ON THE DUALITY OF GUARANTEED CONTROL-ESTIMATION PROBLEMS FOR HIERARCHICAL SYSTEMS

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Abstract
The paper discusses problems of route design and choice for the planar movement of the objects with the limited maneuverability as vessels are. A notion of hierarchical $(i)-$system is considered. Conjugated hierarchical systems provide dual descriptions of complex surroundings with multiple obstacles and route tube formed by the chain of cylindric branches is considered.

Symmetry of models allows to form algorithm based on the duality property of guaranteed control/estimation problems. Quality is evaluated by extremal functionals. A priori procedures of control and estimation are supposed to be determined by the choice of causal (nonanticipative) operators. Particular cases of dual problems for conjugated hierarchical systems are discussed.

Key words
hierarchical systems, problems of guaranteed control and estimation, duality property

1 Introduction

The formation control and team behavior simulation problems are significant items in modern agenda of optimal control and game theories. The problems also include a wide range of statements connected to modelling real-time interaction of teams with finite membership.

The application analysis shows that various interactions of participants could be considered as individual or common movement depending on restricted resources, in particular, information; supply, distribution and transformation of which are under hierarchically organized control.

The motion of participants may be treated in terms of system positions reflecting state, spacial, conceptual and organizational structures, results of observation and management. Participants may change their positions in accordance with consequent control decisions, step-wise formed on a symmetrical and discrete positional grid. Hence the common interaction is split into multiple layers of respectively independent processes for couples of symmetrical systems. Note that logics of business-planning is the same and decision-making tree techniques are evident example. Game models of conflicts in industrial management and production system simulating are similar in ideas, but is out of discussion here.

The problem under consideration in the paper is connected with route planning for the team of objects constrained in dynamics and overcoming obstacles in the common motion.

Research is partly motivated by applications in navigation [Kruglikov, 1994] and net tracing, where practical problems may be stated as geometrical, even planar ones. The crucial point for route planning algorithms design is choice of adequate information structure. The requirement is to describe regularly complex circumstances with multiple obstacles and routes of team motion in case of objects limited in perception and conflicting interests. Further it is convenient to discuss the obstacles as islands.

Different mathematical approaches are known to be examined for route design. The reference presented below does not even sketch the variety of ideas but outlines some restrictions to realize them.

Well-known techniques of the route choice is based on the obstacles description as net cells various in complexity. The route trajectory is drawn nearby the contour vertices. The difficulties of applying this model for the optimal route design are considered in [Shlapobersky, Lyapustina, Nikolaev and Hramov 2007].

Interval analysis techniques [Jaulin, 2001] based on the description of the obstacles as the union of rectangles proves to be efficient. But corresponding constructions depending on the level of investigation are not regularly hereditable, that complicates the hierarchy analysis.

Variation techniques developed in [Berdyshev, Kostousov, 2007] allow full scale route modelling based on contingent constructions. The significant maneuverability is required.

A wide range of situations allows imbedding restrictions of object dynamics and observation in uncertainty of state position. Then the analysis of tubes of admissible trajectories is possible via guaranteed approach. In [Kruglikov, 1994] was shown that a priori choice of optimal tube and parameter approximation of obstacle are symmetrical problems.

The structural symmetry and duality of problem statements and solutions for conjugate systems are essential properties in the optimal control theory under uncertainty [Kurzhanski, 2006]. The duality property for control problems with set-membership description of disturbances and integral / extremal performance index [Kruglikov, 1997] have been investigated on the base of operator presentation.

The possible application of results on duality of guaranteed estimation and control problems for the route design and choice is considered below.

Presented models, based on a notion of hierarchical $(i)-$system, below may provide the unified description of organizational structure, routes and geography. Then the problem of shoreline description is dual with the problem of admissible route design. Main constructions are based on finite combination of chains with cylindric branches. Complex shape of shoreline may be described in advance in the form of sea graph and in the consequence the admissible routes may be chosen on the graph. The admissible route corresponds to the tube of trajectories, sections of which involve accumulated errors.

2 Problem statement.

Assumption 1. Suppose that the following properties hold.

$i)$ Sets $\mathbf{G0}, \mathbf{S0} \subseteq \mathbf{B}(V)$ present the basic zero-level information for description of obstacles and sea. Here $V, \mathbf{B}(V)$ denote metric space (V, dc) and power set respectively. Subsets $G0 = \cup\{G|G \in \mathbf{G0}\}$, $S0 = \cup\{S|S \in \mathbf{S0}\}$ are nonintersecting, $G0 \cap S0 = \emptyset$; $V = G0 \cup S0$.

$ii)$ An admissible trajectory is approximated by a chain with linear branches $\{L_k|\, k \leq K\}$. The inequality below restricts maneuverability

$|L_k| \geq RR[Tan(\alpha_k/2) + Tan(\alpha_{(k-1)}/2)] + d,$

where α_i, RR are angle and radius of return, d minimal length of the straight branch. Accumulated errors of dynamics RR and position observation R^* are estimated by parameter $Rmax$; $Rmax = Max\{R*, RR\}$. Then a chain $CS_{2,K}$ of $(Rmax, L_k)$-cylinder branches is a tube of trajectories corresponding to admissible routes.

$iii)$ Functionals ϕ_K, ϕ_L, ϕ_E estimate quality of $CS_{2,K}$, where $\phi_K(CS_{2,K}) = m(K)$ is amount of branches, $\phi_L(CS_{2,K}) = \sum_{k\leq K} |L_k|$ is a length of tube, $\phi_E(CS_{2,K}) = \phi_L(CS_{2,K})/\phi_K(CS_{2,K})$ index of efficiency.

2.1 The notion of hierarchical $(i)-$system.

Definition 1. The triple $C = \{\mathbf{X}|\mathbf{P}, \mathbf{Q}\}$ is called a hierarchical $(i)-$system if the following components are included.

1.1) *Topological region* $\mathbf{X} = \{X|Sc\}$. Here $X \subseteq (V, dc)$, $Sc = \{cc, rc|\mathbf{e}c\}$ is a polar coordinate system corresponding to the fixed point cc and zero direction $\mathbf{e}c$.

1.2) *Graph of organizational structure* $\mathbf{P} = \{P, \mathrm{P}\}$. Here P is a list of $(i-1)-$systems presenting vertices $P = \{C.m = \{\mathbf{X}|\mathbf{P}, \mathbf{Q}\}.m\}$ and a binary relation P= $\{(C, C^\star)\} \subseteq P \times P$ reflects a structure.

1.3) *Positional approximation* $\mathbf{Q} = \{Q, \mathrm{Q}\}$; links $Q = \{q = L_k\}$ are $Sc-$ordered by index $\theta : Q \rightarrow N$, $\theta(q) = k$; $\theta(Q) = K = \{1, ..., k\} \subseteq \mathbf{N}$.

Following parameter values $[i, kc, K, ok]$ evaluate basic properties of hierarchical system (i)-$C_{kc,K} = (i)\{\mathbf{X}|\mathbf{P}, \mathbf{Q}\}$. Here level i corresponds to the scale $k = [rc/Rmax]$; $k = 2^i$; $kc = m(Q)$ is amount of actual relations, $0 \leq kc \leq 8$. Amount of components is $K = m(P)$.

Typology of $(i)-$systems includes components, subsystems such as chains $CS_{2,K}$ with cylindric branches, arcs, orbits, stars, nets.

2.2 The circumstances description.

Definition 1 allows to form ordered space $\mathbf{B}$ by the operations over hierarchical system:

(+) combination via smooth closure of $(i)-$links qL_k;

(-) Decomposition $(i)C = \{X|L, S\}$, extraction subsystems $(i-1)C.m = \{X|L, S\}.m$, $m = 1, 2$ $(i-1)C.1(+)(i-1)C.2 = (i)C$.

Note that $CS_{2,K}$ is (+) combination of standard branches, $CS_{2,K} = (+|k \in K)CS_{2,1}$.

Set $\mathbf{BG}$ of hierarchical $(i)-$systems $CG = \{\mathbf{X}G|\mathbf{P}G, \mathbf{Q}G\}$ provides the construction of obstacles as a combination of islands.

1.G) $(i)\mathbf{X}G = (i)\{XG|Sc\}$, where $XG \cap G0 \neq \emptyset$, and $Sc = \{cc, rc|\mathbf{e}c\}$ is an internal polar coordinate system.

2.G) $(i)\mathbf{P}G = (i)\{PG, \mathrm{PG}\}$. The list of $(i-1)-$systems presents islands $(i)PG = \{(i-1)CG.m = \{\mathbf{X}G|\mathbf{P}G, \mathbf{Q}G\}.m\}$, and a binary relation PG$\subseteq PG \times PG$ reflects a structure.

$\mathrm{PG} = \{(CG, CG^\star)|\Pi G(\pi G(CG), \pi G(CG)^\star) \leq \delta\}$, where $\pi G(CG) \leq \pi G(CG^\star)$.

An injective mapping $\pi G : PG \rightarrow \mathbf{N}, \pi G(CG) = m$; gives an ordering of islands with respect to Sc. $\pi G(PG) = MG = \{1, ..., mG\}, mG \in \mathbf{N}$.

$\Pi G : N \times N \rightarrow R^1$; $\Pi G(m1, m2) = (i)\delta.m1m2$ describes the separation of $(i-1)$ systems.

3.G) $\mathbf{Q}G = (i)\{QG, QG\}; (i)QG = \{qG\}$ is a $Sc-$ordered list $L_k = \{l, r|\mathbf{l}\}_k$ of links, chains forming a border of the main system and $(i)qG'$ is for borders of nearby systems of the same level (i). Borders of

level (i) are combinations of $(Rmax, L_k)-$cylindrical branches, ordered by $\theta G : QG \to \mathbf{N}$,
$\theta G(qG) = k;\ \theta G(QG) = KG = \{1, ..., kG\} \subseteq \mathbf{N}$.

2.3 The admissible route tubes description.

Set **BS** of hierarchical $(i)-$systems CS gives description for sea and admissible route tubes if $CS = \{\mathbf{X}S|\mathbf{P}S, \mathbf{Q}S\}$ satisfies the following properties.

1.S) $\mathbf{X}S = \{XS|SS\}$, where $XS \cap S0 \neq \emptyset$. An external polar coordinate system $SS = \{cS, rS|\mathbf{e}0\}$ corresponding to the point cS and zero direction **e**0 is fixed.

2.S) Graph $\mathbf{P}S = \{PS, \text{PS}\}$ provides a network description. Here $PS = \{CS.m = \{\mathbf{X}S|\mathbf{P}S, \mathbf{Q}S\}.m\}$, and PS is a binary relation for structure.

3.S) *position*: $\mathbf{Q}S$ presents a $SS-$ordered list $L_j = \{l, r|\mathbf{l}\}(j)$ of links, describing bays, straits, fjords.

In the case of admissible route tubes the chain with cylindric branches may be presented as follows.

$(S.1)$ $\forall QS_2 =< L1, L2 >\in QS\ \exists CS_{2,K} = (+|k \leq K)CS_{2,K}$; $P_K = \{S(k)|1 \leq k \leq K\} \subseteq PS$: $\forall 1 \leq k < K\ (S(k), S(k+1)) \in P\&LO \in QS(1)\&LF \in QS(K)$.

$(S.2)$ $CS_2 = \{L1, L2|(L1, L2) \in QS\} \quad \Leftrightarrow$ $[(l1, lD) * (l2, lD) < 0 \vee L2 = l1l2 + L1_-]$,
where $lD = A(\alpha/2)l12, \quad l12 = l1l2/d(l1, l2)$.

The couple $\{L1, L2\}$ $(LO, LF), LO = \{lO, Rmax|\mathbf{l}O\}, LF = \{lF, Rmax|\mathbf{l}F\}$; is directed, $Sc-$ordered, if $(lO, lD) * (lF, lD) < 0$, where $lD = \mathbf{A}(/2)\mathbf{l}OF, \mathbf{l}OF = lOlF/|lOlF|$. The couple LO, LF is symmetrical; if $(\mathbf{l}O, \mathbf{l}OF) = (\mathbf{l}OF, \mathbf{l}F)$.

Problem 1.1. Suppose that Assumption 1 holds, description of obstacles is given by the set **BG** and LO, LF are fixed. Find an admissible route $(i)CS^* \in$ **BS** : LO, LF optimal according to the sequential criteria: $\Phi_A = \min_{\mathbf{BS}}\{\phi_K((i)CS)$,
$\Phi_W = \min_{\mathbf{BS}}\{\phi_L((i)CS)$,
$\Phi_I = \max_{\mathbf{BS}}\{\phi_E((i)CS)$.

Construction of sets $\mathbf{BG}, \mathbf{BS}$ of hierarchical $(i)-$systems are symmetrical because they are based on the couples of directed links QG, QS. So a notion of conjugated $(i)-$systems $CG = \{XG|QG, PG\}$ and $CS = \{XS|QS, PS\}$ may be given.

Definition 3. $(ig)CG \in \mathbf{BG}$ and $(is)CS^\star \in \mathbf{BS}$ are conjugated hierarchical $(i)-$systems, if
$\forall (is)CS \in \mathbf{BS} : XG \subseteq XS \Rightarrow XS^\star \subseteq XS$;
and $(i)CS^\star$ is the convex hall depended on $(i) - XG, QG$.

The symmetry in extremal problems means duality of problem statements and solutions for conjugate systems.

2.4 Duality of extremal problems.

Further a symmetrical operator representation of extremal a priori problems stated for linear systems with unknown in advance parameters is used.

The notations below are following: $\mathbf{B}(X, Y)$ is the set of linear bounded operators mapping X in Y; $\overline{R} = R^l \cup \{-\infty, +\infty\}$. The symbols $o, *$ stand for a superposition and conjunction respectively. A scalar product in a Hilbert space X is denoted by $< \cdot, \cdot >_X$. E_Y is the unity operator in $Y, E_Y : Y \to Y$.

Assumption 2. Suppose that the following properties hold.

$i)$ $X, Y, Z, \hat{Z}$ are Hilbert spaces.

$ii)$ F, A_0, B, G_0 are fixed causal [9] operators, and B is a strictly causal one. $F \in \mathbf{B}(X, Z)$, $A_0 \in \mathbf{B}(X, Y)$, $B \in \mathbf{B}(\hat{Z}, Y)$, $G_0 \in \mathbf{B}(\hat{Z}, Z)$.

$iii)$ $\mathbf{U}$ is a set of causal operators $U, U \in \mathbf{B}(Y, \hat{Z})$.

$iv)$ Convex proper functionals ϕ and θ, $\quad \phi : Z \to \overline{R}, \quad \theta : X \to \overline{R}$, are such that

$$\inf\{\phi^*(\zeta^*)|\zeta^* \in Z\} > -\infty, \quad \inf\{\theta(\eta)|\eta \in X\} > -\infty;$$

Here ϕ is a closed functional.

Elements ξ and ν, satisfying the linear system

$$\left.\begin{array}{l} \xi = F\eta + G_0\nu; \\ \nu = U\zeta, \quad \xi \in Y, \nu \in \hat{Z}; \\ \zeta = A(U)\eta = A_0\eta + B\nu; \end{array}\right\} \qquad (1)$$

may be interpreted as realizations of an observed signal and control. The control procedure $U, U \in \mathbf{U}$, is fixed before a performance of the system (1) with an uncertain parameter $\eta, \eta \in X$, starts. Then the problem of a priori design of control acting on the base of uncomplete or imperfect observations may be stated as the following one.

Problem 2.1. Under assumption 2 find an operator $U_*, \quad U_* \in \mathbf{U}$, satisfying the condition

$$-\infty < \sup_X\{\Phi(U_*)\} = \min_{\mathbf{U}} \quad \sup_X\{\Phi(U)\} < +\infty,$$

where $\Phi(U) = \phi(F\eta + G_0 oUoA(U)\eta) - \theta(\eta)$.

A functional θ describes the quality restrictions on uncertain parameter η. If θ is defined by $\theta(\eta) = \delta(\eta|W)$, where $\delta(\cdot|W)$ is an indicator function [4] for a convex weakly compact set $W, W \subseteq X$, then problem 2.1 may be interpreted as an a priori problem of ensured control and/or estimation [Kruglikov, 1997].

The assumptions on operators B and $U, U \in \mathbf{U}$, mean that mappings $\Psi_Y(U), \Psi_{\hat{Z}}(U)$;
$\Psi_Y(U) = E_Y - B \circ U, \qquad \Psi_Y(U) : Y \to Y$;
$\Psi_{\hat{Z}}(U) = E_{\hat{Z}} - Uo \circ B, \qquad \Psi_{\hat{Z}}(U) : Z \to Z$.
are homeomorphisms and the equality

$$U \circ \Psi_Y^{-1}(U) = \Psi_{\hat{Z}}^{-1}(U) \circ U,$$

holds for every $U \in \mathbf{U}$. Hence the equality holds
$G_0 oUoA(U) = G(U)oUoA_0$,
where $G(U) = G_0 o\Psi_{\hat{Z}}^{-1}(U)$. Moreover a singleton correspondence between sets $\mathbf{U}$ and $\mathbf{U}_0$ exists. It is defined by equivalent expressions

$\mathbf{U}_0 = \{U_0 = Uo\Psi_Y^{-1}(U) = \Psi_{\hat{Z}}^{-1}(U)oU | U \in \mathbf{U}\}$;
$\mathbf{U} = \{U = U_0 \circ (E_Y + B \circ U_0)^{-1} =$
$= (E_{\hat{Z}} + U_0 \circ B)^{-1} oU_0 | U_0 \in \mathbf{U}_0\}$.

Thus for sets $\mathbf{U}_0$ and $\mathbf{U}$ satisfying the equality above the problem 2.1 is equivalent to the following one.

Problem 2.2. Let the assumption I holds, and an operator set $\mathbf{U}_0$ is given, $\mathbf{U}_0 \subseteq \mathbf{B}(\hat{Z}, Z)$. Find an operator $U_{0*}, U_{0*} \in \mathbf{U}_0$, satisfying the equality

$$-\infty < \sup_X \{\Phi_0(U_{0*})\} = \min_{\mathbf{U_0}} \quad \sup_X \{\Phi_0(U_0)\} < +\infty,$$

where $\Phi_0(U_{0*}) = \phi(F\eta + G_0 oU_{0*} oA_0 \eta) - \theta(\eta)$.

The last statement corresponds to the case of open-loop control systems or problem 2.1 with a closed-loop effect eliminated.

2.5 The basic algorithm design.

Analogy of the problems 1.1 and 2.1 allows to form an algorithm of route design in accordance with the separation principle of control optimal for systems suffering from unknown in advance disturbances. Then solutions of the problem 1.1 may be constructed through a step-wise procedure based on solutions of two separate extremal problems. The first one is a problem of situation estimation in the case of a given control. The other is an optimal synthesis problem for a system with complete measurement data available. In the game theory similar property is known as certainty equivalence. Mention that systems under consideration are non-linear, performance of whose are evaluated through an extremal functional. So the tube formed this way will be admissible, but should not be optimal.

Description of obstacles analogical to the problem 2.2 may be given by the statement below.

Problem 2.2. Suppose that Assumption 1 holds, description of obstacles is given by the finite list PG. Find the set **BG** of hierarchial systems $(i)CG^* \subseteq \mathbf{BG}$ optimal in accordance with the sequential criteria:

$\Phi_A = \min_{\mathbf{BG}}\{\phi_K((i)CG)$;

$\Phi_W = \min_{\mathbf{BG}}\{\phi_D((i)CG)$, where $\phi_D((i)CG)$ evaluates maximal deviation if $\phi_K = K$ is given.

$\Phi_I = \max_{\mathbf{BG}}\{\phi_E((i)CG)$.

The structure of the standard hierarchial system corresponding to the situation without obstacles is given by equalities below.

The couple of links $L2 = \{L1, L2\}LS$ determine the set of routes $\mathbf{BS}(LO, LF) = \{CS_{2,K} = \{LO, \{LO, LF\}_K, LF\} = \{XS | L2, SS = \{S_k | k \in K\}\}(K)\}; Kmin \leq K \leq Krec \leq Kmax$.

Here $\{LO, \{LO, LF\}(K), LF\}$ is a combination of a separate subsystems $CS_{2,K}(LO, LO(K))(+)[$ orbit $\{LO, LF\}(K)]$ $(+)CS_{2,K}(LF(K), LF)$. The result may be codified $f(k) = (k1.k2.k3)$, where $k1, k2$ correspond to efficient and actual set of knots on the $k1-$orbit, $k2 = m(PS); k3 = k-$ knot number; $1 \leq Kmin \leq k3 \leq k2\ Krec \leq k1 = Kef \leq Kmax$. Here values $Krec, Kmax$ may be exactly calculated in particular cases.

Lemma 1. If k is given, then regular surrounding CS^* of the sector $G = S(c, r|\mathbf{e})$ is symmetrical and equally tight in point cm corresponding to directions $\mathbf{A}[\alpha(K-2k0)/K]\mathbf{e}m, k0 = 0, ..., K$. CS^* determines the shortest route. The length of weak surrounding (via the central line) $Lc(m, K) = |LS[G; K]|$ and maximum deviation $c(K)$ from the sector arc, $Lc(\alpha.m, K) = 2rrK \tan(\alpha.m/K)$,

$c(K) = -rr = rr[\cos^{-1}(\alpha.m/K) - 1] = 2rr[\tan^{-1} 2(\alpha.m/2K) - 1] - 1$.

Lemma 2. Extremal via criteria $[E.A|W|I]$ regular surroundings $CS[E](G; K, Rmax)$ of the sector G corresponds to the parameter K values $K* = Kmin|Kmax|Kef$, respectively. Values $Kmax = \min\{KKrec|[2RRtg(\alpha.m/K) + d]Komrr]\}$ provides the minimal length of the linear branch;

Kef is an efficient surrounding $CS[I](G; K) = (+|m)CS2(Gm, 1, Rmax) : |CS(G; Kef)| = \max\{Lc(\alpha.m, K)/K\}$, branch lengthes are no more then $2Rmax$;

$Kef = \min\{K(\alpha.m, r.m), [KK\alpha.m/\phi^*]\}$,

$K(\alpha.m, rm) = \max\{K(\alpha.m), [\alpha.m/\phi^*]\}; \phi^* = 2Arctg(Rmax/(rm + Rmax))$.

Lemma 3. Optimal $[A|W|I]$ routes are standard; $m \subseteq MS(LO, LF)M(LO, LF).\varpi, \omega$. and are of the form

$CS_{2,K} = \varpi\langle L1, LO(K)\rangle(+)[\omega\{LO, LF\}(K)]$
$(+)\varpi\langle LF(K), L2\rangle$.

Approach to the problems of route design and choice based on the analogy with the structural properties of ensured control/estimation is considered. Systems under consideration are non-linear, performance is evaluated by extremal functionals. The simulation tests show the adequacy of presentation in terms of conjugated hierarchical systems and dual problem statements to real situations. The additional theoretical investigation are required.

Acknowledgements

The research was partly supported by the Collaboration Programme in fundamental research of Ural and Siberian Branches of Russian Academy of Sciences "Qualitative and numerical analysis of evolution equations and control systems".

References

Berdyshev, V. I., Kostousov, V. B. (2007). *Extremal problems and models of navigation on geophysical fields*. Ural Branch of RAS. Yekaterinburg. *(in Russian)*

Ivanov, A.O., Tuzhilin, A.A. (2001). *Branching Solutions to One Dimensional Variations Problems*. World Publish Press.

Ivanov, A.O., Tuzhilin, A.A. (2005). Extremal Networks. *Acta Appl. Math.*, **66**, pp. 251–317.

Jaulin, L., Kieffer, M., Dildrit, O., Walter, E. (2005). *Applied Interval Analysis with Examples in Pa-*

rameter and State Estimation, Robust Control and Robotics. Springer-Verlag, London.

Kurzhanski, A.B., Mitchel, I.M., Varaiya, P. (2006). Optimization Techniques for State-constrained Control and Obstacle Problems *Journ. of Optimization Theory and Appl.*, **Vol. 128**(3), pp. 499–521.

Kurzhanski, A.B. (2009). On the problems of compound motion control. In *Abstr. Int. Conf. on Actual Problems of Stability and Control Theory*. Yekaterinburg, Russia, Sept. 21-26. p. 103. *(in Russian)*

Kruglikov S. V. (1994) On the duality principle in a priory problems of ensured control and estimation. *Doklady Akademii Nauk*, **Vol.17**, pp. 570–572. *(in Russian)*

Kruglikov, S. V. (1997) On the operator formalization of the separation property in problems of ensured control and estimation. In *Proc. European Control Conf.* Brussels, Belgium, July 1-4, WE-E-I3.

Kruglikov, S. V. (2005) The Problem of Planning Naval Compound Operations. In *Proc. Conf. on Mathematical and Informational Technologies in Management, Engineering and Education*. Yekaterinburg, Russia, Novem. 20-23. pp. 149– 153. *(in Russian)*

Shlapobersky A. B., Lyapustina O. V., Nikolaev A. A., Hramov M. Y. (2007) Development of a complex preparation of the special cartographic information for the information-control systems. In *Proc. II Inter. Conf. on Mathematical and Informational Technologies in Management, Engineering and Education* . Yekaterinburg, Russia, Novem. 19-20. **Vol.3**, pp. 153–157 *(in Russian)*