Numerical Computing for Ensuring the Trajectory Control of Aerospace Vehicles

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Abstract: A numerical method is under consideration based on the sequential linearization and used to provide the multi-channel control and monitoring, both nominal and commanding, of the aerospace vehicles trajectories. The procedures of the functional differentiation, finite-dimensional approximation, making allowance for the constraints for guidance and motion regimes are described, as well as the methods for the nominal and commanding control.

Keywords: numerical method, sequential linearization, multi-channel control, aerospace vehicles

INTRODUCTION

The space development caused the necessity of the space shuttle launch transport vehicles construction based on the space winged lifting vehicles. The main three engineering concepts of the space shuttle systems construction have been definitely shaped, they are:

- rocket-space systems for launching and orbiting shuttle vehicles,
- shuttle airspace systems using subsonic launcher aircraft and realizing a horizontal takeoff and landing;
- shuttle airspace units ensuring hypersonic speeds of flight in the atmosphere.

The last stage in each system is the aerospace vehicle (AV) – a shuttle flying vehicle capable of fulfilling the controlled motion both in the near-Earth space and in the atmosphere, and having a sufficiently high maximal value of the aerodynamic efficiency $(K_{\max} > 1)$ at hypersonic speeds of flight in the atmosphere.

The improvement and updating of the earth-based and vehicle-borne computing units makes it possible to use still more sophisticated and universal computing technique, such as a sequential linearization method – a direct method of search within the control space, for the AV-trajectories control problems solution. The method consists in the construction of a minimizing succession of controls, requires a great number of calculations and is carried out by universal digital computing devices.

1. STATEMENT OF THE CONTROL PROBLEM

The general problem of the AV-trajectories control is to evaluate the trajectory parameters and the apparatus characteristics, i.e. navigation and identification problems, and to build up the control system for the centroidal motion, i.e. guidance control problem, and for the motion relative to the center of mass (center of gravity), i.e. orientation and stabilization problem. All these problems are being solved concurrently within the control process. The navigation information is required for the guidance problem solution resulting in the formation of control relationships by the channels of the AV-centroidal motion control. These relationships are realized осуществляется in the issue of the orientation and stabilization problems solution. While developing the AV-motion control systems, the problems of navigation and identification, guidance, orientation and stabilization are considered as autonomous ones. For the purpose of a further consideration let's extract from the general control problem the guidance problem, since its solution greatly determines the efficiency of the AV-potential use, control accuracy, reliability and safety of the maneuvers execution. The solution errors of navigation and identification problems, orientation and stabilization problems must not noticeably deteriorate the trajectory control quality. In the paper by the term 'control' the procedure of forming the AV-centroidal motion control is implied.

The AV-control problem has a two-stage solution. During the first stage, i.e. before the motion starts, a nominal (rated) control is formed providing the achievement of the control target in accordance with the selected motion models. At the second stage (in-motion state), on the basis of the nominal control a command or real control is formed providing the goal problem execution under real conditions. Both the nominal and command control of the AV-motion is adjusted for the control constraints, trajectory parameters constraints and restrictions to the unit construction specifications.

The AV-centroidal motion control in the atmosphere is effectively fulfilled owing to the change of the attack angle, high speed bank angle and propulsion system draft. Negligible values of the slip angle and orientation engine propulsion have no significant influence on the motion trajectory in the atmosphere. Within the trajectory extra-atmospheric areas the control can only be effectuated with the aid of propulsion.

The control problem general engineering statement can be verbalized in the following way. The AV-parameters, motion initial conditions and control target are known. It is required to form the motion control, both nominal and command, basing on the channels of the attack angle, high speed bank angle and propulsion system draft adjusted for the restrictions of control, motion regimes and trajectory parameters, so that the formed control would optimize the control quality selected criterion.

The control problem under consideration has the following mathematical formulation. The motion mathematical model is given in the form of a set of ordinary differential equations

$$\dot{x} = f(x,u) \tag{1}$$

with the initial condition

$$x(0) = x_0\,, \tag{2}$$

where $f = (f_1,...,f_n)$ - is a vector-function of the n - dimension second members,

$x = (x_1,...,x_n)$ - is a vector of n-dimension phase coordinates,

$u = (u_1,...,u_r)$ - is a vector of r - dimension control actions.

It is demanded to evaluate the $u(t)$ control within the time interval $[0,T]$ for system (1) with initial condition (2), satisfying the restrictions of the control

$$u(t) \in U\,, \quad \dot{u}(t) \in \dot{U} \quad \text{for all values of } t \in [0,T]\,, \tag{3}$$

restrictions of the functional (function of function)

$$F_j[u(t)] \le 0\,, \ (j = 1,2,...,m) \tag{4}$$

and minimizing the functional

$$F_o[u(t)]\,. \tag{5}$$

The functionals F_j $(j = 1,2,...,m)$ are considered as implicit dependences of control actions $u(t)$, therefore in a general case the expression $F_j[u(t)]$ implies a fundamental possibility to calculate F_j from a known dependence $u(t)$.

2. METHOD OF A SEQUENTIAL LINEARIZATION

A sequential linearization method is meant to form an approximately optimal control under the restrictions of the problem functionals and of the control relationships. This is a typical method of descent in the control space and consists in building up of the minimizing succession of controls. An explicit description of the sequential linearization method, as well as the issues related to its numeric realization is given in Fedorenko (1978). The method modifications, research on its application for tasks of control formation for the AV-motion in the atmosphere and the results of some concrete problems solutions are described in Golubev and Khierullin (1987), Lazarev (1994, 1996a, 1996b), Balakin et al. (1996).

A sequential linearization method consists in building up of the control improvement iterations sequence. First the initial approximation of the reference (support) control $u(t)$ is assigned, then it is successively improved during the search process in order to satisfy all the problems conditions, (3) – (5). And at each iteration a small finite increment $\delta u(t)$ of the reference control $u(t)$ is calculated, allowing to make a passage to a new improved control $u(t) + \delta u(t)$.

Provided there is a certain reference control $u(t)$, then the increment $\delta u(t)$ will be calculated in the following way:

1. System (1) with the reference control $u(t)$ is integrated. The reference solution $x(t)$ and problem functionals F_j $(j = 0,1,...,m)$ are calculated.

2. For the motion reference law $\{u(t), x(t)\}$ functional derivatives $\omega^{(j)}(t)$ from the functionals F_j are calculated by the control $u(t)$:

$$\omega^{(j)}(t) = \frac{\partial F_j[u(t)]}{\partial u(t)} \quad (j = 0,1,...,m)\,. \tag{6}$$

3. A small neighborhood δU of the reference control $u(t)$ is introduced. The conditions to be met in this case are:

first, neighborhood δU of the reference control $u(t)$ must be included into the permissible control modification area, U, i.e. $u(t) + \delta U(t) \in U$;

secondly, within the neighborhood δU the functionals increments ΔF_j $(j = 0,1,...,m)$ must be described with a sufficient accuracy by the first-order formulae

$$\Delta F_j \approx \delta F_j[\delta u(t)] = \int_0^T \omega^{(j)}(t)\delta u(t)dt\,;$$

thirdly, the δU-neighborhood must not be too small to ensure a fast transfer from the initial approximation of the reference control to the unknown target control being sought and satisfying the problem conditions (3) – (5).

4. The increment $\delta u(t)$ is evaluated that is the solution of the initial problem linear approximation, (3) – (5) within the motion reference law neighborhood $\{u(t), x(t)\}$. In accordance with this, δu must meet the following conditions:

$$\delta u(t) \in \delta U \quad \text{for all } t \in [0,T], \tag{7}$$

$$F_j[u(t)] + \delta F_j[\delta u(t)] = F_j[u(t)] + \int_0^T \omega^{(j)}(t)\delta u(t)dt \le 0,$$

$$(j = 1,...,m), \tag{8}$$

$$\min_{\delta u(t)} \delta F_0[\delta u(t)] = \min_{\delta u(t)} \int_0^T \omega^{(0)}(t)\delta u(t)dt. \tag{9}$$

5. The fulfillment of the search completion conditions is checked. Provided the obtained value of the improved control $u(t) + \delta U(t)$, meets all the conditions of the initial problem (3) – (5), then the search of the unknown control sought-for is considered to be complete. In case of the conditions non-fulfillment the next iteration of the control improvement is performed, starting from item 1. And now the improved control value $u(t) + \delta U(t)$ is assumed to be the reference control.

3. PROCESS OF FUNCTIONALS DIFFERENTIATION

The main tool for the optimal control problems theoretical analysis and development of numeric methods for their approximate solution is the calculation of derivatives (6) of the functionals by control included into the problem statement. During the iteration procedure of the sequential linearization method, the data on the functional derivatives values is the basis for the transition to the improved control.

There's a known procedure (Fedorenko, 1978) of differentiating the functionals determined in the controlled system trajectories, in the form:

$$F[u(t)] = \int_0^T \Phi[x(t), u(t)]dt, \tag{10}$$

$$F[u(t)] = \Phi[x(t')], \tag{11}$$

where Φ - is a given sufficiently smooth function of its arguments;

t' - is a given point in $[0,T]$.

The functionals of the type

$$F[u(t)] = \max_t \Phi[x(t), u(t)], \tag{12}$$

$$F[u(t)] = \int_0^T |\Phi[x(t), u(t)]| dt \tag{13}$$

available in the control problems, are differentiated by the directions in the functional area (Fedorenko, 1978) and in problems numeric solution are replaced with one or approximated with several functionals of the type (10) or (11).

The functionals (10), (11) differentiation procedure is reduced to the computation by the following dependences.

The matrix elements $\omega(t)$ of the functionals partial derivatives m by r control actions of the $r \times m$ dimension are calculated by the formula

$$\omega(t) = f_u(t)\psi(t) + \Phi_u, \tag{14}$$

where $f_u(t) = f_u[x(t), u(t)]$ is a conjugate matrix of the $r \times n$ - dimension of partial derivatives of the second members of equations (1) by u;
Φ_u - is the matrix of $r \times m$-dimension of Φ-functions partial derivatives by u.

The dual variables matrix elements ψ of the $n \times m$-dimension represent the solution of the conjugate set of differential equations:

$$\dot{\psi} = -f_x(t)\psi(t) - Y(t), \tag{15}$$

where $f_x(t) = f_x[x(t), u(t)$ - is a conjugate matrix of the $n \times n$-dimension of partial derivatives of the second members of equations (1) by phase coordinates x;
$Y(t)$ - is the $n \times m$-dimension matrix.

For the functionals of type (10) $Y(t) = \Phi_x(t)$, where Φ_x - is the conjugate matrix of the $n \times m$-dimension of Φ-functions partial derivatives by phase coordinates $\mathbf{X}$. Set of equations (15) is integrated from the right to the left with the boundary condition $\psi(T) = 0$.

For the functionals of type (11) $Y(t) = 0$, $\Phi_u = 0$, set of equations (15) is integrated from the right to the left with the boundary condition $\psi(t') = \Phi_x(t')$, and $\psi(t) \equiv 0$ for *t*-values satisfying the inequality $t' \le t \le T$.

In problems pertaining to the control formation for the AV-motion in the atmosphere, the functionals of the type

$$F[u(t)] = \int_0^{t'} \Phi[x(t), u(t)]dt \,, \tag{16}$$

are very important, since with the aid of them the restrictions of phase coordinates and motion regimes are specified at any point of the trajectory.

For these functionals the elements of the functional derivatives and dual variables matrix are computed in accordance with (14) and (15), and $Y(t) = \Phi_x(t)$. Set of equations (15) is integrated from the right to the left with the boundary condition $\psi'(t) = 0$, and it should be noted that $\psi(t) \equiv 0$ for the *t*-values $t' \le t \le T$.

4. FINITE-DIMENSIONAL APPROXIMATION OF THE PROBLEM

A numeric implementation of the sequential linearization method is carried out with the help of a finite-dimensional approximation, allowing of reducing the control improvement process to a sequential solution of standard problems of a linear programming. The linear programming mathematic tool ensures an effective solution of constrained problems.

While performing the control improvement iteration by the sequential linearization method, the initial problem is transformed to a finite-dimensional one owing to the replacement of differential equations of motion (1) with finite-difference ones at their numeric integration. During the numeric integration procedure within the time interval $[0, T]$ the points t_i $(i = 1,2,...,N)$ are situated, these points are nodes that contain all the necessary information for the problem linear approximation solution (7) – (9).

After placing the nodes t_i in these points, the values of phase coordinates x_i, dual variables ψ_i and functional derivatives ω_i are calculated, and the control relationships values u_i are registered. Further on these values are used for the relationships approximation by the phase coordinates time, dual variables, functional derivatives and control actions. Thus a continuous problem (7) – (9) is transformed into a finite-dimensional one that can be numerically solved.

As the result of a finite-dimensional approximation made in each control improvement iteration conditions (7) – (9) are represented in the form of a standard problem of linear programming. For this purpose all the used relationships represented by a finite set of values in the knots are approximated under a definite rule. The calculation procedure of the reference control improvement iteration in case of a piecewise-linear approximation is formed based on the following dependencies.

The control $u(t)$ is a vector-function of the r-dimension. Let each component $(k = 1,2,...,r)$ $u^{(k)}$ of the reference control $u(t)$ is approximated by a piecewise-linear function with $u_i^{(k)}$-values in the nodal points t_i $(i = 1,2,...,N)$. Later on the index "k" won't be indicated, and the control $u(t)$ will mean either r–dimension vector-function or its k th component.

Then the k th component of the control $u(t)$, referred to the class of piecewise-linear functions, can be at each moment **t** calculated by the formula

$$u(t) = u_i + \frac{u_{i+1} - u_i}{t_{i+1} - t_i}(t - t_i)\,,$$

$$t_i \le t \le t_{i+1}\,, \quad (i = 1,...,(N-1))\,.$$

The disturbance $\delta u(t)$ of each k th component of control $u(t)$, represented at the same class of functions has the form

$$\delta u(t) = \delta u_i + \frac{\delta u_{i+1} - \delta u_i}{t_{i+1} - t_i}(t - t_i)\,, \quad t_i \le t \le t_{i+1}\,,$$
$$(i = 1,...,(N-1))\,,$$

where δu_i, δu_{i+1} are constant values denoting variations of a continuous piecewise-linear control at the nodal points.

Under these assumptions, conditions (7) – (9) come to the following problem of linear programming relative to the unknowns $\delta u_1,...,\delta u_N$:

$$\delta u_i^- \le \delta u_i \le \delta u_i^+ \quad (i = 1,2,...,N)\,, \tag{17}$$

$$F_j + \sum_{i=1}^{N} \delta u_i h_i^{(j)} \le 0 \quad (j = 1,2,...,m)\,, \tag{18}$$

$$\min_{\delta u_i} \sum_{i=1}^{N} \delta u_i h_i^{(0)}\,, \tag{19}$$

where F_j are the functionals values calculated for the motion reference law $\{u(t), x(t)\}$; δu_i^+, δu_i^- are small given values.

Coefficients $h^{(j)}$ are computed from the integral correlations [2]. Provided the functional derivatives $\omega^{(j)}$ are known at the nodes t_i $(i = 1,2,...,N)$, then by using a piecewise-linear approximation of relationships $\omega^{(j)}(t)$ it is feasible to get the formulae for the $h^{(j)}$ coefficients calculation at the approximation nodes (Lazarev, 1994).

Subject to the complexity or the stage of the problem solution, the approximation nodes can have a uniform distribution in time, a uniform distribution by special functions or in accordance with the methods providing the optimal distribution.

5. ALLOWANCE FOR THE CONTROL CONSTRAINTS

In problems of control of the AV- motion in the atmosphere, the constraints to the size and velocity of the control actions change are taken into consideration. In each iteration of the search made by the successive linearization method, after the improved control values u_i calculation, the following operations are fulfilled.

At the beginning, starting with the first node a successive check of implementing the inequalities $u_{i\min} \le u_i \le u_{i\max}$ $(i = 1,...,N)$ is carried out. In the nodes where these constraints are not complied, the control relationships values are replaced with $u_{i\min}$ or $u_{i\max}$. Then, starting with the interval between the first and second nodes, a consecutive check of the inequalities

$$\dot{u}_{i\min} \le \left|(u_{i+1} - u_i) / t_{i+1} - t_i)\right| \le \dot{u}_{i\max} \ (i = 1,..., N-1)$$

fulfillment is conducted. The values $\dot{u}_{i\min}$ и $\dot{u}_{i\max}$ correspond to the interval $(i, i+1)$. At the intervals where these restrictions are not fulfilled, the replacement of the control relationships values takes place at the end of the interval. If $u_{i+1} - u_i > 0$, then $u_{i+1} = u_i + \dot{u}_{i\max}(t_{i+1} - t_i)$ or $u_{i+1} = u_i + \dot{u}_{i\min}(t_{i+1} - t_i)$. If $u_{i+1} - u_i < 0$, then $u_{i+1} = u_i - \dot{u}_{i\max}(t_{i+1} - t_i)$ or $u_{i+1} = u_i - \dot{u}_{i\min}(t_{i+1} - t_i)$.

6. ALLOWANCE FOR CONSTRAINTS TO MAXIMAL PARAMETERS

During the control formation for the AV motion in the atmosphere, the restrictions of several functionals of the type (12) are concurrently taken into consideration. Approximation procedures (Fedorenko, 1978) of these functionals are based on the substitution of several functionals of type (11) for one functional (12). In order not to enlarge the functionals number it is proposed to replace each functional of type (12) with one functional (11). In each control improvement iteration for $m1 \le m$ functionals of type (12), time moments t'_{i1} $(i1 = 1,..., m1)$ are registered corresponding to the nodes $n^{(i1)}$, where the functionals achieve extremum points. The functionals are considered as those of type (11). The linear programming problem is solved relative to δu_i.

In problems with one functional of type (12) obviously depending on control, it is additionally assumed in the nodes $n-1$, n, $n+1$ that: $\delta u_{n-1} = \delta u_n = \delta u_{n+1} = K\delta U sign(h_n)$, where $K > 1$ is the coefficient of the reference control small neighborhood extension. Therefore the rate of the functional change is increased and a probable change of the controlled node number is taken into consideration at the next iteration of search.

7. NOMINAL CONTROL FORMING

The nominal control is formed under the condition of practically unlimited period of time, hence the computational algorithm can contain not a predetermined number of operations. In this case the allowance is made for constraints of control, terminal conditions, current trajectory parameters, and the optimization problem can be solved.

The sequential linearization method use is reduced to the choice of the initial control, problem finite-dimensional approximation, computation of functionals and their derivatives by control , and to the linear programming problem solution and obtaining of the improved control that is assumed to be nominal, provided all the problems conditions are met. Otherwise the procedure is iterated, and the improved control serves as the reference one. The nominal control obtained is the function of time.

While forming the nominal control it’s possible to use the search efficiency rise technique: a temporal “freezing” of the approximation nodes lay-out, separate control search by each channel, temporal removal of control for one or several functionals change, modification of the range of acceptability (tolerance region) for the control relationships increments, search procedure interruption, return to the previously obtained results, change of the computational algorithm parameters and continuation of search.

8. COMMAND CONTROL FORMING

The command control forming is real-time, that’s why the computational algorithm contains a predetermined number of operations. The more real conditions are not coinciding with the modeling requirements during the nominal control creation, the more the command control differs from the nominal one.

The command control operability under the conditions of the a priori uncertainty of perturbation actions is ensured owing to the feedback. The multistage control use (Okhotsimskyi et al., 1975) makes it possible to obtain the feedback on the whole, thus closing the control system in case of using at each stage the method that forms the control as the time-varying function. The command control is obtained by the end of each stage by the results of the apparatus motion prognostication made on the basis of the initial information available by the moment of the motion start. For prognostication the data on the phase coordinates values, apparatus parameters, atmosphere characteristics and previously obtained control is used.

During the sequential linearization method the same operations as for the nominal control formation are fulfilled at each stage, but the number of the control improving iterations is predetermined. As the initial control, the one obtained at the previous stage is considered (at the first stage the nominal control is assumed to be the initial one).

9. MATHEMATICAL MODELING

The mathematical modeling was carried out at every stage of the computational method development for the AV-trajectory control, and resulted in solution of three main problems. First, the mathematical modeling was an obligatory stage for the computational method development. It is explained by the necessity of both working off and updating of computational procedures during the concrete problems decisions, and defining recommendations on the choice of parameters computational values for these procedures.

Secondly, the confirmation of operability and effectiveness of the developed technique for ensuring the multichannel nominal and command AV-trajectory control is grounded on the mathematical modeling results. The mathematical modeling provided the development of the numerical method application technique for various classes of problems.

And thirdly, new problems of the AV-trajectories control were solved during the mathematical modeling. This result of the developed numeric method use is of a particular importance, since the obtained outcomes prove the AV great resource during complex advanced maneuvers fulfillment, including emergency situations.

Results of concrete problems of the nominal and command AV-trajectories control formation for the cases of the atmospheric reentry, occurrence of off-optimum situations in the trajectory of the AV launch to the Earth satellite orbit, and also at the change of inclination of the AV orbital plane in the atmosphere, solved by means of the numeric method based on the sequential linearization are shown in Lazarev (1994, 1996a, 1996b, 2007), Balakin, Lazarev and Fillipov (1996), Geraskin and Lazarev (2001). The AV-aerodynamic characteristics and atmospheric density in all the problems were given in tabular form. The motion model was adjusted for the gravitational field eccentricity of the Earth and its spinning motion. The description of the results includes the data specifying the physical aspect of the obtained final results, as well as data on the solution progress, characterizing the efficiency of the numeric methods and algorithms applied.

During the numeric modeling process two types of problems were being solved. The first type covers the problems of nominal control formation that have a known solution. The comparison of the results obtained by the sequential-linearization-based numeric method with the known (reference) solutions of the same problems obtained with the aid of the maximum principle is used for proving the practical applicability of the developed method, as a draft approximation.

The second type problems are more complicated ones of the nominal and command control making. The effectiveness of the developed numeric method and ways of the nominal and command control formation is proved on the basis of solving concrete problems containing restrictions for the phase coordinates, motion regimes and control.

CONCLUSIONS

The results of the control problems solution, their comparison with those obtained out of the same problems solved basing on the maximum principle, as well as the results of new problems afford ground to conclude that the developed numeric method based on the sequential linearization is operable and efficient for solving the AV-trajectories control development problems.

The described method advantages are a low sensitivity to the initial control, possibility of taking into account of various restrictions and constraints, of controlling the search process and influence on it, relative simplicity of computational procedures re-adjustment under the problem conditions change, including the additional constraints emergence.

The experience of the sequential linearization-based numeric method allows of defining it as a universal approach to a wide range of the AV-trajectories control problems.

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