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COMBINING MAGNETIC AND LORENTZ ATTITUDE CONTROL SYSTEMS TO SOLVE FIVE SATELLITE STABILIZATION PROBLEMS

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Abstract

A satellite moving in the gravitational and magnetic fields of Earth is considered. The possibility of designing an integrated attitude control system that combined magnetic and Lorentz control systems is being studied. The expediency of such an association is shown. The effectiveness of the constructed electrodynamic attitude control system for stabilization of different programmed motions of the satellite is confirmed. An algorithm for constructing control torques is shown for each of the considered problems. The results of the computer simulation are presented. The three-axial stabilization of a satellite in the orbital frame requires restoring and dissipative components of control torques. In general case for monoaxial stabilization of a satellite in the orbital frame the control parameters can be constructed as a sum of restoring, dissipative, and compensating components. Stabilization in two-axis programmed rotation requires one to compensate for the specified gyroscopic torque, and an additional term can be introduced into one of the control vectors. For the problem of three-axial stabilization in the Koenig frame, the gravity gradient torque can be compensated by means of the Lorentz torque. Finally, the three-axial stabilization of a satellite in the magneto-Lorentz frame also involves the creation of a compensating torque.

Key words

Satellite, attitude stabilization, magnetic torque, Lorentz torque, electrodynamic attitude control system

1 Introduction

A lot of forces and torques act on a satellite in near-Earth space [Beletsky, 1966; Ivanov et al., 2020]. Among them are the forces and torques caused by the influence of the gravitational and magnetic fields of the Earth [Lovera and Astolfi, 2006; Zhou et al., 2017; Ovchinnikov and Roldugin, 2019], rarefied near-Earth plasma [Sarychev et al., 2007; Barinova et al., 2023; Somov et al., 2024], and the light pressure of the Sun [Mashtakov et al., 2018; Somov et al., 2022b]. The practice of creating and developing systems for satellite attitude stabilization indicates that despite attempts to use a wide variety of forces and torques arising in near-Earth space, systems based on the use of gravitational and magnetic fields of the Earth remain the most common. Interesting options for creating integrated satellite attitude control systems using force factors of different nature have been known almost since the very beginning of the space age. First of all, these are systems that use both the gravitational and magnetic fields of the Earth. As is known, the gravity gradient torque provides the possibility of passive orientation of the satellite in the orbital coordinate system [Beletsky, 1966]. However, the gravity gradient torque alone is not enough to stabilize the satellite in the orbital coordinate system. A libration damping system, passive or active, is also required [L. Gaul and Sachau, 1998]. In many cases, damping systems rely on the capabilities provided by the Earth's magnetic field and are implemented on the basis of spherical liquid dampers, hysteresis rods, and dampers based on Foucault currents.

In addition, the Earth's magnetic field provides an opportunity to implement an active magnetic attitude control system using the interaction of the Earth's magnetic field (with magnetic induction $\vec{B}$) with a controlled intrinsic magnetic moment $\vec{I}$ [Arslanova et al., 2023] of the satellite by means of the magnetic torque $\vec{M}_M$ (Fig. 1).

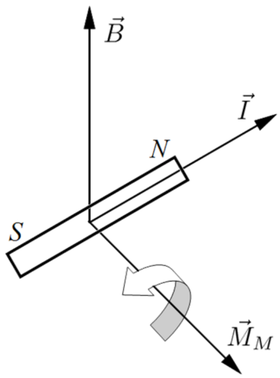

Figure 1. $\vec{M}_M$ torque

Active magnetic attitude control systems are capable of solving various and quite complex problems [Ovchinnikov and Roldugin, 2021; Roldugin, 2023]. These systems have the following advantages: high reliability, low power consumption, small size and mass. The disadvantage of active magnetic attitude control system include the limitation on the magnitude of the control torque, which does not allow quick satellite reorientation. Moreover, active magnetic attitude control system cannot create a control torque along the vector $\vec{B}$.

Another possibility for creating a control torque using the Earth's magnetic field is related to the presence of a charge Q on the satellite. Such a charge can be caused not only by precipitation from the rarefied near-Earth plasma but also by an artificial charging process, for example, in order to provide active anti-radiation protection. As shown in articles [Tikhonov and Tkhai, 2016], the charge can significantly affect the dynamics of the rotational motion of the satellite, especially if the satellite center of mass (point C) does not coincide with the satellite center of charge (point O). And the presence of a charge controlled by magnitude Q and position $\vec{\rho}_0 = \overrightarrow{CO}$ can be the basis for creating a Lorentz attitude control system, also called a magneto-Coulombic attitude control system [Giri and Sinha, 2014; Giri and Sinha, 2017; Aleksandrov and Tikhonov, 2020; Prabhat et al., 2022]. The Lorentz torque can also be realized in a different way when an electrically neutral satellite as a whole carries opposite charges separated in opposite directions (Fig. 2).

This paper focuses on the analysis of an integrated satellite attitude control system based on magnetic and Lorentz systems. It is shown that such an approach to the design of an attitude control system solves the underactuation problem in magnetic and Lorentz attitude control systems. Thus, in [Antipov and Tikhonov, 2007], the concept of an electrodynamic control system was proposed, which quickly became popular. In subsequent years, using an electrodynamic control system, a number of relevant applied problems related to the satellite attitude control were solved. These studies are reflected in the papers [Aleksandrov and Tikhonov, 2012; Aleksandrov and Tikhonov, 2013; Aleksandrov et al., 2016; Klyushin et al., 2024] and cited therein.

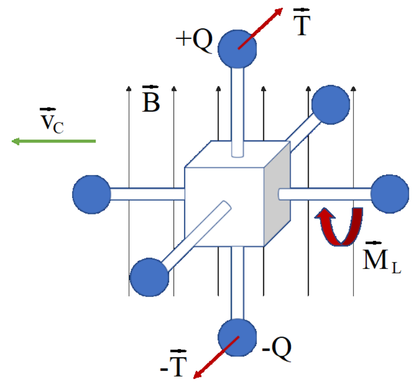

Figure 2. $\vec{M}_L$ torque

However, the mentioned papers did not consider the problem of how much control efficiency increases if an electrodynamic control system is used instead of a magnetic or Lorentz control system taken separately. The range of problems, the solution of which is possible only on the basis of an integrated electrodynamic control system, has also not been studied. This work is aimed at filling these shortcomings.

2 Programmed attitude motion and control design

In accordance with general idea of electrodynamic attitude control system [Antipov and Tikhonov, 2007], there are two control torques originated from the geomagnetic field with magnetic induction $\vec{B}$: the Lorentz torque $\vec{M}_L$ and the magnetic torque $\vec{M}_M$ which have, respectively, the following forms

$$\vec{M}_L = \vec{P} \times \vec{T}, \qquad \vec{M}_M = \vec{I} \times \vec{B}. \tag{1}$$

Here, $\vec{P} = Q\vec{\rho}_0$, $\vec{T} = \vec{v}_C \times \vec{B}$, and $\vec{v}_C$ is velocity of the satellite mass center with respect to the Greenwich reference frame [Somov et al., 2023].

The torques (1) can be used to solve various problems of satellite attitude control. In this connection, the control torques (1) are constructed in different ways depending on the specific programmed motions discussed in this section.

2.1 Threeaxial stabilization of a satellite in the orbital frame

Consider a satellite whose center of mass moves in the Newtonian central Earth's gravitational field along Keplerian circular orbit of radius R. We study satellite attitude motion with respect to the orbital frame $C\xi\eta\zeta$, which is often chosen as the base frame due to convenience in solving a number of practically important problems [Somov et al., 2022a]. The axis $C\xi(\vec{\xi}_0)$ is directed along the tangent to the orbit towards the motion, the axis $C\eta(\vec{\eta}_0)$ is directed along the normal line toward the orbit plane, the axis $C\zeta(\vec{\zeta}_0)$ is directed along the local vertical. Orientation of principal central axes of inertia $Cxyz$ of the satellite with respect to axes $C\xi\eta\zeta$ is defined by matrix $\mathbf{A}$ of direction cosines α_i, β_i, γ_i $(i = 1, 2, 3)$ so that the equalities

$$\begin{aligned}\vec{\xi}_0 &= \alpha_1\vec{i} + \alpha_2\vec{j} + \alpha_3\vec{k},\\ \vec{\eta}_0 &= \beta_1\vec{i} + \beta_2\vec{j} + \beta_3\vec{k},\\ \vec{\zeta}_0 &= \gamma_1\vec{i} + \gamma_2\vec{j} + \gamma_3\vec{k}\end{aligned}$$

are valid. The programmed satellite orientation in the orbital frame is defined by matrix $\mathbf{A}_0$ of direction cosines. Let $\vec{\omega}' = p\,\vec{i} + q\,\vec{j} + r\,\vec{k}$ be satellite angular velocity with respect to the orbital frame, the satellite position for which

$$\mathbf{A} = \mathbf{A}_0, \quad \vec{\omega}' = \vec{0} \tag{2}$$

is considered as the programmed satellite attitude motion. The control torques (1) which ensure the motion (2) are constructed in [Antipov and Tikhonov, 2007] by means of controlled parameters

$$\vec{P} = \vec{P}_{rest} + \vec{P}_{diss}, \quad \vec{I} = \vec{I}_{rest} + \vec{I}_{diss}, \tag{3}$$

where $\vec{P}_{rest} = Qk_L\vec{T}_0$ and $\vec{I}_{rest} = k_M\vec{B}_0$ support generation of restoring components of the torques $\vec{M}_L$ and $\vec{M}_M$ respectively, whereas $\vec{P}_{diss} = Qh_L\,\vec{\omega}' \times \vec{T}$ and $\vec{I}_{diss} = h_M\,\vec{\omega}' \times \vec{B}$ support generation of dissipative components, providing active damping [Shelenok, 2024] using the same torques. Thus, the control torques (1) take on the form

$$\begin{aligned}\vec{M}_L &= Qk_L\vec{T}_0 \times \vec{T} + Qh_L(\vec{\omega}' \times \vec{T}) \times \vec{T},\\ \vec{M}_M &= k_M\vec{B}_0 \times \vec{B} + h_M(\vec{\omega}' \times \vec{B}) \times \vec{B}.\end{aligned}$$

The coefficients k_L, k_M, h_L, h_M may be scalar functions of time, for example as follows:

$$\begin{aligned}k_L &= \frac{k_{L0}}{Q\|\vec{T}(t)\|^2}, \quad k_M = \frac{k_{M0}}{\|\vec{B}(t)\|^2},\\ h_L &= \frac{h_{L0}}{Q\|\vec{T}(t)\|^2}, \quad h_M = \frac{h_{M0}}{\|\vec{B}(t)\|^2}.\end{aligned}$$

Here, k_{L0}, k_{M0}, h_{L0}, h_{M0} are positive constants that can be chosen. Consider a typical satellite that moves in orbit with radius $R = 7 \cdot 10^6$ m and inclination $i = 60^o$. Gravitational and electrostatic properties of the satellite are described by the values of principal central moments of inertia $A = 1000\,\mathrm{kg{\cdot}m^2}$, $B = 600\,\mathrm{kg{\cdot}m^2}$, $C = 1400\,\mathrm{kg{\cdot}m^2}$ and the charge $Q = 5 \cdot 10^{-3}\,\mathrm{C}$. The control torque $\vec{M}_L$ is characterized by coefficients $k_{L0} = 2.5 \cdot 10^{-3}$, $h_{L0} = 0.2$, and the control torque $\vec{M}_M$ is characterized by coefficients $k_{M0} = 2 \cdot 10^{-3}$, $h_{M0} = 1.0$.

The results of computer modeling (Figs. 3, 4, 5) make it evident that in the case where Lorentz and magnetic attitude control systems alone cannot solve the problem, the integrated electrodynamic attitude control system solves the problem successfully. The "airborne" angles φ, θ, ψ (roll, pitch, and yaw) are equal to zero in the programmed satellite attitude motion with $\mathbf{A}_0 = \mathrm{diag}(1, 1, 1)$. The dimensionless variable $u = \omega_0 t$ is an argument of latitude of the satellite.

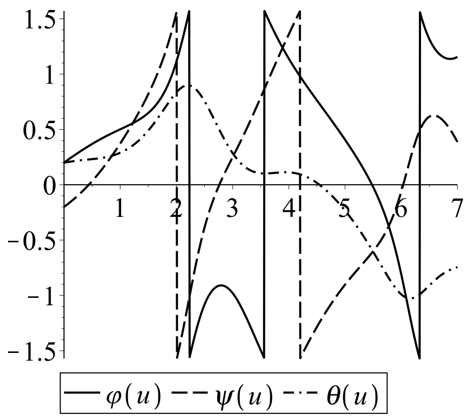

Figure 3. M_L only operating

2.2 Monoaxial stabilization of a satellite in the orbital frame

Let $\vec{\sigma}_0$ be the ort of some axis fixed in the orbital frame. In projections on the xyz axes, this ort, hereinafter referred to as $\vec{s}_0$, has the form

$$\mathbf{A}^\top\vec{\sigma}_0 = \vec{s}_0 = c_1\vec{s}_1 + c_2\vec{s}_2 + c_3\vec{s}_3,$$

where $c_1 = $ const, $c_2 = $ const, $c_3 = $ const. It is unchanged in the orbital frame. Let $\vec{r}_0$ be some ort

$$\vec{r}_0 = x_0\vec{i} + y_0\vec{j} + z_0\vec{k},$$

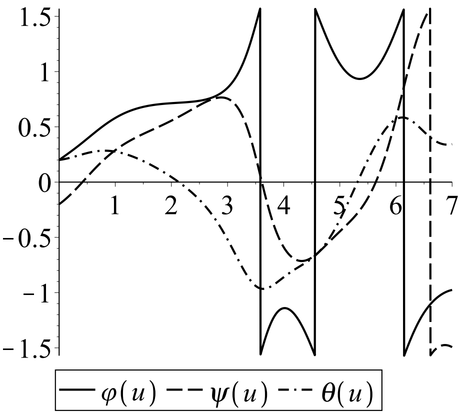

Figure 4. M_M only operating

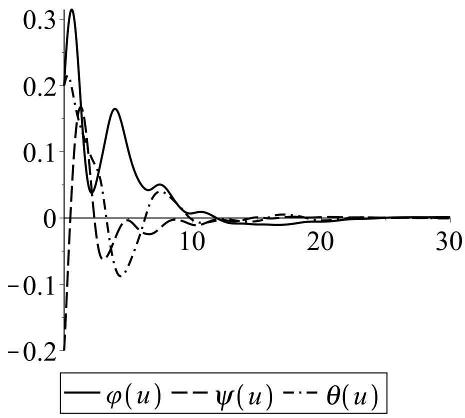

Figure 5. Both M_L and M_M

where x_0 = const, y_0 = const, z_0 = const, which is unchanged in the coordinate system $Cxyz$ rigidly connected to the satellite. The problem is to construct control torques (1) that ensure the existence and asymptotic stability of such a position of the satellite in the orbital frame in which

$$\vec{r}_0 = \vec{s}_0, \quad \vec{\omega}' = 0. \tag{4}$$

On the basis of dynamic Euler equations, it can be shown [Aleksandrov and Tikhonov, 2013] that controlled parameters taken in the form (3) can not solve the problem since the gravity gradient torque and inertial term result in the torque

$$\vec{g} = 3\omega_0^2 \vec{s}_3 \times (\mathbf{J}\vec{s}_3) - \omega_0^2 \vec{s}_2 \times (\mathbf{J}\vec{s}_2) \tag{5}$$

that destroys any non-straight relative equilibrium position of the satellite in the orbital frame. In order to compensate for the torque (5), the controlled parameters can be constructed in the form

$$\vec{P} = \vec{P}_{comp} + \vec{P}_{rest} + \vec{P}_{diss}, \ \vec{I} = \vec{I}_{comp} + \vec{I}_{rest} + \vec{I}_{diss},$$

where

$$\vec{P}_{comp} = g_1 \frac{(\vec{B} \times \vec{T})}{|\vec{B}||\vec{T}|^2}, \ \vec{I}_{comp} = g_3 \frac{\vec{T}}{|\vec{B}||\vec{T}|} - g_2 \frac{(\vec{B} \times \vec{T})}{|\vec{B}|^2|\vec{T}|},$$

and g_1, g_2 and g_3 are the projections of the torque $\vec{g}$ on the axes with orts

$$\vec{b} = \frac{\vec{B}}{|\vec{B}|}, \quad \vec{t} = \frac{\vec{T}}{|\vec{T}|}, \quad \vec{w} = \frac{\vec{B} \times \vec{T}}{|\vec{B}||\vec{T}|}.$$

The same procedure can be applied for the construction of the restoring component $\vec{M}_{rest}$ of the control torque. Let $\vec{M}_{rest}$ have the form $\vec{M}_{rest} = k_0 \vec{r}_0 \times \vec{s}_0$, then it can be found as the sum

$$k_0 \vec{r}_0 \times \vec{s}_0 = \vec{P}_{rest} \times \vec{T} + \vec{I}_{rest} \times \vec{B},$$

where [Aleksandrov and Tikhonov, 2013]

$$P_{rest1} = P_{rest2} = 0, \ P_{rest3} = -k_0 \vec{b}(\vec{r}_0 \times \vec{s}_0)/|\vec{T}|,$$
$$I_{rest1} = 0, \ I_{rest2} = -k_0 \vec{w}(\vec{r}_0 \times \vec{s}_0)/|\vec{B}|,$$
$$I_{rest3} = k_0 \vec{t}(\vec{r}_0 \times \vec{s}_0)/|\vec{B}|.$$

The dissipative components of control torques can be created as suggested in [Antipov and Tikhonov, 2007] by choosing the following components of controlled parameters: $\vec{P}_{diss} = h_\Lambda \vec{\omega}' \times \vec{T}$, $\vec{I}_{diss} = h_M \vec{\omega}' \times \vec{B}$.

As a result, the control torques are as follows:

$$\begin{aligned} \vec{M}_L &= (\vec{P}_{rest} + \vec{P}_{diss} + \vec{P}_{comp}) \times \vec{T}, \\ \vec{M}_M &= (\vec{I}_{rest} + \vec{I}_{diss} + \vec{I}_{comp}) \times \vec{B}. \end{aligned} \tag{6}$$

Therefore, in the problem under consideration, the programmed motion (4) can be implemented only with the simultaneous action of both restoring torques. As for dissipative torques, they can be used independently or together at the same time. For computer modeling consider a satellite with principal central moments of inertia $A = 1000\,\text{kg}\cdot\text{m}^2$, $B = 700\,\text{kg}\cdot\text{m}^2$, $C = 800\,\text{kg}\cdot\text{m}^2$ which moves in a circular equatorial orbit with radius $R = 7 \cdot 10^6\,\text{m}$. The other parameters are as follows: $Q = 5 \cdot 10^{-3}\,\text{C}$, $k_0 = 0.1$, $h_{L0} = 0.0983$, $h_{M0} = 0.4966$. The programmed motion is the equilibrium $\vec{r}_0 = \vec{s}_0$ in the orbital frame, where

$$\vec{r}_0 = (\gamma_1, \gamma_2, \gamma_3)^\top = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})^\top.$$

The results of computer modeling (Figs. 6, 7, 8) make it evident that dissipative components of Lorentz (Fig. 6) and magnetic (Fig. 7) torques taken alone can solve the problem, but the integrated electrodynamic attitude control system (Fig. 8) solves the problem more effectively, as it reduces the convergence time.

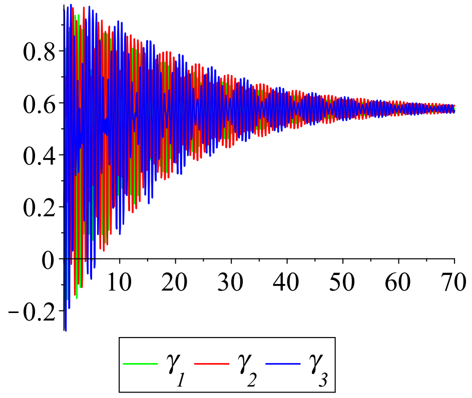

Figure 6. M_{Ldiss} only operating

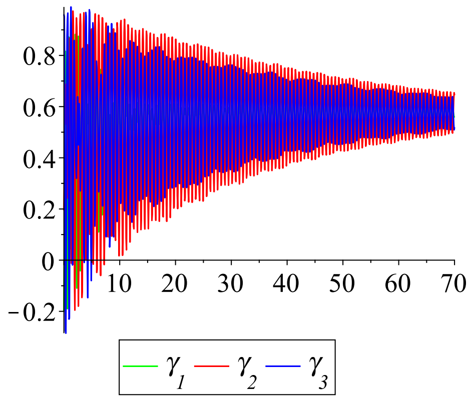

Figure 7. M_{Mdiss} only operating

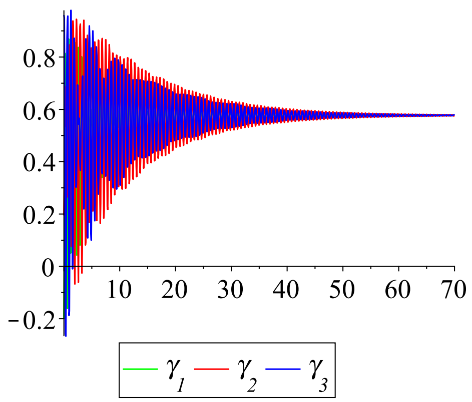

Figure 8. M_{Ldiss} and M_{Mdiss}

2.3 Stabilization of an Earth-pointing satellite in the two-axis programmed rotation

Further, we will consider the case where the Cz axis of the dynamic symmetry of the satellite is stabilized along the local vertical $C\zeta$, and the satellite itself performs a uniform rotation around this axis [Aleksandrov and Tikhonov, 2012] with an angular velocity of $\vec{\omega}_0' = \mu\vec{k} = (0, 0, \mu)^\top$. For this mode of motion,

$$\vec{r}_1 = (\cos(\mu t), -\sin(\mu t), 0)^\top,$$
$$\vec{r}_2 = (\sin(\mu t),\ \cos(\mu t),\ 0)^\top,$$
$$\vec{r}_3 = (0, 0, 1)^\top.$$

The control torques (1) should ensure the existence and asymptotic stability of the following regime of two-axis programmed rotation of the satellite:

$$\vec{\omega}\,' = \vec{\omega}\,'_0\,, \quad \vec{s}_i = \vec{r}_i\,, \quad i = 1, 2, 3. \tag{7}$$

To ensure uniform rotation of the satellite in the rotating orbital coordinate system, it will be necessary to compensate for the resulting gyroscopic torque directed along the $C\zeta$ axis. In order to compensate for the specified gyroscopic torque, an additional term $k_{L1}\vec{r}_2$ can be introduced into vector $\vec{P}$ or an additional term $k_{M1}\vec{r}_3$ into vector $\vec{I}$. The coefficients in these terms, as it is easy to verify, are selected from the condition of the existence of two-axis programmed rotation (7) based on the dynamic Euler equations [Aleksandrov and Tikhonov, 2012], and are respectively equal

$$k_{L1} = C\omega_0\mu/(v_{C\xi}B_\eta), \quad k_{M1} = -C\omega_0\mu/B_\eta.$$

In the case when two additional terms are introduced ($k_{L1}\vec{r}_2$ in the vector $\vec{P}$ and $k_{M1}\vec{r}_3$ in the vector $\vec{I}$), each of them must contain an additional multiplier, and the sum of these multipliers should be equal to one. Taking into account the restoring and dissipative components, the control torques are as follows:

$$\vec{M}_L = k_L\vec{T}_0 \times \vec{T} + h_L(\vec{\omega}_r' \times \vec{T}) \times \vec{T}$$
$$+\varepsilon_L k_{L1}\vec{r}_2 \times \vec{T},$$
$$\vec{M}_M = k_M\vec{B}_0 \times \vec{B} + h_M(\vec{\omega}_r' \times \vec{B}) \times \vec{B}$$
$$+\varepsilon_M k_{M1}\vec{r}_3 \times \vec{B}.$$

Here $\varepsilon_L + \varepsilon_M = 1$. Each of the control torques $\vec{M}_L$ and $\vec{M}_M$ taken alone can solve the stabilization problem (7). Consider these cases and compare the corresponding solutions with those in the case where both control torques $\vec{M}_L$ and $\vec{M}_M$ operate simultaneously in the integrated control system. In the last case, let us choose $\varepsilon_L = \varepsilon_M = 0.5$.

For computer modeling consider a satellite [Aleksandrov and Tikhonov, 2012] with parameters $A = B = 1000\,\text{kg}\cdot\text{m}^2, C = 500\,\text{kg}\cdot\text{m}^2$, $Q = 5\cdot 10^{-3}\,\text{C}$ which moves in a circular equatorial orbit with radius $R = 7\cdot 10^6$ m. The programmed motion is the two-axis rotation $\varphi = 0$, $\theta = 0$, $\psi = \mu t$, where $\mu = 3\,\omega_0$. The results of computer modeling (Figs. 9, 10, 11) are shown in terms of quaternion components calculated for one and the same parameters and initial values. For convenience and clarity, the components of the quaternion are given in deviations from the programmed motion (7).

It can be seen that the Lorentz control torque $\vec{M}_L$ cannot stabilize (Fig. 9) the programmed motion (7). Magnetic control torque $\vec{M}_M$ performs well (Fig. 10). However, both control torques $\vec{M}_L$ and $\vec{M}_M$ in an integrated control system provide higher stabilization accuracy at the end of the time interval (Fig. 11).

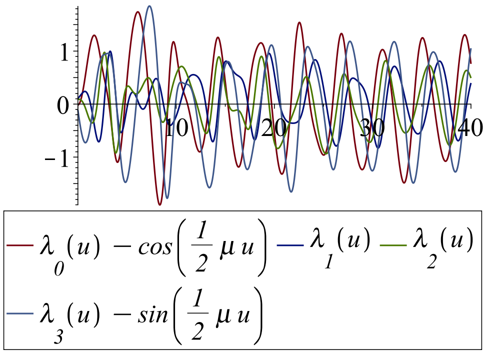

Figure 9. M_L only operating

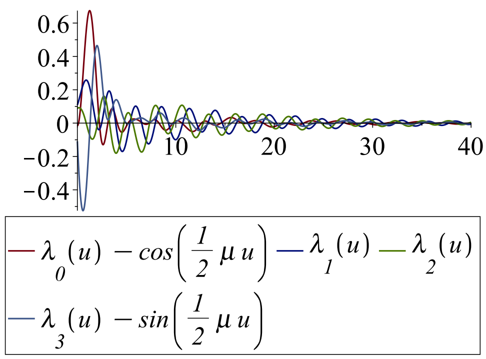

Figure 10. M_M only operating

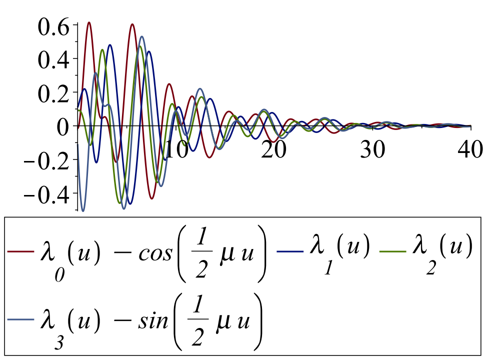

Figure 11. Both M_L and M_M

2.4 Threeaxial stabilization of a satellite in the Koenig frame

In this subsection, we consider the problem of satellite attitude stabilization in the Koenig frame [Aleksandrov et al., 2016]. This problem is relevant not only for satellites but also for different mechanical systems modeled by rigid bodies. Let us consider the Koenig coordinate system $CXYZ$, where the axis CY is orthogonal to the orbital plane. Without loss of generality, let us assume that in the programmed orientation, the main central axes of inertia of the satellite coincide with the axes of the Koenig frame of the same name. Then, the programmed attitude motion of the satellite is described by the following equations:

$$\begin{gathered} \vec{s}_1 = \vec{r}_1 = (\cos u, 0, -\sin u)^\top, \\ \vec{s}_2 = \vec{r}_2 = (0, 1, 0)^\top, \\ \vec{s}_3 = \vec{r}_3 = (\sin u, 0, \cos u)^\top, \ \vec{\omega} = \vec{0}. \end{gathered} \tag{8}$$

The gravity gradient torque occurs when the constantly acting disturbing torque is in the programmed motion (8). It is shown in [Aleksandrov et al., 2016] that the gravity gradient torque can be compensated by means of the Lorentz torque. Thus, the control torques are constructed in the form

$$\begin{gathered} \vec{M}_L = (\vec{P}_{rest} + \vec{P}_{diss} + \vec{P}_{comp}) \times \vec{T}, \\ \vec{M}_M = (\vec{I}_{rest} + \vec{I}_{diss}) \times \vec{B}. \end{gathered} \tag{9}$$

However, despite the fact that compensation for the constantly acting perturbing gravitational torque can only be provided by the Lorentz torque, it turns out that the Lorentz torque is not enough to implement the programmed motion (8). For example, consider a satellite with parameters $A = 1000\,\text{kg}{\cdot}\text{m}^2$, $B = 1700\,\text{kg}{\cdot}\text{m}^2$, $C = 800\,\text{kg}{\cdot}\text{m}^2$, $Q = 5{\cdot}10^{-3}$ C. As shown in Fig. 12, no stabilization was observed in the case of only the Lorentz torque operating. Stabilization of the programmed motion (8) is also absent in the case of exposure only to magnetic restoring and dissipative torques (Fig. 13), despite the presence of a compensating Lorentz torque. Only in the presence of all terms in the control torques (9) does the convergence of the control process with the programmed motion (8) occur (Fig. 14).

Here, as in the previous subsection, the results of computer modeling are expressed in terms of quaternion components calculated for one and the same parameters and initial values.

2.5 Threeaxial stabilization of a satellite in the magneto-Lorentz frame

The coordinate system naturally combined with the electrodynamic attitude control method is the magneto-Lorentz coordinate system, the orts of which $\vec{b}$, $\vec{t}$, $\vec{w}$ are directed, respectively, along the vectors $\vec{B}$, $\vec{T}$, $\vec{B} \times \vec{T}$ [Tikhonov, 2021]. Therefore, it can be used as a basic coordinate system for solving a number of dynamic

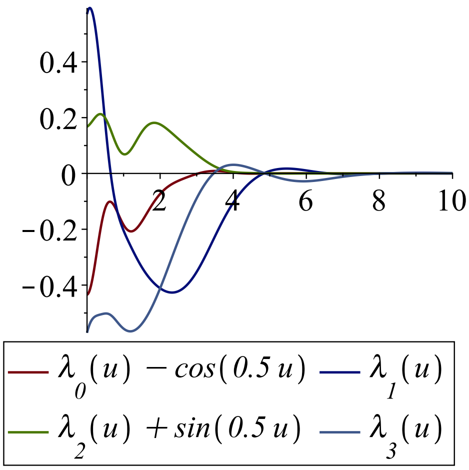

Figure 14. Both M_L and M_M

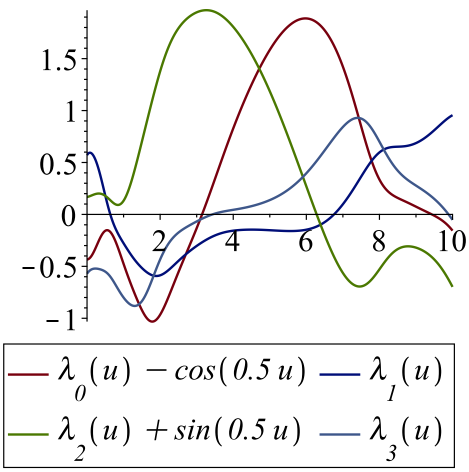

Figure 12. M_L only operating

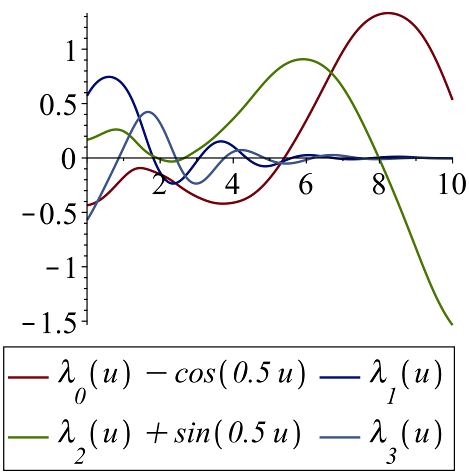

Figure 13. M_M only operating

problems of a satellite, equipped with intrinsic magnetic moment and electric charge [Aleksandrov and Tikhonov, 2023]. This subsection discusses the issues of satellite attitude stabilization in the magneto-Lorentz coordinate system.

Let the unit vectors $\vec{i}$, $\vec{j}$, $\vec{k}$ coincide, respectively, with the unit vectors $\vec{w}$, $\vec{b}$, $\vec{t}$ of the basic coordinate frame in the programmed attitude motion. Denote $\vec{\sigma}_1$, $\vec{\sigma}_2$, $\vec{\sigma}_3$ the unit vectors $\vec{w}$, $\vec{b}$, $\vec{t}$ of the basic coordinate frame, expressed in the basis $\vec{i}$, $\vec{j}$, $\vec{k}$. Then the programmed attitude motion of the satellite is defined by the equations

$$\begin{aligned}
\vec{\sigma}_1 &= \vec{\rho}_1 = (1,0,0)^\top, \\
\vec{\sigma}_2 &= \vec{\rho}_2 = (0,1,0)^\top, \\
\vec{\sigma}_3 &= \vec{\rho}_3 = (0,0,1)^\top, \\
\vec{\omega}' &= \vec{0}.
\end{aligned} \tag{10}$$

In the programmed motion (10), the satellite performs an attitude motion with an angular velocity $\vec{\omega}_b$ of the magneto-Lorentz coordinate system. An explicit expression for the angular velocity $\vec{\omega}_b$ as a function of time is obtained in [Tikhonov, 2021]. As follows from the Euler dynamic equations, the gravitational moment $\vec{M}_G$ and the inertial terms form a constantly acting disturbing torque $\vec{M}_d = \vec{M}_G - \mathbf{J}(\dot{\vec{\omega}}_b)_{xyz} - \vec{\omega}_b \times (\mathbf{J}\vec{\omega}_b)$, where $(\dot{\vec{\omega}}_b)_{xyz}$ is the local derivative of $\vec{\omega}_b$ over time calculated in the coordinate system $Cxyz$. Therefore, the construction of a stabilizing electrodynamic control that ensures the operation of the satellite in mode (10) includes the creation of compensating torques. In [Aleksandrov and Tikhonov, 2023] it is shown that additional components $\vec{P}_{comp} = |\vec{T}|^{-1}(M_{db}\vec{\rho}_1 - M_{dw}\vec{\rho}_2)$ and $\vec{I}_{comp} = -|\vec{B}|^{-1}M_{dt}\vec{\rho}_3$ should be added to controlled vectors $\vec{P}$ and $\vec{I}$, respectively. As a result, the control torques have the form (6). Therefore, in the problem under consideration, the programmed motion (10) can be implemented only with the simultaneous action of the both compensating torques $\vec{M}_{Lcomp}$ and $\vec{M}_{Mcomp}$.

As for restoring and dissipative components, they can be used independently or together at the same time. For computer modeling, consider the same satellite as in [Aleksandrov and Tikhonov, 2023]. It moves in a circular orbit with radius $R = 7 \cdot 10^6$ m and inclination $i = 30^o$. Inertial parameters and electric charge are as follows: $A = 1000\,\text{kg}{\cdot}\text{m}^2$, $B = 600\,\text{kg}{\cdot}\text{m}^2$, $C = 1400\,\text{kg}{\cdot}\text{m}^2$, $Q = 5 \cdot 10^{-3}$ C.

The case when only the Lorentz restoring and dissipative components are operating is shown in Fig. 15. Obviously, there is no stabilization process. A similar result is observed in the case where only magnetic restoring and dissipative torques act (Fig. 16). Only when Lorentz and magnetic torques are combined, the process of stabilization of the satellite programmed motion (10) is observed (Fig. 17).

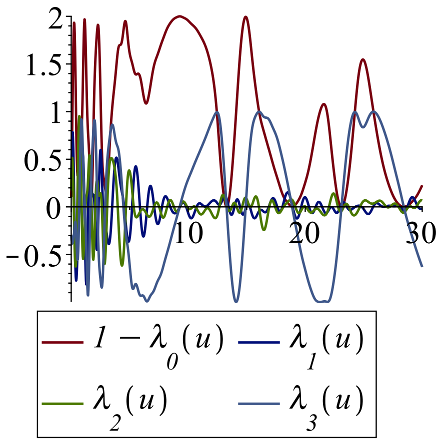

Figure 15. M_L only operating

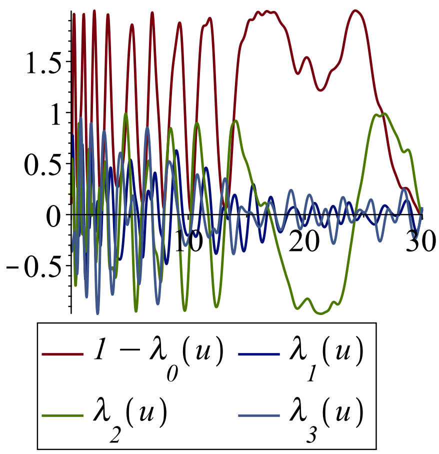

Figure 16. M_M only operating

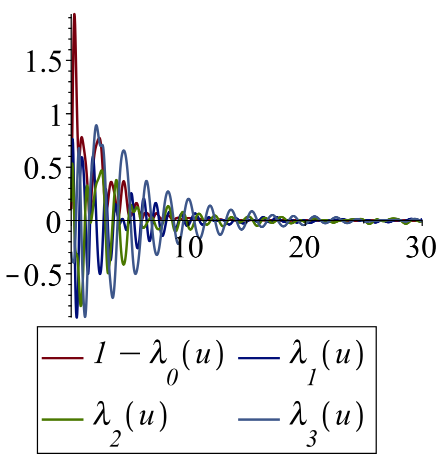

Figure 17. Both M_L and M_M

3 Conclusion

Five problems of satellite attitude stabilization are considered, which are successfully solved using an integrated electrodynamic attitude control system based on the simultaneous use of the Lorentz torque and the torque of magnetic interaction. It is established that attempts to simplify the control system based on the refusal to use any one of the mentioned torques lead either to the failure of the mission or to a less effective solution. It is also established that, for some of the problems discussed above, the realization of the desired attitude motion of the satellite is fundamentally impossible without the simultaneous use of the Lorentz torque and the torque of magnetic interaction. In all the problems of satellite attitude stabilization considered above, the solution was made taking into account the disturbing effect of the gravitational torque, which is the most significant for typical satellites in medium-altitude orbits.

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References

Aleksandrov, A. Y., Antipov, K. A., Platonov, A. V., and Tikhonov, A. A. (2016). Electrodynamic Stabilization of Artificial Earth Satellites in the Konig Coordinate System. *Journal of Computer and Systems Sciences International*, **55** (2), pp. 296–309.

Aleksandrov, A. Y. and Tikhonov, A. A. (2012). Electrodynamic stabilization of program rotation of artificial Earth satellite in the orbital coordinate system. *Vestnik St. Petersburg University. Mathematics*, **50** (4), pp. 335–340. in Russian.

Aleksandrov, A. Y. and Tikhonov, A. A. (2013). Monoaxial electrodynamic stabilization of Earth satellite in the orbital coordinate system. *Automation and Remote Control*, **74** (8), pp. 1249–1256.

Aleksandrov, A. Y. and Tikhonov, A. A. (2020). Averaging technique in the problem of Lorentz attitude stabilization of an Earth-pointing satellite. *Aerospace Science and Technology*, **104**.

Aleksandrov, A. Y. and Tikhonov, A. A. (2023). Natural magneto-velocity coordinate system for satellite attitude stabilization: Dynamics and stability analysis. *Journal of Applied and Computational Mechanics*, **9** (2), pp. 513–520.

Antipov, K. A. and Tikhonov, A. A. (2007). Parametric control in the problem of spacecraft stabilization in the geomagnetic field. *Autom. Remote Control*, **68** (8), pp. 1333–1345.

Arslanova, D., Gavrish, Y., Gapionok, E., Grosheva, A., Knyazev, N., Kukhtin, V., Lamzin, E., Nezhentzev, A., Ovsyannikov, A., Ovsyannikov, D., Sytchevsky, S., and Znamenshchikova, N. (2023). Feasibility study of permanent magnet dipoles for sila facility. *Cybernetics and Physics*, **12** (4), pp. 252–256.

Barinova, E. V., Belokonov, I. V., Elisov, N. A., Kramlikh, A. V., Lomaka, I. A., Nikolaev, P. N., and Timbai, I. A. (2023). Some Features of Dynamics and Attitude Control of Nanosatellites in Low Orbits. *Gyroscopy and Navigation*, **14** (3), pp. 183–204.

Beletsky, V. V. (1966). *Motion of an Artificial Satellite about its Center of Mass*. Israel Program for Scientific Translation, Jerusalem.

Giri, D. K. and Sinha, M. (2014). Magneto-Coulombic Attitude Control of Earth-Pointing Satellites. *Journal of Guidance, Control, and Dynamics*, **37** (6), pp. 1946–1960.

Giri, D. K. and Sinha, M. (2017). Three-axis attitude control of Earth-pointing isoinertial magneto-Coulombic satellites. *International Journal of Dynamics and Control*, **5** (3), pp. 644–652.

Ivanov, D. S., Ovchinnikov, M. Y., Roldugin, D. S., Tkachev, S. S., Trofimov, S. P., Shestakov, S. A., and Shirobokov, M. G. (2020). Software package for simulating the angular and orbital motion of a satellite. *Mathematical Models and Computer Simulations*, **12** (4), pp. 561–568.

Klyushin, M. A., Maksimenko, M. V., and Tikhonov, A. A. (2024). Electrodynamic attitude stabilization of a spacecraft in an elliptical orbit. *Aerospace*, **11** (11).

L. Gaul, J. L. and Sachau, D. (1998). Active damping of space structures by contact pressure control in joints. *Mechanics of Structures and Machines*, **26** (1), pp. 81–100.

Lovera, M. and Astolfi, A. (2006). Global magnetic attitude control of spacecraft in the presence of gravity gradient. *IEEE Transactions on Aerospace and Electronic Systems*, **42**, pp. 796–805.

Mashtakov, Y., Ovchinnikov, M., Petrova, T., and Tkachev, S. (2018). Attitude and relative motion control of satellites in formation flying via solar sail with variable reflectivity properties. In *Proceedings of the International Astronautical Congress, IAC, Bremen*, 10.

Ovchinnikov, M. Y. and Roldugin, D. S. (2019). A survey on active magnetic attitude control algorithms for small satellites. *Progress in Aerospace Sciences*, **109**, pp. 100546.

Ovchinnikov, M. Y. and Roldugin, D. S. (2021). Magnetic attitude control and periodic motion for the in-orbit rotation of a dual-spin satellite. *Acta Astronautica*, **186**, pp. 203–210.

Prabhat, H., Mukherjee, B. K., Giri, D. K., and Sinha, M. (2022). Fault-tolerant sliding mode satellite attitude stabilization using magneto-coulombic torquers. *Aerospace Science and Technology*, **121**, pp. 107316.

Roldugin, D. S. (2023). Extensive numerical simulation of a magnetically actuated satellite rotation around the sun direction with sun sensors data only. *Mathematical Models and Computer Simulations*, **15** (5), pp. 792–801.

Sarychev, V., Mirer, S., Degtyarev, A., and Duarte, E. (2007). Investigation of equilibria of a satellite subjected to gravitational and aerodynamic torques. *Celestial Mechanics and Dynamical Astronomy*, **97**, pp. 267–287.

Shelenok, Y. (2024). Combined repetitive control system for active damping of forced vibrations. *Cybernetics and Physics*, **13** (1), pp. 69–76.

Somov, Y., Butyrin, S., and Somov, S. (2023). Guidance, navigation and attitude control of mini-satellites in a low earth orbit constellation for areal survey. *Cybernetics and Physics*, **12** (2), pp. 145–151.

Somov, Y., Butyrin, S., and Somov, S. (2024). Spatial guidance and control of a space robot during flights in a low-orbit constellation of earth observation mini-satellites. *Cybernetics and Physics*, **13** (1), pp. 77–81.

Somov, Y., Butyrin, S., and Somova, T. (2022a). Physical hysteresis of the piezoelectric drive in precise image velocity stabilization system built into the earth observing orbit telescope. *Cybernetics and Physics*, **11** (3), pp. 157–164.

Somov, Y., Butyrin, S., and Somova, T. (2022b). A space robot control at approaching and inspecting a geostationary satellite state. *Cybernetics and Physics*, **11** (1), pp. 30–36.

Tikhonov, A. A. (2021). Natural magneto-velocity coordinate system for satellite attitude stabilization: The concept and kinematic analysis. *Journal of Applied and Computational Mechanics*, **7** (4), pp. 2113–2119.

Tikhonov, A. A. and Tkhai, V. N. (2016). Symmetrical oscillations of charged gyrostat in weakly elliptical orbit with small inclination. *Nonlinear Dynamics*, **85** (3), pp. 1919–1927.

Zhou, K., Huang, H., Wang, X., Sun, L., and Zhong, R. (2017). Magnetic attitude control for Earth-pointing satellites in the presence of gravity gradient. *Aerospace Science and Technology*, **60**, pp. 115–123.