Robust Stability of Nonlinear Gyromoment Attitude Control System for Flexible Spacecraft

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Abstract: The paper discusses a method of synthesis of a nonlinear orientation system with gyro-flywheels. The system must guarantee the robustness of the stability property of the flexible spacecraft (FS) movement. Two-stage procedure of the FS parameters correction and further extension of the base control algorithm was suggested. In the extended algorithm the estimations of the unstable modes of the FS elastic oscillations were used. Some results of computer simulation of the suggested orientation system were obtained. These results confirm the efficiency of the control algorithm and the stability robustness of the FS construction elastic oscillations.

Keywords: Aerospace control, Flexible spacecraft, Orientation system, Robust stability

1. INTRODUCTION

At present, the spacecraft orientation systems are usually discrete ones. It is due to the requirements for reliability and economic functioning of the orientation systems and because of the on-board computers for realization control algorithms are used. The discontinuous character of the control actions is the main cause of the arising elastic oscillations of the FS construction. A priori uncertainty of the FS parameters and their variation stipulated by the presence of moving masses (for example, panels of the solar batteries, antennas and so on) can lead to increase of these elastic oscillations and instability of the feedback system motion.

In Krutova (2001), it was shown that the forms of the "rapid" components $\tilde{x}_i(t),\ (i=\overline{1,n})$ envelopes depend on the value of the control discreteness interval $T_0 \in [T_{0\min}, T_{0\max}]$. At this point in the course of the object functioning at some fixed range of the value T_0 the following situation may occur: one part of the motion $\tilde{x}_i^-(t) \subset \tilde{x}^-$ is stable, the other part $\tilde{x}_i^0(t) \subset \tilde{x}^0$ is neutral (amplitudes of the elastic oscillation $\tilde{x}_i(t)$ are constant) and the last part $\tilde{x}_i^+(t) \subset \tilde{x}^+$ is unstable. Even if the frequency of one mode of the elastic oscillation is close to the value $f_u = T_0^{-1}$ or divisible by f_u then it will be a resonance and the system motion will be unstable (Rutkovsky et al., 1998).

One of the main tasks of this paper is the synthesis of the control algorithm that guarantees the stability of both slow ("rigid") and rapid motions taking into account the uncertainty of the FS parameters. The solution of this task is effected on the basis of the robustness realization of the suggested regulator that guarantees the extension of the motions $\tilde{x}^-$ and $\tilde{x}^0$ domains. The unstable modes ($\tilde{x}^+$) are estimations (Sukhanov et al., 2003) and are damped out by the regulator. Such an approach can be called the synthesis of the control system that has the property of the robustness with respect to stability (Kosut et al., 1983).

2. THE ANGULAR MOTION EQUATIONS OF THE FS WITH FLYWHEELS

The dynamics of angular motion of the considered type mechanical systems is usually described by the Lagrange equations in the finitely element form:

$$A\ddot{q} + Bq = Q\,, \tag{1}$$

where q is the $(n'\times 1)$-dimensional vector of generalized coordinates that defined the FS position and configuration, $n' = n+3$; A, B are the symmetric matrices $(n'\times n')$ of masses and flexibility; $Q = KM$ is the vector of generalized forces, K is the $(n'\times l)$ -dimensional vector of the actuator device influence coefficients, $M = (M_1, M_2, M_3)^{\mathrm{T}}$ is the vector of control action.

Usually, it is required to have the small values of the deviations from the stationary ones $q^0(t) \equiv 0$ (regime of stabilization). In this case, the linearized model can be written as following

$$A^*\ddot{x} + Bx = K^*M\ , \tag{2}$$

where $x \doteq (q - q^0) \le \varepsilon \to 0$, A^*, K^* are the constant matrices corresponding to A and K .

Equation (2) can be represented in the modal-physical form (Glumov et al., 1998) that defines the rotational motion $x = (x_i)$, $i = 1,2,3$, of an object in the inertial space

$$\ddot{\bar{x}} = Nm(u);\ \ddot{s} + \omega^2 s = Km(u);\ \tilde{x} = Ls;\ x = \bar{x} + \tilde{x}, \qquad (3)$$

where $s = (s_j)^{\mathrm{T}}$ is the n'-dimensional vector of normal coordinates; $\omega^2 = diag(\tilde{\omega}_j^2),\ j = \overline{1,n}$; $\tilde{\omega}_j$ are the fundamental frequencies of the FS elastic oscillations; $m(u) = I^{-1}M(u)$, $I = diag(I_x, I_y, I_z)$; $N = A_{11}^{-1}I$; L is the matrix of connection of the coordinates $\tilde{x}$ and s, see Glumov et al. (1998); $\bar{x}$ is the vector of the object coordinates, when it is rigid; $\tilde{x}$ is the vector of additional displacements of the coordinates $x(t)$ due to the object construction elasticity. The elements of the vector $x = (x_i),\ i = 1,2,3$ are considered as the Euler angles.

As an actuator device in the considered FS the flywheels are used. They are one-dimensional gyro with control rate of rotation (Raushenbakh and Tokar, 1974). The control actions $M(u)$ in (3) are the moments of dynamic reaction forces $M_{\mathrm{r}x}, M_{\mathrm{r}y}, M_{\mathrm{r}z}$ that appear at acceleration and deceleration flywheels

$$M_{\mathrm{r}x} = -J_x\dot{\Omega}_x,\ M_{\mathrm{r}y} = -J_y\dot{\Omega}_y,\ M_{\mathrm{r}z} = -J_z\dot{\Omega}_z, \qquad (4)$$

where J_x, J_y, J_z are the moments of the flywheels inertia (usually $J_x = J_y = J_z = J$); $\Omega_x, \Omega_y, \Omega_z$ are the projections of absolute rate of rotations on the fixed-body coordinate system.

Let $\Omega_{x0}, \Omega_{y0}, \Omega_{z0}$ be the moments of the flywheels rate of rotation that correspond to no perturbed motion of the FS in an orbit with the rate $\omega_{0z} = -\omega_e$. At small displacements φ, ψ, ϑ of the fixed body coordinate system from base one the FS angular velocity will be

$$\omega_x = \dot{\varphi} + \omega_e\varphi,\ \omega_y = \dot{\psi} - \omega_e\psi,\ \omega_z = \dot{\vartheta} - \omega_e. \quad (5)$$

The rates of rotation of the flywheels are written as following

$$\Omega_x = \Omega_{x0} + \Delta\Omega_x,\ \Omega_y = \Omega_{y0} + \Delta\Omega_y,\ \Omega_z = \Omega_{z0} + \Delta\Omega_z, \quad (6)$$

where $\Delta\Omega_x, \Delta\Omega_y, \Delta\Omega_z$ are the small increments that are used for eliminating the orientation errors.

Usually, $\dot{\varphi}, \dot{\psi}, \dot{\vartheta}$ are much more then ω_e. In this case, the terms with ω_e in (5) can be omitted. If gyro moments are small, the FS three-dimensional motion can be considered as the sum of three independent ones (roll, course and pitch). In this case system (3) is separated on three independed ones in the modal-physical form (Glumov et al., 1998). Each system is described the FS angular motion with respect to any orthogonal axis of inertia coordinate system

$$\begin{aligned} \ddot{x} &= -\omega x + km(u), \\ x^{\Sigma} &= ex, \end{aligned} \qquad (7)$$

where $x = (\bar{x}, \tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)^{\mathrm{T}}$; x^{Σ} is output coordinate of the FS angular position; $\omega = \mathrm{diag}\,(0, \tilde{\omega}_1^2, ..., \tilde{\omega}_n^2)$, $k = (1, \tilde{k}_1, ..., \tilde{k}_n)^{\mathrm{T}}$, $e = (1,1,...,1)$ is $(n'+1)$-dimensional unit vector, $m \equiv m(u)$ is scalar control action, $u(x, t, \lambda_v)$ is the control signal of the flywheel input, λ_v is a varied coefficient of the control algorithm.

Represented in the scalar form the FS modal-physical model (MPM) of one-dimensional angular. motion (7) (for example on the plane of pitch, where $x \doteq \vartheta$) can be written as follows

$$\begin{cases} \ddot{\bar{x}} = m_z(u),\ m_z(u) = M_{\mathrm{r}z}(u)I_{\Sigma}^{-1},\ I_{\Sigma} = (I_z + J_z); \\ \ddot{\tilde{x}}_i + \omega_i^2\tilde{x}_i = \tilde{k}_i m_z(u);\ i = \overline{1,n_1};\ \tilde{x}_i \subset \tilde{x}^C, \\ \tilde{k}_i m_z(u)\omega_i^{-2} = \tilde{x}_{ci}; \end{cases} \qquad (8a)$$

$$\begin{aligned} &\ddot{\tilde{x}}_j + \omega_j^2\tilde{x}_j = \tilde{k}_j m(u);\ j = \overline{n_1+1,n};\tilde{x}_j \subset \tilde{x}^R, \\ &\tilde{k}_j m(u)\omega_j^{-2} = \tilde{x}_{cj}; \end{aligned} \qquad (8b)$$

$$x = \bar{x} + \tilde{x},\ \tilde{x} = \sum_{i=1}^{n_1}\tilde{x}_i + \sum_{j=n_1+1}^{n}\tilde{x}_j = \tilde{x}^C + \tilde{x}^R. \qquad (8c)$$

In equations (8): $x \doteq x^{\Sigma}$ is the measured coordinate of FS; $\bar{x}$ is the coordinate of the "rigid" object motion, $\tilde{x}$ is the coordinate of the FS additional motion due to the elasticity of the object construction; $\omega_i,\ i = \overline{1,n}$, are fundamental frequencies of the FS; $\tilde{k}_i$ are the excitability coefficients. The equations (8a) define the main part of the FS motion $\bar{x}, \tilde{x}^C$. They describe the motion of FS "kernel" and consist of the equation for "rigid" motion and equations of the elastic modes contributory essential influence on the FS dynamics. Among these modes some unstable ones can be. Equations (8b) describe the motion $\tilde{x}^R$ that due to the modes with high frequencies and small degrees of excitability ($\tilde{x}_{cj} \ll \tilde{x}_{ci}$). Usually these modes are stable and have small influence on the FS dynamics. Because of this, the sum of these modes can be often considered as a noise of the object.

Torque $M_{\mathrm{r}z}(u)$ is the contour action of the flywheel; $u = u(x, \dot{x}, t)$ is the control law. Index z defining motion with respect to the pitch is omitted further.
The moment of the gyro-flywheel dynamic reaction is defined in the form (Raushenbakh and Tokar, 1974)

$$M_{\mathrm{r}} = -M_{\mathrm{d}} + M_{\mathrm{f}}, \qquad (9)$$

where M_d is the moment of the flywheel engine, M_f is the moment of resistance on the engine shaft.

From (9) for increments of the moments we have

$$\Delta M_{\mathrm{r}} = -\Delta M_{\mathrm{d}} + \Delta M_{\mathrm{f}}. \qquad (10)$$

For *dc*-motor and also for *ac*-motor (Alekseev and Bebenin, 1964) we have the model

$$\Delta M_{\text{d}} = k_u \Delta u - k_\Omega \Delta\Omega\,, \tag{11}$$

where $k_u, k_\Omega > 0$ are the constant coefficients, $\Delta\Omega$ is the increment of the flywheel rate of rotation that corresponds to the signal Δu.

The moment of momentum of the mechanical system "FS-flywheel" for unperturbed motion with respect to the *oz* axis is the following:

$$K_0 = -I\omega_e + J\Omega_0\,. \tag{12}$$

And for small displacements it is correctly to write

$$K = I(\dot{x} - \omega_e) + J\Omega_0 + J\Delta\Omega\,, \tag{13}$$

where $\dot{x} \doteq \dot{\vartheta}$. On the basis of the law of conservation of momentum it is $K_0 = K$. From this and from (12) and (13) we can obtain

$$\Delta\Omega = -\frac{I}{J}\dot{x}\,. \tag{14}$$

Substituting (14) in (11), we obtain the expression for the engine moment

$$\Delta M_{\text{d}}(u) = k_\omega \dot{x} + k_u \Delta u,\;\; k_\omega = \frac{I}{J}k_\Omega > 0\,. \tag{15}$$

Further taking into account (10) and assuming $\Delta M_{\text{f}} \approx 0$, we shall get the equation for the FS control moment

$$M_{\text{r}}(u) = -(k_\omega \dot{x} + k_u u)\,. \tag{16}$$

Now from (8a), (8c) and (16) we obtained MFM of the FS with a flywheel

$$\begin{aligned}
&\ddot{\bar{x}} = -[k_\omega(\dot{\bar{x}} + \sum_{i=1}^{n_1}\dot{\tilde{x}}_i) + k_u u,]I_\Sigma^{-1},\\
&\ddot{\tilde{x}}_i + \tilde{\omega}_i^2 \tilde{x}_i = -\tilde{k}_i[k_\omega(\dot{\bar{x}} + \sum_{i=1}^{n_1}\dot{\tilde{x}}_i) + k_u u]I_\Sigma^{-1},\;\; i = \overline{1, n_1},\\
&x = \bar{x} + \tilde{x},\;\; \tilde{x} = \sum_{i=1}^{n_1}\tilde{x}_i.
\end{aligned} \tag{17}$$

Equations (17) can be rewritten in the form

$$\begin{aligned}
&\ddot{\bar{x}} + k'_\omega \dot{\bar{x}} = k'_u u - \tilde{f}_1,\\
&\ddot{\tilde{x}}_i + \tilde{k}_i k'_\omega \dot{\tilde{x}}_i + \tilde{\omega}_i^2 \tilde{x}_i = \tilde{k}_i k'_u u - \tilde{k}_i k'_\omega(\dot{\bar{x}} + \sum_{j=1,\, j\neq i}^{n_1}\dot{\tilde{x}}_i),\;\; i = \overline{1, n_1},\\
&x = \bar{x} + \tilde{x},\;\; \tilde{x} = \sum_{i=1}^{n_1}\tilde{x}_i.
\end{aligned} \tag{18}$$

where $k'_\omega = k_\omega I_\Sigma^{-1} > 0,\;\; k'_u = -k_u I_\Sigma^{-1},\;\; \tilde{f}_1 = k'_\omega \tilde{x}$ is the disturbance due to the elastic oscillations.

From equation (18) it is shown that using wheels as a control device leads to appearance of damping in the control system. This fact affects positively on the robustness of stability property with respect to elastic oscillations. But the coefficient $k'_\omega = k_\omega I_\Sigma^{-1}$ is very small. Therefore, this damping is also small and the control system motion can be unstable because of the terms $\tilde{f}_1$ and $f_2 = \tilde{k}_i k'_\omega(\dot{\bar{x}} + \sum_{j=1,\, j\neq i}^{n_1}\dot{\tilde{x}}_i)$ in (18). This means that to increase the stability margin of the system and the stability robustness, it is necessary to have active control.

The control law is chosen as a *PD*-algorithm (Alekseev and Bebenin, 1964). It is the simplest algorithm and it has the following form

$$u(t) = -k_0[k_1(x - x^*) + k_2\dot{x}]\,. \tag{19}$$

Here k_0, k_1, k_2 are the constant coefficients, x^* is the required value of the angle x.

This algorithm is called as a base one. If control law is realized with the help of on-board computer, algorithm (19) must be rewritten in a digital form

$$u(t_k) = -k_0[k_1\hat{x}(t_k) + k_2\Delta\hat{x}(t_k)],\;\; k = 0,1,2,\ldots, \tag{20}$$

where $\hat{x}(t_k) = \hat{x}_k = S^{-1}\sum_{s=1}^{S} x_{s,k-1},\;\; k = 0,1,2\ldots$ is the estimation of the coordinate $x(t)$ on the discreteness of the k-th interval T_0. For the $\hat{x}_k$ estimation s values of the coordinate $x(t_{s,k-1}),\, s = \overline{1, S}$, are used during the previous the $(k-1)$-th interval T_0. The values $\Delta\hat{x}(t_k) = \Delta\hat{x}_k = T_0^{-1}(\hat{x}_k - \hat{x}_{k-1})$ are calculated as the first difference of the coordinate $\hat{x}(t_k)$. For discrete system the signal $u(t_k)$ is constant on the interval T_0 and the control law $u(t_k)$ and control action $M_{\text{r}}(u)$ are discontinuous.

3. PARAMETRIC ENSURING OF ROBUST STABILITY

As control law (20) is discrete one, systems (18), (20) are nonlinear. So to investigate its stability, the method of simulation is used.

As a control object we consider FS with the constant main parameters ($I \approx 10^4\, kg\cdot m^2,\;\; \tilde{k}_1 = [(I/I_0) - 1] \geq \pi^2/4 \approx 10$). In equations (18) the value $n_1 = 1$ was chosen. It corresponds to the case when only one mode is taken into consideration, but by convention it is possible to consider that the frequency $\tilde{\omega}_i$ at its increasing will be as the frequency of the second the third modes and so on. Realizing the simulation the transient processes of the system (18), (20) at $T_0 = \text{var},\, \tilde{\omega}_1 = \text{const}$ and using the method of calculating index λ of the envelope $\text{Env}[\tilde{x}_i(u(t))] \approx e^{\lambda(\tilde{\omega}_i)t}$ (Glumov et al., 1998), it is possible to define the regions ΔT_0^-, ΔT_0^+, ΔT_0^0 where $\tilde{x}_i(t,T_0)$ corresponds to $\tilde{x}^-, \tilde{x}^+, \tilde{x}^0$ (Fig 1).

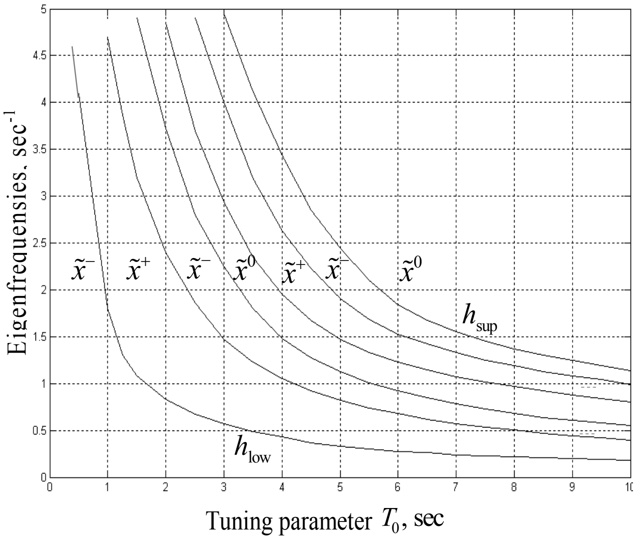

Fig. 1. Stability regions at the coefficient $k_\omega = 0$.

For example, at $\tilde{\omega}_i = 1{,}5 s^{-1}$ these regions are the following: $\Delta T_0^- = 1{,}2$ for $T_0 = (0 \div 1{,}2)$; $\Delta T_0^+ = 1{,}8$ for $T_0 = (1{,}2 \div 2{,}9)$; $\Delta T_0^- = 1{,}0$ for $T_0 = (2{,}9 \div 3{,}9)$; $\Delta T_0^0 = 0{,}9$ for $T_0 = (3{,}9 \div 4{,}8)$; $\Delta T_0^+ = 1{,}3$ for $T_0 = (4{,}9 \div 6{,}2)$ and so on.

The results shown in Fig 1 (the coefficients $k_1 = 7{,}5$, $k_2 = 275$ were chosen for the conditions of the control quality of the " rigid" motion) were obtained when the parameter $k_\omega = 0$. This case corresponds to the situation when as the control devices use was made, for example, of gas nozzles that do not give damping of the elastic oscillations. The aforementioned results show that the boundaries of the regions $\tilde{x}^-, \tilde{x}^+, \tilde{x}^0$ are very clear. Really, these boundaries are some fuzzy ones because the procedure of the parameter λ estimation is approximate.

The boundaries h_{low} and h_{sup} isolate the regions where at the left of the curve h_{low} the component $\tilde{x}_i(\tilde{\omega}_i t)$ at all T_0 belongs to $\tilde{x}^-$ and at the right of the curve h_{sup} the component $\tilde{x}_i(\tilde{\omega}_i t)$ at all T_0 belongs to $\tilde{x}^0$.

It should be noted that increasing the number of the modes that are taken into account does not change the position of all boundaries on the plane $(\tilde{\omega}_i, T_0)$.

The FS motion stability of the system (18), (20) at $k_\omega = 0$ can be ensured only when inside the interval $T_0 \in [T_{0\min}, T_{0\max}]$ it is possible to have the value $T_0 = T_0^*$ for which all modes $\tilde{x}_i(\tilde{\omega}_i t)$ do not belong to the region $\tilde{x}^+$. In particular, in the considered example of $T_0^* = 4s$ the FS motion is stable if $\tilde{\omega}_i \notin \{[0{,}42 \div 1{,}06], [1{,}95 \div 2{,}65]\}$ (Fig 1). It is clear that inaccuracy of knowing the frequencies $\tilde{\omega}_i$, their varying, fuzziness of the regions boundaries and relatively smallness of the regions $\tilde{x}^-$ lead to low stability robustness.

Now let us consider the case of using gyro-flywheel as the control device, so that $k_\omega > 0$. Mathematical simulation shows that even at small values $k_\omega > 0$ the regions $\tilde{x}^0$ are substituted for the regions $\tilde{x}^-$ because the modes damping appeared in the system. At this boundary h_{sup} isolates from the right the region of weakly damped modes $\tilde{x}^-$ for all $\tilde{\omega}_i$, $i = \overline{1, n_1}$ that comply with the condition $\tilde{\omega}_i \geq h_{\text{sup}}(T_0)$. So in this case, the system can possess the property of the stability robustness.

In the region that is situated from the left boundary h_{low} the elastic modes are damped much better but the degree of the stability robustness is lower than in previous case.

At further increasing the coefficient k_ω robust stability region widen (Fig 2). The single region of unstable $\tilde{x}^+$ is constructed step by step.

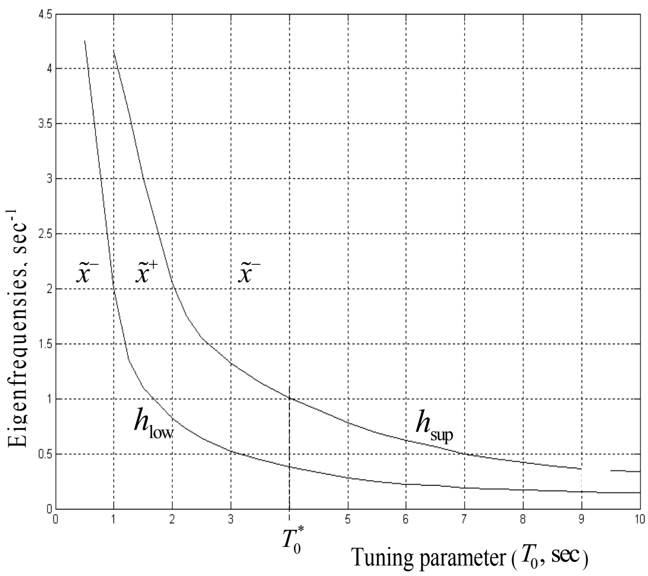

Fig. 2. Stability regions at coefficient $k_\omega = 0{,}02$.

In principle, increasing the value of the coefficient k_ω (coefficient of moment of the flywheel motor) is limited (Alekseev and Bebenin, 1964). Because of this the region of instability $\tilde{x}^+$ is presented by all means. Thus, the special precautions must be taken.

4. ROBUST STABILITY AT THE UNSTABLE MODES

Let us assume that the first step of the robust algorithm synthesis is fulfilled and taking into account some requirement, the value of the discreteness interval $T_0 = T_0^*$ has been chosen. For this value in the region $\tilde{x}^+$ that is limited by the curves $h_{\sup}$ and h_{low} the "instability interval" of the frequencies $[\tilde{\omega}_{\sup}, \tilde{\omega}_{\text{low}}]$ exists. Then at $\tilde{\omega}_{\sup} \geq \tilde{\omega}^{+} \geq \tilde{\omega}_{\text{low}}$, there is at least one unstable mode with the frequency $\tilde{\omega}^{+}$, that is, $\tilde{x}_i(t, \tilde{\omega}^{+}) \subset \tilde{x}^{+}$.

For damping unstable modes with the frequencies $\tilde{\omega}_j^{+} \in [\tilde{\omega}_{\text{low}}, \tilde{\omega}_{\sup}]$, $j = \overline{1,k}$, it is possible to use a modified *PD*-algorithm of the FS orientation (Krutova and Sukhanov, 2009). Moreover in the control system the Kalman filter is introduced. In the filter input the coordinate $x(t)$ and the signal $M_{\text{r}}(u)$ that is the output of the control device (16) model $M_{\text{r}}^{\text{m}}(u)$ are given.

The Kalman filter outputs (Sukhanov et al., 2003) are the estimation of unstable modes $\hat{\tilde{x}}_j(t), \dot{\hat{\tilde{x}}}_j$, $j = \overline{1,k}$ that are obtained on-line. The evident condition $k \ll n_1$ permits having the Kalman filter of lowered order that guarantees high velocity and quality estimations (Sukhanov et al., 2003).
The estimations $\hat{\tilde{x}}_j(t), \dot{\hat{\tilde{x}}}_j$, $j = \overline{1,k}$, are used for forming an additional signal $\tilde{u} = f(\hat{\tilde{x}}_j, \dot{\hat{\tilde{x}}}_j)$ that is additively supplemented to the main one (20). The additional signal structure can be different depending on the requirements for the process of unstable modes damping. One possibility of the control law is the discrete *PD*-algorithm (2). Its coefficients are chosen according to the method suggested in (Krutova and Sukhanov, 2009).

Then the control law of the FS stabilization can be written as follows.

$$u(t_k) = u_0(\bullet) + \sum_{j=1}^{k} \tilde{u}_{jk}(\hat{\tilde{x}}_{jk}, \Delta\hat{\tilde{x}}_{jk}) = -k_0(k_1 x_k + k_2 \Delta x_k) - \\ -\sum_{j=1}^{k} k_{0j}(k_{1j}\hat{\tilde{x}}_{jk} + k_{2j}\Delta\hat{\tilde{x}}_{jk}). \qquad (21)$$

Here $u_0(\bullet)$ is the base algorithm of the FS control in the form of (20); $\tilde{u}_j(t)$ are the components of the additional control law by unstable modes damping; k_{0j}, k_{1j}, k_{2j} are the constant coefficients. Their values depend on the frequencies $\tilde{\omega}_j^{+}$ and they are chosen according to the requirements imposed on the control time t_{pr} at the FS turn for prescribed angle. This control time must be less than the admissible value t_{pr}^{*} (Krutova and Sukhanov, 2009).

5. COMPUTER SIMULATION

Let us consider the results of two examples for computer simulation. They are illustrated by the FS orientation system stability robustness with the considered two-step correction of the control algorithm. In the first case, the FS stabilization system with control algorithm (20) was considered. In the second case, control algorithm (21) was used.

The parameters of object (8) were the following:
$I \approx 10^4\, kg \cdot m^2$, $I_0 \approx 263\, kg \cdot m^2$, $\tilde{\omega}_1 = 0,2\, s^{-1}$; $\tilde{\omega}_2 = 0,7\, s^{-1}$;, $\tilde{\omega}_3 = 1\, s^{-1}$; $\tilde{\omega}_4 = 2\, s^{-1}$; $\tilde{k}_1 = 10$, $\tilde{k}_2 = 9$, $\tilde{k}_3 = 10$, $\tilde{k}_4 = 8$.

As a control device was flywheel with *dc*-motor. Its coefficient k_{ω} that is limited by small value and the coefficients of algorithm (20) must be chosen so that on the plane $(\tilde{\omega}_i, T_0)$ the regions' configuration coincides with one shown in Fig. 2. The first step of algorithm (20) correction will be fulfilled and elastic oscillations stability robustness at $\tilde{\omega}_i \geq h_{\sup}$ is attained. In the example under consideration we have obtained:
$k_1 = 7,5$; $k_2 = 275\, s$; $T_0^{*} = 4\, s$, $k'_{\omega} = 0,02$.
Figure 2 shows that $\tilde{\omega}_{\sup} = h_{\sup}(T_0^{*}) = 1,05\, s^{-1}$ and $\tilde{\omega}_{\text{low}} = h_{\text{low}}(T_0^{*}) = 0,38\, s^{-1}$.
Taking into account the values $\tilde{\omega}_i$, $i = \overline{1,4}$, it is obvious that the elastic modes $\tilde{x}_2(t, \tilde{\omega}_2^{+})$ and $\tilde{x}_3(t, \tilde{\omega}_3^{+})$ are unstable ones. These results are verified by computer simulation (Fig. 3). So it is required to have the second step of the FS control algorithm correction.

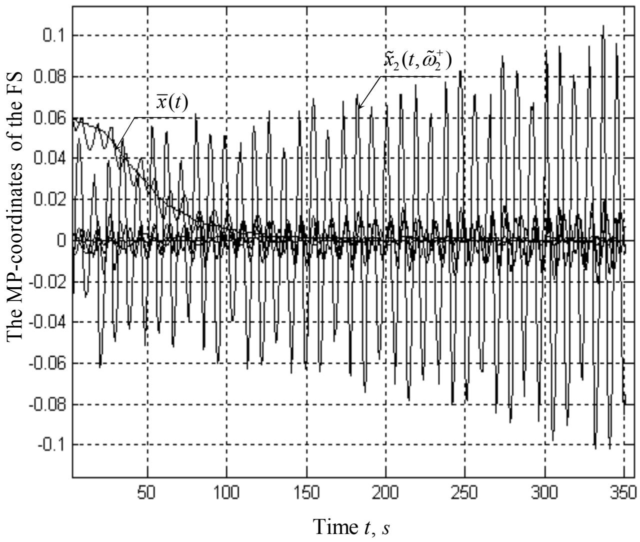

Fig. 3. Transient process at base algorithm (20).

For that in control (20) two additional components are introduced. They are the following

$$\tilde{u}_1(t_k) = -k_{01}(k_{11}\hat{\tilde{x}}_{1k} + k_{21}\Delta\hat{\tilde{x}}_{1k})\text{; d}$$
$$\tilde{u}_2(t_k) = -k_{02}(k_{12}\hat{\tilde{x}}_{2k} + k_{22}\Delta\hat{\tilde{x}}_{2k}),$$

where $\hat{\tilde{x}}_{1k}$, $\hat{\tilde{x}}_{2k}$, $\Delta\hat{\tilde{x}}_{1k}$, $\Delta\hat{\tilde{x}}_{2k}$ are the estimations of the first and second elastic modes and their first differences. The suggested realization of such kind of the control algorithm extension $u_2(t_k) = u_0(t_k) + \tilde{u}_1(t_k) + \tilde{u}_2(t_k)$ and choosing coefficients k_{0j}, k_{1j}, k_{2j}, $j = 1,2$ optimal values takes it possible to get system with high damping of the elastic modes and robust stability with respect to these modes (Fig. 4).

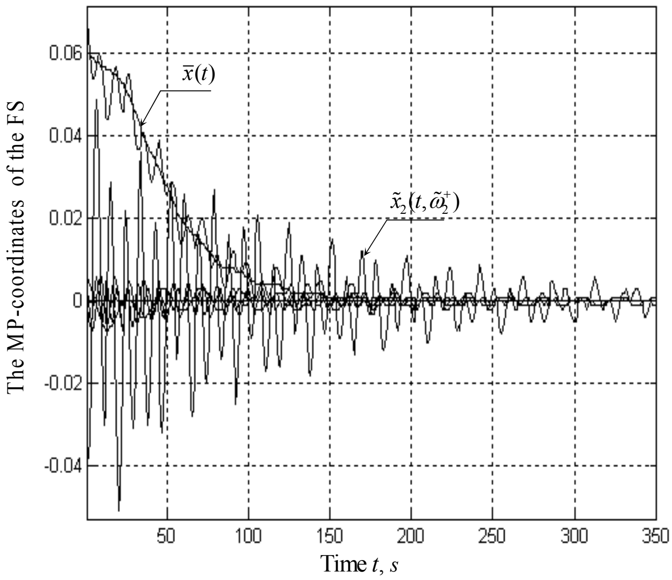

Fig. 4. Transient processes at extended algorithm $u_2(t)$.

6. CONCLUSION

The approach suggested for ensuring the FS control orientation robust stability is distinguished by two-step correction of the control algorithm: first is the correction of the base algorithm according to the limited value of the flywheel amplification coefficient and after that the correction of the algorithm structure. As the result we have the system that possesses high control quality and the property of the stability robustness.

Such two-step approach to synthesis of the FS orientation system requires a close-loop control. But the degree of this meshing is not high because of the Kalman filter is used with a lowered order.

ACKNOWLEDGEMENT

The work reported in this paper is a contribution to Project 08-08-00512, funded by the Russian Foundation for Basic Research and by the Program for Fundamental Research No 15 of Division on the EMMCP of RAS.

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