# INTERNAL CONTROL IN NONEQUILIBRIUM PHYSICAL PROCESSES

**Tatiana Khantuleva, Kamil Abdullaev**
Department of Mathematics and Mechanics
Saint-Petersburg State University
Saint-Petersburg, 198504, Petrodvoretz, University Avenue 28
Russia
khan47@mail.ru

**Abstract**
New theoretical approach proposed to the nonequilibrium processes in open systems combines methods of statistical mechanics and cybernetic physics in order to construct closed mathematical model of a system with indeterminacy and random factors, self-organization and self-regulation by means of internal close-loops at an intermediate scale level. In scope of the approach a new class of dynamical systems with partially internal and external control has been originated.

**Key words**
Uncertain, indeterminacy, incompleteness, transition, nonequilibrium, self-organization, structure, mesoscale, close-loop, self-regulation.

## 1 Introduction

Dynamical systems described by a set of nonlinear differential equations are conventionally used for mathematical modeling in physics and mechanics for a long time. However, last decades in nonequilibrium statistical mechanics it had been found out that for a wide range of nonequilibrium high-rate physical processes the correct mathematical models should be integro-differential [Bogolyubov (Jr.), Sadovnikov and Shumovsky, 1989; Zubarev, 1971]. Moreover, the models should not be rigid; they should be able to change themselves according to the modifying type of the interaction mechanisms in between the system elements under external loading. However, it is impossible to control all factors that can change a behavior of a system under dynamic loading. As a result of an external action upon a complicated system there arise indefinite and random factors that affect the system behavior. Nonequilibrium uncertain processes inside the system could not be described using conventional models as far as the set of variables become incomplete. The external action upon the system and originated transport processes will be followed by relaxation of any degrees of freedom that in turn influences the system behavior. It is clear that in order to construct an adequate model for such nonequilibrium transition process one must involve an internal control into the model by means of a close-loop. Methods of the control theory involving internal and external close-loops were actively used in many problems [Fradkov, 2005; Fradkov, 2007]. However, for the transition processes the close-loop insertion is the necessary attribute of mathematical modeling to complete the problem formulation under indefinite conditions. In physics and mechanics problems on nonequilibrium transport in open systems can be referred to the transition ones, and only the use of the control theory for their modeling allows a prediction of the macroscopic system behavior with uncertain factors included. New theoretical approach to nonequilibrium transport developed on the base of nonequilibrium statistical mechanics and cybernetic physics proposes a way of structurization and discretization of the system under dynamic external loading. The forming structure elements have sizes related to the intermediate scale level between macro- and microscopic scales. In high-rate process the internal structure evolution going at this mesoscopic scale appears to be described using the control theory methods. Being the information carriers the mesoscopic structure elements introduce the internal information-control close-loop into the system. It had been shown that the speed-gradient method allows description of the internal structure evolution and results in a new type dynamical system at mesoscale with partially internal and external controls.

So, the self-organization and self-regulation should be necessarily included into the mathematical model of an open system far from equilibrium to reveal indeterminacy and uncertain effects. That is why one can confirm that as long as physicists would use differential rigid models for high-rate uncertain processes without involving of the internal structure evolution, the gap between fundamental science and practice will not be overcome.

## 2 Nonequilibrium Transport

Transport of mass, momentum and energy follows all real physical processes. For distributed systems the mass $\rho$, momentum **p** (**p** /$\rho$=**v**), and internal energy $E$ densities are used to describe the transport processes in media. The densities should satisfy the general transport equations

$$\frac{\partial \rho a}{\partial t} + \nabla \cdot \mathbf{J} = I, \quad a = 1, \mathbf{v}, E.$$

Here **J**, $I$ are densities of the momentum and energy fluxes and sources respectively. If the fluxes are expressed through the densities and the sources are given, the transport equations govern the macroscopic reaction of the system to an external action. However, the problem on the relationships between the fluxes and densities completing the set of transport equations is solved only in two limiting cases: for small velocity gradients or strain-rates (classical hydrodynamics for fluids) and small deformations (linear theory of elasticity for solids). In both limits the transport equations are the partial differential ones derived under the conventional continuum mechanics condition $l << |a| \,/\, |\nabla a|$. It means that the typical linear size of the internal structure element $l$ can be neglected compared to the typical scale of the velocity field inhomogeneity. For high-rates and large deformations a medium demonstrates both hydrodynamic and elastic properties and moreover, the internal structure effects too. So, the processes can be referred to the transition ones, and the problem on the determining relationships is not solved in general till present.

On the condition $l \sim |a| \,/\, |\nabla a|$ a new problem arises, how to determine the concept of the macroscopic density in nonequilibrium. The classical definition looses its conventional meaning and it is necessary to involve the probability concepts and mathematical statistics. In scope of the statistical theory the averaged values are defined independently on the scale parameters in terms of statistical distributions characterized by the mathematical expectation and dispersion. Supposing the conventional scale condition satisfied and the statistical distribution in continuum mechanics close to the normal one, macroscopic densities are determined as the first statistical moments. This case corresponds to the case near local thermodynamic equilibrium where at microscale chaotic heat fluctuations take place. It had been experimentally found out that as the strain-rates growing a synchronization of the fluctuations begins. The fluctuations cease being independent and there appears a correlation between states of the system in different space regions and in different time intervals. In the nonequilibrium statistics there exists a concept of the correlation function for two characteristics of the system *f, g* at points *x, y* and for the different times [Bogolyubov (Jr.), Sadovnikov and Shumovsky, 1989]

$$\langle f(x,t) g(y,\vartheta)\rangle \xrightarrow[t\to\infty]{} \langle f(x,t)\rangle \langle g(y,\vartheta)\rangle .$$

After the relaxation the events become independent and the state of the system becomes equilibrium. The difference between the left and the right parts characterizes the deviation from equilibrium.

## 3 Statistical Mechanical Base

From the point of view of nonequilibrium statistical mechanics the general transport equations are not entirely localized under essentially nonequilibrium conditions [Bogolyubov (Jr.), Sadovnikov and Shumovsky, 1989]. The most profound result for the incomplete description of high-rate processes obtained in scope of the nonequilibrium statistical mechanics consists in the general integral governing relationships between the conjugate thermodynamic fluxes *J* and forces *G* (gradients of macroscopic variables) valid through the relaxation regimes [Zubarev, 1971]. In scope of the general relationships the dependence of the flux components **J** on the gradient components *G* has an integral form

$$J(\mathbf{r},t) = \int_{-\infty}^{t} dt' \int_{V} d\mathbf{r}' \Re(\mathbf{r},\mathbf{r}',t,t') G(\mathbf{r}',t'). \quad (1)$$

It means that a state of the system in a point **r** at an instance $t$ is determined by all the history of the system evolution all over the space region full of a medium. The main meaning of the relationship (1) is connected with the fact that nonetuilibrium statistical ensemble at the microscale gives rise to correlation function $\Re(\mathbf{r},\mathbf{r}',t,t')$ at the intermediate scale between micro- and macro- levels. This function plays a role of a weight function that determines the contributions of mesoscale phenomena into the macroscopic medium behavior. From the general properties of the correlation function one knows its behavior in the limiting cases.

At the initial instance *t=0* the system had been subjected to an external loading during a time interval $t_R$, the typical stress relaxation time for the given medium under the loading conditions is $t_r$. The relaxation proceeds by stages. At the initial stage of frozen relaxation the correlation function is constant $\Re(\mathbf{r} = \mathbf{r}_\Gamma, \mathbf{r}', t = 0, t') = K = const$ defined by correlation with the boundary and initial conditions. In this case the dissipation can be neglected and the transport mechanism should be reversible. At this stage on the condition $t_R \leq t << t_r$ the system remembers its prehistory. Later it will be shown that the medium reaction initially corresponds to the solid-like behavior. The typical duration of the initial stage determines the memory properties of the medium. The correlation is not entirely fixed in real situation. There exist heat fluctuations at microscale. The fluctuations being neglected make the system description determinate but the growing fluctuations and weakening correlation make the system uncertain.

The final near-equilibrium hydrodynamic stage $t_r << t \leq t_R$ corresponds to the case where the memory and nonlocal effects can be entirely neglected. Then the transport relaxation kernels determine the transport coefficients in scope of the linear thermodynamics of irreversible transport processes [De Groot and Mazer, 1963]. For the momentum transport the irreversible part of the stress tensor is linearly connected to the velocity gradient

$$\mathbf{J}(\mathbf{r},t) \approx k_0(\mathbf{r},t) G(\mathbf{r},t) \equiv \mathbf{J}^{N-S}(\mathbf{r},t),$$

$$k_0(\mathbf{r},t) \equiv \int_{-\infty}^{t} dt' \int_V d\mathbf{r}' \Re(\mathbf{r},\mathbf{r}',t,t'). \qquad (2)$$

At this stage the system forgets its prehistory. The transport coefficients ${k_0}^{(0)}$ near the local equilibrium present the medium macroscopic property of viscosity conditioned by the medium structure at the microscale. For the momentum transport this limit results the well-known Navier-Stokes equations that present a theoretical base for the classical hydrodynamics. Navier-Stokes equations are the second order partial differential equations of the parabolic type describing the diffusive transport mechanism. So, the limit of the neglected correlations corresponds to a fluid-like medium behavior. As far as a flow velocity grows the viscosity coefficient first ceases being constant and depending on the size and geometry of the system, then it becomes insufficient to describe the momentum transport in the transition regime.

At the transition stage the space-time correlations are not enough small to be neglected and cannot consider being constant. The macroscopic medium reaction on an external loading should be entirely determined by the time evolution of the finite-size correlations. In this case the combined (both solid-like and fluid-like) medium reaction on the external loading should be expected. Moreover, the system could demonstrate effects of its internal structure. The transport equations are integro-differential and describe the both mechamisms of reversible and dissipative transport. So, Eqn. (1) resulted from the nonequilibrium statistical mechanics confirms that without taking into account the space nonlocal and memory effects in between the limiting situations it is impossible to describe the nonequilibrium processes in a correct way and the gradient-type theories allow only an asymptotic description near the limiting situations.

The principle difficulty consists in that the correlation function is unknown nonlinear functional of the macroscopic gradients $G$ [Bogolyubov (Jr.), Sadovnikov and Shumovsky, 1989; Zubarev, 1971]. If the correlation function would be determined by the gradients the transport equations entirely determine the nonequilibrium mass, momentum and energy transport. However, because of an arbitrary external loading it is impossible to derive from the nonequilibrium ensemble even approximate closed form of the functional. So, the only way to deal with the relationships (1) is a modeling of the correlation function on the base of the general invariance principles and the known asymptotic behavior. However, a substitution of simple space-time dependencies into the kernels doesn't allow an adequate description of nonequilibrium processes and a satisfaction to the natural boundary conditions for the continuum description. Many decades this circumstance was an obstacle for the nonlocal models to apply to nonequilibrium transport in real media. The transition models necessarily should include a functional dependence on the gradients $G$ connected to an external loading that determines the medium reaction on the nonequilibrium conditions. Without this close-loop between the external loading propagation and the medium reaction on it any description of nonequilibrium processes would be not only incomplete but incorrect.

## 4 Self-organization During Nonequilibrium Transport

In real media the velocity of the perturbation propagation is always finite $d\mathbf{r}/dt = \mathbf{u}$. It means that there exists a trajectory of the momentum propagation $\mathbf{r} = \mathbf{r}(t)$. The equation doesn't govern a real motion of the medium element but phase (or group for dispersed media) velocity of the wave propagation. In equilibrium for small perturbation the velocity $u \to C$ tends to the sound velocity. In this case there is no mass transport and no dissipation that follows it.

So, the integration over space and time in Eqn. (1) formally can be reduced to an integral either over space or time

$$\begin{aligned} J(\mathbf{r},t) = \int_{-\infty}^{t} dt' \int_V d\mathbf{r}' \Re(\mathbf{r},\mathbf{r}',t,t') G(\mathbf{r}',t'). = \\ = \int_{\Omega(t)} d\mathbf{r}' \Re(\mathbf{r},\mathbf{r}') G(\mathbf{r}'). \end{aligned} \qquad (3)$$

The integration in Eqn. (3) is going over the volume $\Omega(t)$ embraced by the perturbation that has already come from a boundary $\Gamma$. Resolution of the function $G$ by Tailor near the point $\mathbf{r}' = \mathbf{r}$, and its substitution into Eqn. (3) result an infinite order differential operator

$$\begin{gathered} \int\limits_{\Gamma}^{\Gamma+\Omega(t)} d\mathbf{r}'\Re(\mathbf{r},\mathbf{r}')G(\mathbf{r}') = k_0(\mathbf{r},t)G(\mathbf{r})+ \\ +k_1(\mathbf{r},t)\frac{\partial G}{\partial \mathbf{r}}(\mathbf{r}) + \frac{1}{2}k_2(\mathbf{r},t)\frac{\partial^2 G}{\partial \mathbf{r}^2}(\mathbf{r}) + ... \end{gathered} \tag{4}$$

The first moments of the nonequilibrium statistical distribution of the space nonlocal correlation have definite physical sense related to the medium internal structure. The 0-order moment $k_0(\mathbf{r},t)= \mu$ defines effective values of the transport coefficients for the medium with internal structure. The 1-order moment $k_1(\mathbf{r},t)$ defines a mathematical expectation of the vector $\langle \mathbf{r}' - \mathbf{r} \rangle_{\Re} = \gamma$ that is not equal to zero by definition if the space correlation exists and the statistical distribution differs from $\delta\,(\mathbf{r}' - \mathbf{r})$function. There appear new direction and new length defined by the vector $\gamma$ generated by nonequilibrium transport in a real medium. It means that any nonequilibrium distribution gives rise to the medium polarization along the direction of the vector $\gamma$ under an external loading through the medium boundaries. This vector determines a shoulder of the force acting on a finite size medium element in the inhomogeneous velocity and stress fields. It means that such an element should rotate as a whole and can be considered as an element of the dynamic internal medium structure. The resulted structured medium is characterized by internal rotations and asymmetrical stress tensor (like in turbulent motion). In general case the resulted structure size depends on boundary conditions.

The second moment $k_2(\mathbf{r},t) = \left\langle (\mathbf{r}' - \mathbf{r})^2 \right\rangle = \varepsilon^2 - 2\mathbf{r}\gamma$ determines the dispersion of the space correlation distribution $\left\langle \mathbf{r}'^2 \right\rangle - \mathbf{r}^2 = \varepsilon^2$ when $\mathbf{r}\gamma$=0 and introduces the share of the nonlocal effects as a degree of the deviation from the local equilibrium state of the system. Really, in the case where $\varepsilon \to 0,\ \gamma \to 0$ the nonlocal effects can be neglected, the distribution function tends to the $\delta$-function. This case corresponds to the fluid reaction to perturbations near equilibrium. In the opposite case where $\varepsilon \to \infty$ the distribution doesn't decay with the distance from the point $\boldsymbol{r}$ remaining constant for any finite product $\mathbf{r}\gamma$. In this case the medium in a whole is embraced by correlation propagating from the loading region, its reaction corresponds to the solid one. In both limits the medium doesn't demonstrate the internal structure effect. Its behavior corresponds to the response in scope of the continuum mechanics validity.

In the intermediate case $t_R \cong t \cong t_r$ under a pulse loading the medium reaction is like one of a multi-phase dispersed mixture of solid phase grains with rigid correlation and viscous liquid with weak correlation. So, correlation inside the medium under external loading originates new space (or time) scales of the internal structure formation.

## 5 Internal Structure Formation

Then, the nonlocal model for the correlation function depending only on the first moments that have a meaning of the internal structure characteristics is introduced [Khantuleva and Mescheryakov, 1999a; Khantuleva and Mescheryakov, 1999b; Khantuleva, 1999; Khantuleva, 2000; Khantuleva, 2003; Khantuleva, 2013]. The first moments approximation for the correlation function results a new governing relationship between the flux and the gradient with the medium structure parameters included

$$J(\mathbf{r},t) = \frac{1}{\varepsilon} \int\limits_{\Omega(t)} d\mathbf{r}' \exp\left\{ -\frac{\pi(\mathbf{r}' - \mathbf{r} - \gamma)^2}{\varepsilon^2} \right\} G(\mathbf{r}'). \tag{5}$$

Here the Gaussian model function determines a type of the space correlation decaying with a distance from the point $\mathbf{r}$. The higher moments of the statistical distribution in Eqn. (5), characterizing fluctuations of the medium structure parameters are not disappeared in the model and can be expressed through out the first moments. Following an analogy with the well-known local-equilibrium statistical distribution in velocities the nonlocal distribution becomes normal near the hydrodynamic limit near the local equilibrium while for growing values of the nonlocal parameter the integral kernel can take asymmetrical form.

For the nonlocal model (5) there arises a problem on the model parameters determination taking into account that the relaxation kernel should be a nonlinear functional of the gradients G and the external loading conditions. The only real possibility for the model is a definition of the structure parameters as being functionally depending on $G$. Then the structure parameters can be determined by using boundary and integral conditions imposed on the system including the external loading. At arbitrary model parameters it is impossible to satisfy these conditions that are natural for a continuum description. So, the boundary and integral conditions can be satisfied only on account of the parameters and therefore determine them

$$\Phi_i \left[ G(\mathbf{r},t), \mu, \varepsilon, \gamma \right] \Big|_{\mathbf{r} = \mathbf{r}_\Gamma} = 0,\ i = 1, \ldots, 5. \tag{6}$$

According to Eqn. (6) the relationships for the space nonlocal models determine the structure parameters $\varepsilon, \gamma(t)$as time-depending functions while for the memory models, to the opposite, should depend on coordinates.

The nonlinear relationships (6) complete the model and due to their nonlinearity determine the time evolution of a spectrum of internal structure scales. In general nonequilibrium case the scale spectrum is discrete and in limiting cases it becomes continuous defining the validity region of continuous mechanics. The bifurcation points for the branching equations (6) point out the structure transitions in the system and determine the threshold values of the loading parameters. It means that the condition (6) allow the self-organization effects included into the nonlocal model.

It is very important to note that the governing equation (5) under the conditions (6) complete the transport equations (1) in a self-consistent way. It means that macroscopic fields in a medium depend on the medium internal structure and the structure evolution is determined by the macroscopic fields and external loading conditions. There arises a close-loop in a nonequilibrium system that introduces a self-regulation included into the nonlocal model.

## 6 Internal Control and the Structure Evolution

However, the evolution of the structure parameters are not entirely defined by boundary conditions. A part of them tend to their equilibrium values as for the isolated systems. The drift of the parameters makes the process irreversible and for isolated systems corresponds to the general evolution principle [Nicolis and Prigogine, 1977]. This criterion defines the rate of the entropy production and pointes out a direction to equilibrium. Last decade the entropy production criteria are often used to describe a system evolution. The integral entropy production in scope of the nonlocal theory is determined as follows

$$\dot{S}(t) = \int\limits_V dx J(x,t) G(x,t). \tag{7}$$

According to the Prigogin's theorem the nonequilibrium stationary states are characterized by the minimal entropy production [10]. There is no proved results for states far from equilibrium. One knows only that the system evolution is entirely related to the evolution of their internal structure.

Another approach determining a direction of a process has been developed in the control theory and based on the speed-gradient method [Fradkov, 2007]. According to this method all ways among of a process there is the only one realized with the control parameters evolving proportionally to the gradient of some goal functional. If to choose the minimal integral entropy production as a goal functional and the structure parameters as the control ones the speed-gradient algorithm in a finite form defines the system evolution by a set of $1^{st}$ order differential equations with respect to the structure parameters $s$

$$\dot{s} = -g \nabla_s \dot{S}. \tag{8}$$

The differential form of the algorithm results the set of the $2d$ order equations

$$\ddot{s} = -g \nabla_s \dot{S}. \tag{9}$$

The constant value $g > 0$ determines the rate of the structure evolution if the structure evolves much more slowly than the macroscopic fields. Only on this condition the concept of the internal structure has its conventional meaning. The 2d order equations of evolution describe inertia properties of a medium structure. One can show that even out of the validity of the continuum mechanics for isolated systems the general criterion of evolution holds

$$\dot{s} = -g \nabla_{\dot{s}} \ddot{S} = -g \nabla_{\dot{s}} (\nabla_s \dot{S} \dot{s}) = -g \nabla_s \dot{S}. \tag{10}$$

For open systems boundary conditions make the evolution nonmonotonous. Self-organization leads to the entropy decreasing.

So, the self-organization and self-regulation appear to be the necessary components in order to make the nonlocal model complete and to describe nonequilibrium transport in a correct way.

## 7 Application to Wave Propagation in Condensed Media

Consider an elastic-plastic plane wave propagation induced by a shock at the velocity $V_0$ in semi-space full of condensed matter. When a loading stress is below an elastic limit an elastic waveform propagate in a matter without mechanical energy dissipation into a heat. If the stress exceeds an elastic limit, the nonstationary wave propagation is followed by the irreversible mass transport, the shear relaxation and the energy loss. In solids under shock loading a two-wave front is forming during the elastic-plastic wave propagation: elastic precursor below the elastic limit and retarding plastic flow over the limit. In this regime the elastic precursor is going at the constant longitudinal sound velocity $C$ ( $\rho_0 C^2 = \rho_0 {C_0}^2 + \frac{4}{3} G$, $\rho_0 {C_0}^2$ and $G$ are spherical and shear elastic modulus) and relaxing up to its stationary value. The nonstationary plastic front is moving irregularly. In the transition regime between elastic and ideal plastic waves for short shocks instead of the mechanical energy dissipation a dynamic structure formation arises. All the processes are not described by continuum mechanic models. In the reference connected to the elastic precursor $\zeta = \frac{1}{t_R}\left(t - \frac{x}{C}\right)$, $\xi = \frac{x}{L}$ ($t_R$ is the loading time, and $L$ is a target width) mass and momentum transport equations in the linear approximation with respect to the small parameter $V_0/C << 1$, when the matter density slightly declines from its initial value $\rho = \rho_0 + \rho_1,\ \rho_1/\rho_0 << 1$ are as follows

$$\frac{\partial \rho_1}{\partial \zeta} - \frac{\rho_0}{C}\frac{\partial v}{\partial \zeta} + \frac{\varepsilon}{\tau}\frac{\partial \rho_0 v}{\partial \xi} = 0,$$

$$\frac{\partial v}{\partial \zeta} - \frac{1}{C}\frac{\partial C\Pi}{\partial \zeta} + \frac{\varepsilon}{\tau}\frac{\partial \Pi}{\partial \xi} = 0. \quad (11)$$

The nonlocal model (5) in the new reference is used to close the set (11)

$$\Pi = -\int_0^{Ct} d\xi' \int_o^{\omega} d\zeta' \Re(\zeta,\zeta';\tau)\delta(|\xi-\xi'|)\left[-\frac{\partial v}{\partial \zeta'} + \frac{\varepsilon}{\tau}\frac{\partial v}{\partial \xi}\right] =$$

$$= \int_o^{\omega} d\zeta' \Re(\zeta,\zeta';\tau)\left[\frac{\partial v}{\partial \zeta'} - \frac{\varepsilon}{\tau}\frac{\partial v}{\partial \xi}\right], \quad (12)$$

$$\omega(\zeta) = \begin{cases} \zeta, & \zeta < 1, \\ 1, & \zeta \geq 1. \end{cases}$$

Here the model parameters are introduced: $\tau = \frac{t_r}{t_R}$, $\theta = \frac{t_m}{t_R}$, determining relative relaxation and retardation typical times. The parameter $\varepsilon = \frac{Ct_r}{L}$ determines a typical relaxation length. If $\frac{\varepsilon}{\tau} = \frac{Ct_R}{L} << 1$, the new variables are separated $\tau\frac{\partial}{\partial\zeta} \gg \varepsilon\frac{\partial}{\partial\xi}$: high-rate variable describes the wave front, and the slow one the front evolution. On the condition $\frac{\varepsilon}{\tau} << 1$ Eqns. (11)–(12) can be reduced to a one integral equation

$$v = \int_o^{\omega} d\zeta' \exp\left\{-\frac{\pi(\zeta-\zeta'-\vartheta(\xi))^2}{\tau(\xi)^2}\right\}\frac{\partial v}{\partial \zeta'}. \quad (13)$$

Eqn. (13) due to the smoothing integral operator describes a spreading out of the front during its propagation because of the mass velocity pulsations like in the wave packet.

While the stress is proportional to the strain the first stage corresponds to an elastic process when $\tau \to \infty$, $\Re \to 1$, $\omega = \zeta < 1$. When the stress would exceed an elastic limit the elastic stage stops and the stress begins depend on the strain-rate. Unlike the conventional concept of the ideal elastic-plastic wave where the loading proceeds during all the wave, in the integral model (13) the loading stops on the elastic precursor top, and the retarding plastic front results from the medium inertia. Supposing $\frac{\partial v}{\partial \zeta} = 1$ during the loading Eqn. (13) has an explicit approximate solution

$$v(\zeta;\tau,\theta) = \begin{cases} \frac{\tau}{2}\left(erf\frac{\sqrt{\pi}(\zeta-\vartheta)}{\tau} + erf\frac{\sqrt{\pi}\vartheta}{\tau}\right), \zeta < 1, \\ \frac{\tau}{2}\left(erf\frac{\sqrt{\pi}(\zeta-\vartheta)}{\tau} + erf\frac{\sqrt{\pi}(1-\zeta+\vartheta)}{\tau}\right), \zeta \geq 1. \end{cases} \quad (14)$$

The solution (14) describes all set of experimentally found dependencies between stress, strain and strain-rate in the wide range of the shock velocities and target widths.

The parameters together with the waveform are evolving during the wave propagation in the medium. In order to complete Eqn. (14) it is necessary to describe the wave front evolution based on the control theory of adaptive systems. The minimal integral entropy production inside the waveform can be chosen as the goal functional

$$\sigma(\xi) = \int_0^{\infty} d\zeta\frac{\partial v}{\partial \zeta}\int_0^{\omega} d\zeta' \exp\left\{-\frac{\pi(\zeta-\zeta'-\theta(\xi))^2}{\tau^2(\xi)}\right\}\frac{\partial v}{\partial \zeta'}. \quad (15)$$

The Eqn. (15) defines a generalized entropy production which can result $\sigma = 0$ for the elastic waves ($\tau \to \infty$), $\sigma > 0$ for the dissipative processes ($\tau \to 0$) and $\sigma < 0$ for the synergetic structure formation in the transition regime $\tau \sim 1$.

The model parameters $\tau, \theta$ play a role of the control ones. The speed-gradient method [Fradkov, 2007] in a finite form defines the structure evolution by the following set of nonlinear differential equations for the parameters $\tau(\xi), \theta(\xi)$.

$$\frac{\partial \theta}{\partial \xi} = -g\nabla_\theta \sigma, \quad \frac{\partial \tau}{\partial \xi} = -g\nabla_\tau \sigma. \quad (16)$$

The set (16) describes the structure evolution in accordance with the leading of the entropy production to its minimal value. The set (16) depending on the velocity profile itself introduces a close-loop into the system and makes it complete and self-regulated.

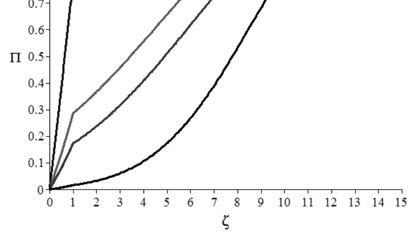


Fig. 1. Nonstationary waveform evolution.

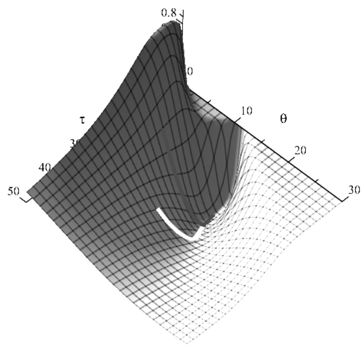


Fig. 2. Gradient trajectories on the surface $\sigma(\tau, \theta)$ [Khantuleva and Litvinov, 2011].

The nonlinear set (16) determines the structure transitions occurring inside the waveform that is followed by the pulse amplitude loss due to the kinetic energy going to mesoscale. The structure formation at mesoscale in the form both shear and rotations had been found out in experiments on the shock loading of different metals [Mescheryakov and Divakov, 1994; Mescheryakov et al., 2008].

## 8 Conclusion

So, the self-organization and regulation by the internal close-loop are the necessary attributes of the modeling for a wide range of the transition processes in different kind of systems including the living ones. New possibilities for a control in the transition uncertain processes can have an important meaning for the development of new thin technologies, biomechanics and medicine and for prediction of multi-scale catastrophic phenomena.

## References

Bogolyubov (Jr.), N.N., Sadovnikov, B.I. and Shumovsky, A.S. (1989). *Mathematical Methods in Statistical Mechanics of Model Systems*, Moscow: Nauka (in Russian).

De Groot, S. and Mazer, P. (1963). *Nonequilibrium Thermodynamics*, Amsterdam: North-Holland publ. Co.

Fradkov, A.L. (2005). Application of cybernetic methods in physics. *Physics-Uspekhi*, **48**(2), pp. 103–127.

Fradkov, A. (2007). *Cybernetical Physics: From Control of Chaos to Quantum Control*, Springer, Berlin.

Khantuleva, T.A. (1999). Modelling of fast high-gradient processes on the base of self-consistent nonequilibrium distribution function, *Matematicheskoe Modelirovanie*, **11**(6), 1999, pp. 17–24 (in Russian).

Khantuleva, T.A. (2000). Non-local theory of high-rate straining followed by structure formations, *Journal of Physics IV France*, EDP Sciences, Les Ulis, **10**, pp. 485–490.

Khantuleva, T.A. (2003). The shock wave as a nonequilibrium transport process, In: *High-pressure Compression of Solids VI: Old Paradigms and New Challenges* (Y.Horie, L. Daison, N.N. Thadhani, Eds.), Springer, pp. 215–254.

Khantuleva, T.A. (2013). *Nonlocal Theory of Nonequilibrium Transport Processes*. St. Petersburg: St. Petersburg State Univ., 276 p.

Khantuleva, T.A. and Litvinov, A.V. (2011). Effect of strain rate on the plastic properties of solids, *Proc. of the conf. Advanced Problems in Mechanics*, Part 2. pp. 309–312.

Khantuleva, T.A. and Mescheryakov, Yu.I. (1999a). Nonlocal theory of the high-strain-rate processes in a structured media, *International Journal of Solids and Structures*, **36**, pp. 3105–3129.

Khantuleva, T.A. and Mescheryakov, Yu.I. (1999b). Kinetics and nonlocal hydrodynamics of forming of a mesostructure in dynamically deformable media, *Fizicheskaya Mezomekhanika*, **2**(5), pp. 5–17 (in Russian).

Mescheryakov, Yu. I. and Divakov, A. K. (1994). Multi-scale kinetics of microstructure and strain-rate dependnence of materials, *DYMAT Journal*, No 1, pp. 271–276.

Mescheryakov, Yu. I., Divakov, A. K., Zhigacheva, N.I., Makarevich, I.P. and Barakhtin, B.K. (2008). Dynamic structures in shock-loaded copper. *Physical Review B.*, **78**, pp. 64301–64316.

Nicolis, G. and Prigogine, I. (1977). *Self-Organization in Nonequilibrium Systems. From Dissipative Structure to Order Through Fluctuations,* N.Y.: Wiley.

Zubarev, D.N. (1971). *Nonequilibrium Statistical Thermodynamics*, Moscow: Nauka, 1971 (in Russian).