tracking trajectory may be described by the equation

$$\ddot{x}_{1ref} = -\omega^2(x_{1ref} - \gamma) \quad (25)$$

with  $x_{1ref}(t_0) = A_1 + \gamma$ ;  $\gamma = \frac{x_{1h} - x_{1l}}{2}$  is the middle of the *admissible range* for  $x_1$ , with an upper limit  $x_{1h}$  and a lower limit  $x_{1l}$ ;  $A_1$  is the amplitude of the signal.

Hence,

$$x_{1ref}(t) = A_1 \sin(\omega t) + \frac{x_{1h} - x_{1l}}{2} \quad (26)$$

#### 4.2 Optimization

Since there is no systematic way concerning the choice of adjustment parameters  $k_i$  involved in the command  $u$  designed through backstepping technique, nor one concerning the parameters  $\alpha_{ij}$  (7) when designing a sliding surface for high order systems, so an optimization technique is employed in order to get the *optimal* tuning parameters.

The problem now consists of proposing the appropriate *cost function*  $J$ , and then determining the optimal parameters, by using existing Matlab/Simulink toolbox. One has four adjustment parameters for command  $u$  (24), each one is introduced in the design procedure in order to guarantee the stability:  $\alpha$  from (18),  $k_1$  from (17) and  $k_{21}, k_{22}$  from (23). Thus, an initial optimization problem may be:

$$\min_{\alpha, k_1, k_{21}, k_{22}} \left( \int_0^{\infty} \rho_1 \varepsilon_1^2 dt + \rho_2 h(\max |u| - U_{\max}) \right) \quad (28)$$

where  $h(y) = \begin{cases} y^2 & \text{if } y > 0 \\ 0 & \text{if } y \leq 0 \end{cases}$  is the penalty function;  $\varepsilon_1$  has

been defined in (14),  $U_{\max}$  is the maximum admissible voltage across the actuator coil; let us denote by  $J$  the function between brackets in (28). One can use two penalty functions  $\rho_1$  and  $\rho_2$  in order to specify the importance of

each term of the cost function. For the numerical simulation, it has been used  $\frac{\rho_2}{\rho_1} \approx 10^2$ .

Since it is virtually impossible to satisfy the constraint  $|u| < U_{\max}$  during the transient regime (according to our simulations), it then may be considered:

$$\min_{\alpha, k_1, k_{21}, k_{22}} \left( \int_0^{\infty} (\rho_1 \varepsilon_1^2 + \rho_2 h(|u| - U_{\max})) dt \right) \quad (29)$$

where the performance criterion penalizes the violation of the constraint  $|u| < U_{\max}$  throughout all simulations time domain.

## 5. SIMULATION RESULTS

In order to validate the proposed design control procedure applied to the micro-pump, it is used simplex algorithm from Matlab functions, one may thus check an optimal solution for  $\alpha, k_1, k_{21}, k_{22}$ . From the numerical simulations, it turns out that the cost function from (29) displays many local minima, so the choice for the initial starting point is important. For this reason, by reviewing the desired performance/robustness ratio of the pump-system, in conjunction with controller simplicity need, it may be tried an initialization of the search-algorithm with  $k_{21} = 0$ ,  $k_{22} = 0$  (because of the discussion drawn at the end of Section 4 regarding the *chattering effect*).

The simulations results which are shown hereafter have been performed having as priority the overall robustness, so it is chosen  $k_{21} = 10k_{22}$ . The simulation time is 0.3 seconds, and furthermore, it has been added a *white noise* of zero mean value, within the range  $\pm g$ ,  $g = 9.81m/s^2$ , into the second equation of (11), within the time interval 0.1–0.2 seconds. Since this equation corresponds to the acceleration of the piston, it makes sense to give a physical meaning to the additive unstructured perturbation. Below, the simulation results are shown with normalized values: sample perturbation signal (Fig.1), piston position and piston reference signal (Fig.2), tracking error i.e.  $\varepsilon_1 = x_1 - x_{1ref}$  (Fig.3), the command  $u$  i.e. the voltage applied across the actuator coil (Fig.4); current intensity (Fig.5) and the cost function  $J$  as defined in (29) (Fig.6).

One may observe the *efficient* and *robust* behaviour of the overall system in spite of the presence of the perturbation; the command  $u$  and the current intensity  $I$  remain within the physical boundaries.

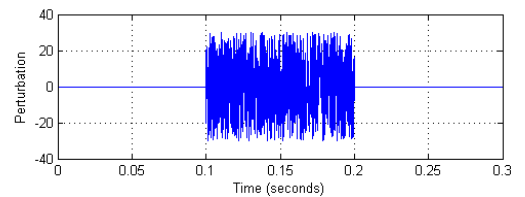


Fig. 1. Additive white noise perturbation

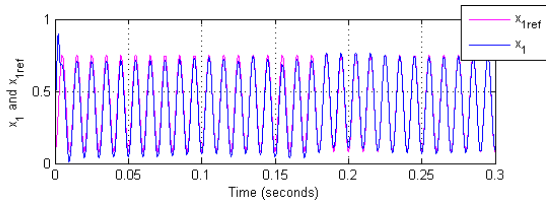


Fig. 2. Normalized piston position and its reference signal

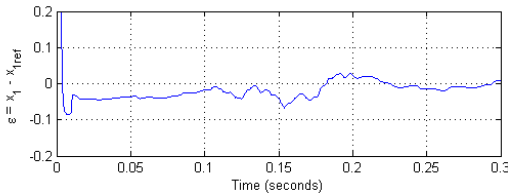


Fig. 3. Normalized tracking error signal

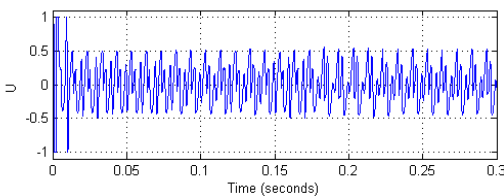


Fig. 4. Normalized voltage (command  $u$ )

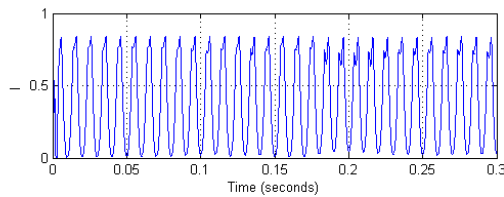


Fig. 5. Normalized current intensity

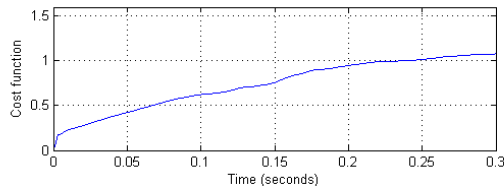


Fig. 6. Cost function  $J$

*Remark:* In Fig.4, the fact that there is no *chattering effect* in spite of choosing  $k_{22} \neq 0$  suggests that the sliding surface has not been reached throughout the simulation time domain of 0.3 seconds.

## 6. CONCLUSION

An efficient control technique applied to a new concept of micro-pump for small satellite propulsion system has been presented, consisting of mixing backstepping with sliding modes in conjunction with a simplification hypothesis (decoupling the system into two subsystems: *slow* and *fast* one). The choice of the adjustment parameters involved in the command is made by using an appropriate optimization technique. Theoretical concepts have been presented together with the simulation results which are shown to be quite satisfactory in nominal case as well as in case where a perturbation is acting, this is in order to check about the level

efficiency of the robustness property. The command remains within the physical boundaries and the manufacturer's imposed constraints are met. Nevertheless, the major drawback of this controller is related to its relative complexity, at least for the actual available implementation device sampling time in our experimental laboratory.

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