Locally Optimal Control for Resonance Oscillations in a Nonlinear System with Bounded Perturbations

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Locally optimal control is designed to minimize the phase and frequency deviations from resonance in a nonlinear system with bounded perturbation. The control strategy is independent of the shape of perturbations and the structure of the conservative subsystem. As an example, the phase and frequency control of forced oscillations in a system of weakly coupled oscillators is constructed.

I. Introduction

For a wide range of oscillatory models the analysis of weakly perturbed motion in the neighbourhood of resonance is reduced to the analysis of "an equivalent pendulum" [1, 2]. The phase and frequency deviations from the resonance surface correspond, respectively, to the phase and the frequency of the pendulum oscillations. The phase plane of the pendulum is divided into the domains of libration and rotation separated by the separatrix of resonance. The domain of libration is interpreted as an admissible domain; the passage through the separatrix is associated with failure of resonance. The control task is thus to keep the system within this domain.

The pendulum-like model allows making use of the well-developed asymptotic methods of control of oscillations [3, 4]. The asymptotic solutions of the resonance control problems for deterministic and stochastic single-degree-of-freedom systems have been discussed in [5, 6, 7]. However, the solution of similar problems for multidimensional systems is problematic.

Locally optimal control strategy is considered as an alternative approach to motion control. It has been shown [3, 8] that locally optimal control closely corresponds to the solution of the maximum residence time problem. However, the locally optimal control design is much simple than the direct solution of the optimal control problem.

In Section II we write the equations of the near-resonance motion, introduce the criterion of local optimality, and construct the solution of the control problem. Then we specify the phase and frequency criteria of local optimality and define the phase and frequency control associated with these criteria. We show that the phase and frequency control is asymptotically equivalent to locally optimal control.

Section III demonstrates the applicability of the approach developed to a multidimensional system. As an example, we construct the phase and frequency control for a nonlinear system of coupled oscillators.

II. Basic Methodology

2.1. The equations of motion

For brevity, we consider a two-frequency system with a scalar slow variable, scalar perturbation and scalar control. The MIMO system may be studied in a similar way.

A typical model of a two-frequency system is associated with a nonlinear quasi-conservative single-degree-of-freedom system with weak harmonic excitation. The system dynamics is characterized by the impulse-phase variables [1]; in the quasi-conservative system the impulse is the slow variable, the phase is the fast variable. The natural frequency of the nonlinear system explicitly depends on the impulse. A resonance relationship between the system and excitation frequencies determines the frequency and the impulse of motion. In practice, the excitation frequency is constant; we consider a more general case of two frequencies depending on the slow variable.

The equations of motion are reduced to the standard form

$$\dot{x} = \varepsilon f(x,\theta_1,\theta_2) + \varepsilon^{n} F(x,\theta_1,\theta_2)u + \varepsilon\Delta(x,\theta_1,\theta_2,\xi(t)), \qquad (1)$$

$$\dot{\theta}_i = \omega_i(x) + \varepsilon f_i(x,\theta_1,\theta_2) + \varepsilon^{n} G_i(x,\theta_1,\theta_2)u + \varepsilon\Delta_i(x,\theta_1,\theta_2,\xi(t)),$$

$i = 1, 2.$

Here $x \in X$ is the slow variable, $\theta_i (\mathrm{mod}\ 2\pi)$ are the fast scalar phases, control $u \in U$. The domain X is an open set in R^1, U is a compact in R^1, $\varepsilon > 0$ is a small parameter. The coefficient ε^{n} shall be so chosen that control would remain weak but counteracting the external perturbation. The choice of the parameter n for different types of systems is discussed later.

We presume that

1°. The right-hand sides of system (1) are 2π – periodic in θ_1, θ_2, and smooth enough in all variables;

2°. The perturbation $\xi(t)$ is uniformly bounded, $|\xi(t)| \le \xi_0$, $-\infty \le t \le \infty$.

Following [1, 2], we specify the resonance relationships between the system frequencies. Consider the subsystem

$$\dot{x} = \varepsilon f(x, \theta_1, \theta_2),\ \dot{\theta}_i = \omega_i(x). \quad (2)$$

Define the time average $\langle f \rangle$ of the function $f(x, \theta_1, \theta_2)$ as the function of the slow variable x and the frequencies ω_1, ω_2

$$\langle f \rangle = \Phi(x, \omega_1, \omega_2) = \lim_{T\to\infty} \frac{1}{T}\int_0^T f(x, \omega_1 t, \omega_2 t)\mathrm{d}t.$$

The function $\Phi(x, \omega_1, \omega_2)$ is assumed to be continuous uniformly in almost all $x \in X$, $\omega_{1,2}$ except for a set (ω_1, ω_2) of the solutions of the equation

$$\rho(x) = m_1\omega_1(x) + m_2\omega_2(x) = 0, \quad (3)$$

where m_1 and m_2 are some integers, $m_1^2 + m_2^2 \neq 0$. Formula (3) determines the resonance frequencies of system (2).

Let Eq. (3) have a unique solution x^* such that

$$\rho(x^*) = 0, \quad \mathrm{d}\rho(x^*)/\mathrm{d}x = r \neq 0. \quad (4)$$

In addition to 1°, 2°, we assume

3°. The perturbation $\xi(t)$ does not yield a new resonance relationship similar to (3) in a neighbourhood of $x = x^*$.

Assumptions 1°-3° imply that the solution of system (1) exists and the requisite transformations remain valid in a neighbourhood of resonance (4) for any admissible control $u \in U$.

The control task is to keep the frequencies in the near-resonance domain in the presence of perturbation. We shall state this requirement as a control problem.

Following the standard approach [1, 2], we introduce the variables v and φ characterizing the frequency and phase deviations in the near-resonance domain. As known [1, 2], the frequency deviations in the near-resonance domain are of order $\mu = \varepsilon^{1/2}$. Thus we write

$$\mu v = \rho(x) = m_1\omega_1(x) + m_2\omega_2(x), \quad \varphi = m_1\theta_1 + m_2\theta_2. \quad (5)$$

Conditions (4), (5) allows the representation of the phase variables in the near-resonance domain as

$$x = X(\mu v) = x^* + \mu x_1 + \mu^2,\ x_1 = r^{-1}v,$$

$$\theta_1 = \theta,\ \theta_2 = m_2^{-1}(\varphi - m_1\theta). \quad (6)$$

Inserting (5), (6) into system (1), we obtain the equations of motion in the near-resonance domain in the form

$$\dot{v} = \mu[\beta(\varphi) + b(\varphi, \theta) + \Delta^*(\varphi, \theta, \xi(t))] + \mu^{2n-1}F^*(\varphi, \theta)u + \mu^2[r_1(v, \varphi, \theta, \mu)u + R_1(v, \varphi, \theta, \xi(t), \mu)],$$

$$\dot{\varphi} = \mu v + \mu^{2n}G^*(\varphi, \theta)u + \mu^2[r_2(v, \varphi, \theta, \mu)u + R_2(v, \varphi, \theta, \xi(t), \mu)],$$

$$\dot{\theta} = \omega^* + \mu\kappa v + \mu^{2n}G_1^*(\varphi, \theta)u + \mu^2[r_3(v, \varphi, \theta, \mu)u + R_3(v, \varphi, \theta, \xi(t), \mu)], \quad (7)$$

where the residual terms r_i, R_i vanishes as $\mu \to 0$, and

$$\theta = \theta_1,\ \omega^* = \omega_1(x^*),\ \kappa = \omega_{1x}(x^*),$$

$$\beta(\varphi) = \langle f^*(\varphi, \theta)\rangle,\ b(\varphi, \theta) = f^*(\varphi, \theta) - \beta(\varphi),$$

$$\Phi^*(\varphi, \theta) = r^{-1}\Phi(x^*, \theta, \theta_2(\varphi, \theta)). \quad (8)$$

Here Φ and Φ^* are the vectors with the components (f, Δ, F, G_1) and $(f^*, \Delta^*, F^*, G_1^*)$, respectively, $\langle f^*(\varphi, \theta)\rangle$ denotes the averaging in θ, and

$$G^*(\varphi, \theta) = m_1G_1^*(\varphi, \theta)) + m_2G_2^*(\varphi, \theta)). \quad (9)$$

We now define the admissible domain of motion. Consider the slow subsystem of (3)

$$\dot{v} = \mu\beta(\varphi),\ \ \dot{\varphi} = \mu v. \quad (10)$$

Equations (10) describe motion of the pendulum-like system with energy $E = \mu H(\varphi, v)$, where

$$H(\varphi, v) = U(\varphi) + v^2/2. \quad (11)$$

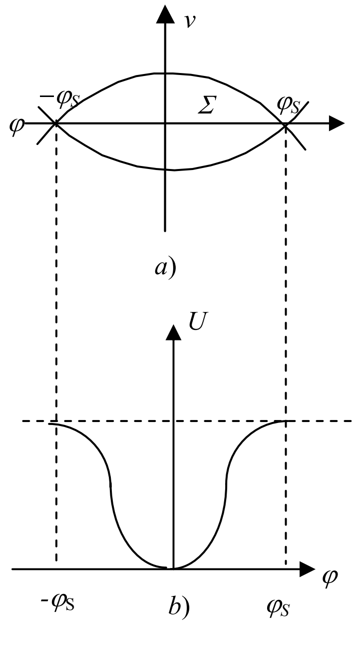

Figure1. *The phase plane* (*a*) *and potential* (*b*) *of system* (10)

in which the *cos*-like potential $U(\varphi)$ is defined by the equation $U_\varphi(\varphi) = -\beta(\varphi)$. The steady-state solution of Eq. (10) is defined by the equalities $v = 0$, $\beta(\varphi) = 0$.

For the typical pendulum-like model we have $U(\varphi) = U(-\varphi)$, $\beta(-\varphi) = -\beta(\varphi)$ [1]. Let $\varphi = 0$ and $\varphi = \pm\varphi^s$ be the solutions of the equation $U_\varphi(\varphi) = -\beta(\varphi) = 0$ corresponding to the minimum and the symmetric maxima of the function $U(\varphi)$, respectively (Fig. 1).

For simplicity we let $U(0) = 0$. It follows from the minimum condition that $U_{\varphi\varphi}(0) > 0$, $\beta_\varphi(0) < 0$. Definition (5) and inequality $\beta_\varphi(0) < 0$ imply that the point O: $\{\varphi = 0, v = 0\}$ is the stable steady-state solution of system (10) associated with the stable resonance in the unperturbed system. From this standpoint, system (7) can be interpreted as the equations of the perturbed motion of the pendulum. In order to depict admissible deviations, we consider the phase plane of system (10) (Fig. 1). The phase plane is divided into the domains of libration and rotation separated by the separatrix; the passage from libration to rotation is associated with failure of resonance [1, 2]. The domain of libration Σ can thus be treated as the reference domain.

The control task is not so much to minimize deviations from the reference point O but to prevent the perturbed system from leaving the safe domain Σ. In general, the direct solution of the maximum residence time problem for system (7) with unknown coefficients is impossible. To simplify the problem, we construct an associated locally optimal control [3, 8]. Further we prove that locally optimal control is closely tied to the solution of the maximum residence time problem.

2.2. The control problem

We construct the function

$$h = v^2/2 + k(\varphi)^2/2 \tag{12}$$

as a measure of deviations from the steady-state point O, $k > 0$ is a weight coefficient. Define the domain $\Sigma^h \in \text{int } \Sigma$ such that $(\varphi, v) \in \Sigma^h \Rightarrow h \in [0, h^*)$. The control task is to keep the function $h(t)$ within the interval $[0, h^*)$ over the maximum time interval. An associated locally optimal control problem [8] is reduced to minimization of the derivative

$$J(u) = \dot{h}(t) \tag{13}$$

at each moment t. The control constraint is taken in the form $|u| \le U_0$. Locally optimal control is defined as

$$u_{\text{opt}} = \arg\min_{|u|\le U_0} J(u). \tag{14}$$

The physical meaning of criterion (13) is quite obvious. Control slows down motion from the core to the boundary of the interval Σ^h if $\dot{h} > 0$, but accelerates motion toward the core of Σ^h if $\dot{h} < 0$. In both cases, control extends the time of the system's residence within Σ^h. Further we demonstrate that control (14) maximize the time until the first exit from Σ^h.

Calculating (13) by virtue of Eqs (7) we find

$$\dot{h} = v\dot{v} + k\varphi\dot{\varphi} =$$

$$[\mu^{2n-1}F^*(\varphi, \theta)v + \mu^{2n}kG^*(\varphi, \theta)\varphi + \mu^2 r(\varphi, v, \theta, \mu)]u + \mu R(\varphi, v, \theta, \xi(t), \mu), \tag{15}$$

where the functions r and R comprise the residual terms and the terms independent of u, respectively.

It follows from (14), (15) that

$$u_{\text{opt}} = -U_0\text{sgn}[\mu^{2n-1}F^*(\varphi, \theta)v + \mu^{2n}k\varphi G^*(\varphi, \theta) + \mu^2 r(\varphi, v, \theta, \mu)]. \tag{16}$$

Introduction of the small parameter μ allows construction of a relatively simple near-optimal control u^* such that $u^* \to u_{\text{opt}}$ as $\mu \to 0$. We consider the near-optimal control for two models of the controlled systems.

2.2.1. The frequency control. Let $F(x, \theta_1, \theta_2) \neq 0$, $n = 1$. In this case the control term in the first equation of system (7) is of the leading order; the control terms in the other equations are negligibly small. This yields

$$u^* = -U_0\text{sgn}F^*(\varphi, \theta)\text{sgn}v \tag{17}$$

as $\mu \to 0$. Near-optimality of control (17) can be proved in the standard way [3, 4].

Under the assumptions accepted in this item, control u^* counteracts the frequency deviations v but it is negligible in the last equations of system (7). This decomposition allows exclusion of the phase dependence from the cost criterion. Introduce the function

$$h^v = v^2/2 \tag{18}$$

as a measure of the frequency deviations from resonance. Let control u^v minimize the derivative $\dot{h}^v(t)$, that is

$$J^v(u) = \dot{h}^v(t),\quad u^v = \arg\min_{|u|\le U_0} J^v(u). \tag{19}$$

In the very same way as before we find

$$u^v = u^* = -U_0\text{sgn}F^*(\varphi, \theta)\text{sgn}v. \tag{20}$$

Equality (20) implies that criterion (18) may substitute for criterion (13). The associated control law (17) can thus be interpreted as the frequency control.

2.2.2. The phase control. If $F(x, \theta_1, \theta_2) = 0$, we take $n = ½$. In this case control u is not involved in the main terms of the first equation in (7) but becomes substantial in the second equation.

It follows from conditions (14), (15) that

$$u^* = -U_0\text{sgn}G^*(\varphi, \theta)\text{sgn}\varphi \tag{21}$$

as $\mu \to 0$. This implies that control (21) directly counteracts the phase deviations. Now we introduce the function

$$h^\varphi = \varphi^2/2 \tag{22}$$

as a measure of the phase deviation and find control u^φ minimizing the derivative $\dot{h}^\varphi(t)$. Calculating $\dot{h}^\varphi(t)$ by virtue of Eqs (7) ($n = ½$) and omitting the higher order terms, we obtain

$$\dot{h}^\varphi = \dot{\varphi}\varphi = \mu[v + G^*(\varphi, \theta)u]\varphi$$

as $\mu \to 0$, and

$$u^\varphi = u^* = -U_0\text{sgn}G^*(\varphi, \theta)\text{sgn}\varphi. \tag{23}$$

Hence, the phase criterion (22) may replace criterion (13) if $F = 0$. The associated control law (21) can be interpreted as the phase controls.

Comment 1. The phase and frequency control is independent of the structure of the conservative part of the system characterizing by the coefficient $\beta(\varphi)$. Also, there is no explicit dependence of the control function on the shape of the perturbation $\xi(t)$.

Comment 2. If the properties of the perturbation $\xi(t)$ are unknown, the precise description of the system dynamics is problematic. The statement can be made that system (7) leaves the safe domain on the time interval $T^u \sim 1/\mu$ but control slows down the motion and increases the time until escape from the safe domain. However, if $\xi(t)$ is a periodic or quasi-periodic process such that

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T \Delta^*(\varphi,\ \omega^* t, \xi(t))\mathrm{d}t = 0,$$

then locally optimal control stabilizes the system near the point *O*. This result can easily be obtained by the averaging method [3, 4].

2.3. Locally optimal control as the solution of the maximum residence time problem

We demonstrate that locally optimal control (14) corresponds to the solution of the maximum residence time problem. Let (φ^u, v^u) be an orbit of system (7) governed by control u, h^u be function (12) calculated along the orbit (φ^u, v^u) starting at the point *O* at $t = 0$. Let T^u be the first moment the function h^u reaches the upper admissible value h^*. In case $u = u_{\text{opt}}$, we denote $h^u = h^0$, $T^u = T^0$. We show that

$$T^0 = \sup_{|u|\le U_0} T^u . \tag{24}$$

To this end, we write

$$h^* = h^u(T^u) = \int_0^{T^u} \dot{h}^u(t)dt .$$

By definitions (13), (14), $h^0(T^u) < h^u(T^u) = h^*$ for any T^u. This means that the locally optimal system does not reach the boundary of the admissible domain by the moment T^u, and proposition (24) holds.

III. Control of coupled oscillators

Equalities (20) and (23) imply that criteria (18) or (21) can replace the general criterion (13). We illustrate this approach by an example.

Consider a linear resonance circuit weakly connected with a nonlinear system. The linear circuit enhances a weak periodic signal of frequency Ω; then the transformed enhanced signal is fed to the input of the nonlinear system. In the absence of perturbations the nonlinear system generates oscillations of a prescribed frequency correlated to Ω. The control task is to maintain the prescribed frequency of nonlinear oscillations in the presence of perturbation. The perturbation can appear either in the nonlinear system directly, or in the resonance and control circuits.

Next we investigate different control strategies.

1. Let the equations of motion have the form

$$\ddot{\psi} + \varepsilon b\dot{\psi} + \Omega^2\psi + \varepsilon\delta_1(\psi, \xi_1(t)) = \varepsilon a \sin \Omega t + \varepsilon s(x, \dot{x}, \psi, \dot{\psi}),$$

$$\ddot{x} + \varepsilon n\dot{x} + \phi(x) + \varepsilon\delta_2(x, \xi_2(t)) = \varepsilon q(x, \dot{x}, \psi, \dot{\psi}) + \varepsilon u . \tag{25}$$

Here $\phi(x) = \mathrm{d}\Pi(x)/\mathrm{d}x$, $\Pi(x)$ is the potential of the conservative counterpart of the nonlinear system. The perturbations $\xi_{1,2}(t)$ satisfy the assumptions of Section 1. The terms $q(x, \dot{x}, \psi, \dot{\psi})$ and $s(x, \dot{x}, \psi, \dot{\psi})$ describe the interaction of the subsystems. Control u is designed by the criteria of Section II.

Reduce system (25) to the standard form. We employ the standard transformation of the variables ψ and $\dot{\psi}$ [2]

$$\psi = R\cos\theta_1, \quad \dot{\psi} = -\Omega R\sin\theta_1. \tag{26}$$

Then we denote $\dot{x} = z$ and introduce the variables y, θ_2 by formulas [1]

$$y = \frac{1}{2}z^2 + \Pi(x),\ z(y,x) = \pm\sqrt{2(y - \Pi(x))},$$

$$\frac{\partial\theta_2}{\partial x} = \frac{\omega(y)}{z(y,x)},\ \omega(y) = \frac{2\pi}{T(y)}, \tag{27}$$

where

$$T(y) = \oint_{y=const} \frac{dx}{z(y,x)} .$$

Formula (27) define the variables $x = X(y,\ \theta_2)$ and $\dot{x} = z(y,x) = Z(y,\ \theta_2)$ as the functions of the variables y, θ_2.

Substituting (26), (27) in (25) and using the notations of Section II, we reduce system (25) to the standard form with two slow variables and three fast phases

$$\dot{R} = -\frac{\varepsilon}{\Omega}[\Psi(v, R, \theta_1, \theta_2, \theta_3) + S(y,\ \theta_2) +$$
$$\Delta_1(R,\ \theta_1,\ \xi_1(t))]\sin\theta_1,$$
$$\dot{y} = \varepsilon\{f(y,\ \theta_2) + [Q(v, R, \theta_1, \theta_2) + u]Z(y,\ \theta_2) +$$
$$\Delta_2(y,\ \theta_2,\ \xi_2(t))\},$$
$$\dot{\theta}_1 = \Omega - \frac{\varepsilon}{\Omega R}[\Psi(v, R, \theta_1, \theta_2, \theta_3) + S(y, \theta_2) +$$
$$\Delta_1(R, \theta_1,\ \xi_1(t))]\cos\theta_1,$$
$$\dot{\theta}_2 = \omega(y) + \varepsilon\frac{\partial\omega}{\partial y}\{f(y,\ \theta_2) + [Q(v, R, \theta_1, \theta_2) + u]Z(y, \theta_2) +$$
$$\Delta_2(y,\ \theta_2,\ \xi_2(t))\},$$
$$\dot{\theta}_3 = \Omega, \tag{28}$$

the coefficients in the right-hand side of system (27) are obtained by the same way as in Section II.

We investigate the main resonance, at which

$$\rho(y^*) = \omega(y^*) - \Omega = 0,$$

$$\mathrm{d}\rho(y^*)/\mathrm{d}y = \mathrm{d}\omega(y^*)/\mathrm{d}y = r \ne 0. \tag{29}$$

As in Section II, we introduce the new variables

$$\varphi = \theta_2 - \theta_3,\ \varphi_1 = \theta_1 - \theta_3,\ \theta_3 = \theta,$$

$$\mu v = \rho(y) = \omega(y) - \Omega,\ \mu = \varepsilon^{1/2}. \qquad (30)$$

Substituting (29), (30) into (28) and ignoring the insubstantial higher-order terms, we obtain the system

$$\dot v = \mu[f^*(\varphi,\theta) + F^*(\varphi,\theta)u + Y^*(v,R,\varphi,\ \varphi_1,\ \theta) + \Delta_2^*(\theta,\xi_2(\mathrm{t}))]$$

$$= \mu[F^*(\varphi,\theta)u + V(R,\ \varphi,\ \theta,\ \xi_2(t))],$$

$$\dot\varphi = \mu v + \mu^2 rV(v,R,\varphi,\ \varphi_1\ \theta,,\xi_2(t)),$$

$$\dot R = \mu^2 P_1(v,\ R,\ \varphi,\ \varphi_1,\ \theta,\ \xi_1(t)),$$

$$\dot\varphi_1 = \mu^2 P_2(v,\ R,\ \varphi,\ \varphi_1,\ \theta,\ \xi_1(t)), \qquad (31)$$

$$\dot\theta = \Omega,$$

in which the coefficients are defined in the same way as in Section II. The precise form of the terms $P_{1,2}$ is insubstantial for the requisite transformation.

Since the function Y^* depends on the slow variables v, $R,\varphi,\ \varphi_1$ system (31) does not allow separation of the simple conservative subsystem similar to (10). However, as seen from (31), control u affects directly the frequency deviation v. Hence control can be chosen by criterion (19). By the same arguments as above, we obtain the frequency control

$$u^v = -U_0 \mathrm{sgn}\, F^*(\varphi,\ \theta)\mathrm{sgn}\, v \qquad (32)$$

coinciding with (20). It is useful to consider the feedback counterpart of control (32). Using the representation $F^*(\varphi,\theta) = r^{-1}Z(y^*, \theta + \varphi)$, $Z(y,\ \theta + \varphi) = \dot x$, we obtain the associated feedback control in the form

$$u^v = -\,U_0\mathrm{sgn}(r^{-1}\dot x)\mathrm{sgn}[\omega(y) - \Omega]. \qquad (33)$$

The only parameter requisite for the control design is $\mathrm{sgn} r = \mathrm{sgn}\omega_y(y^*)$. This parameter can be found without calculating the frequency $\omega(y)$, namely, $r > 0$ if the system is "hard", and $r < 0$ if the system is "soft" in the neighbourhood of the point y^*. The physical meaning of solution (33) is obvious. Let $r > 0$, that is an increase of the nonlinear subsystem energy y entails an increase of the frequency $\omega(y)$. Let $\omega(y) > \Omega$ at some moment t. In this case control (33) takes the form $u^v = -\,U_0\mathrm{sgn}(\dot x)$. Control of this type slows down the motion, diminishes the system energy and results in a decrease of the frequency $\omega(y)$. If $\omega(y) < \Omega$ at some moment t, control $u^v = U_0\mathrm{sgn}(\dot x)$ works in the opposite direction.

2. Let control u acts upon the excitation frequency. The equations of the controlled motion take the form

$$\ddot\psi + \varepsilon b\dot\psi + \Omega^2\psi + \varepsilon\delta_1(x,\xi_1(t)) = \varepsilon a\sin\theta_3 + \varepsilon s(x,\dot x,\psi,\dot\psi),$$

$$\ddot x + \varepsilon n\dot x + \phi(x) + \varepsilon\delta_2(x,\xi_2(t)) = \varepsilon q(x,\dot x,\psi,\dot\psi), \qquad (34)$$

$$\dot\theta_3 = \Omega + \varepsilon^{1/2}u.$$

The right-hand sides of Eqs (34) use the same notation as Eq. (25). The change of variables (26), (27) transforms Eqs (34) to the standard form. Formulas (29), (30) define the resonance relation and the change of variables in the near-resonance domain. In the same way as above we obtain the equations of motion in the near-resonance domain

$$\dot v = \mu V,\quad \dot\varphi = \mu v - \mu u + \mu^2 rV,$$

$$\dot R = \mu^2 P_1,\ \dot\varphi_1 = -\mu u + \mu^2 P_2, \qquad (35)$$

$$\dot\theta = \Omega + \mu u.$$

The functions $P_{1,2}$ and V are defined as in system (31). Control u is involved in three equations of system (34). In case the task is to maintain the resonance oscillations of the nonlinear subsystem regardless the dynamics of the linear circuit, one can consider the subsystem for the variables (φ, v) and construct control minimizing criterion (22). Arguing as above, using formula (21), and considering $G^* = -1$, we obtain

$$u^\varphi = U_0\mathrm{sgn}\varphi. \qquad (36)$$

CONCLUSIONS

Locally optimal control has been designed to minimize the phase and frequency deviations from resonance in a nonlinear system with bounded perturbation. It has been shown that locally optimal control is independent of the shape of perturbations and the structure of the conservative subsystem.

As proved, locally optimal control keeps the system near the stable resonance over the maximum time interval, that is it corresponds to the solution of the maximum residence time problem. The direct solution of the maximum residence time problem for a system with unknown perturbations is prohibitively difficult. The approach developed is effective and simple in use.

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