VIBROIMPACT MOTION OF ROTOR TAKING INTO ACCOUNT FRICTION AT THE CONTACT

Ludmila Banakh
Mechanical Engineering Research Ins.
Russian Academy of Sciences
Sc.Dr., banl@inbox.ru

Andrey Nikiforov
Mechanical Engineering Research Ins.
Russian Academy of Sciences
Ph.D., n.andre@mail.ru

Grigory Panovko
Mechanical Engineering Research Ins.
Russian Academy of Sciences
Sc.Dr., gpanovko@yandex.ru

Abstract
Vibroimpact motion of system "high-speed, flexible shaft with unbalanced disk rotating inside floating sealing ring" is analyzed. It is supposed, that the disk and ring are solid. This rotor system was simulated via Runge-Kutta method. Four vibroimpact regimes were discovered. Spectral analysis of these rotor oscillations is made. Increase of rotor vibration amplitude is calculated. Change of rotor revolutions per minute due to contact is appreciated.

Key words
Rotor, sealing ring, vibroimpact regimes.

1 Introduction

In high-speed rotor machines for prevention of leakages through clearances between rotor and stator elements are widely used floating sealing rings [Childs, 1993]. The rotors rotate inside them with a small radial clearance (0.1-0.3мм), which is actually waterproof.

Because vibration of rotor are inevitable, in particular its cause own residual unbalance, the rotor with floating sealing rings is typical system in which can occur impacts. Given effect increases substantially the vibration level and damages.

2 Vibroimpact system "rotor-ring"

A rather simple calculated model for vibroimpact interaction of high-speed, unbalanced rotor with several floating sealing rings is used. It is system "rotor-ring" with equivalent parameters (Figure 1). Using of such model is possible in consequence of considerable detuning of natural frequencies and because unbalanced loads much exceed weight loads. In result, close to the critical speed a multi-mass rotor behaves approximately as one-mass, and the additive hydrodynamic effect of any quantity of sealing rings is possible to present by one equivalent ring.

Impact oscillations of system "rotor-ring" (between them a liquid passes as continuous flow, see figure 1) are described by equations (1). A pressure field of liquid in the clearance between rotor disk and floating sealing ring δ_0 is simulated as three concurrently acting hydrodynamic forces: elastic (spring) $F_s = k_h \Delta$, damping $F_d = d_h \Delta \dot{\theta}$ and non-conservative $F_n = 0.5 \dot{\varphi} d_h \Delta$. Here k_h is hydrostatic stiffness and d_h is hydrodynamic damping [Nikiforov, Banakh, Panovko and Shohin, 2007]. Projections of these forces from moving axes (u, v) on orthogonal ones (x, y), is written proceeding from expressions: $x_1 - x_2 = \Delta \cos\theta$, $\dot{x}_1 - \dot{x}_2 = -\Delta \dot{\theta} \sin\theta$, $y_1 - y_2 = \Delta \sin\theta$, $\dot{y}_1 - \dot{y}_2 = \Delta \dot{\theta} \cos\theta$.

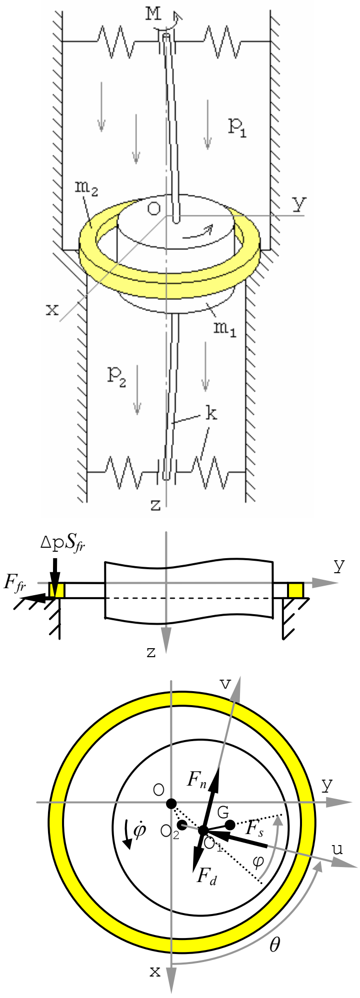

Figure 1. Calculated model "rotor-ring"

$$m_1\ddot{x}_1 + kx_1 + k_h(x_1 - x_2) + d_h(\dot{x}_1 - \dot{x}_2)$$
$$+0.5\dot{\varphi}d_h(y_1 - y_2) = m_1 a\dot{\varphi}^2\cos\varphi,$$
$$m_1\ddot{y}_1 + ky_1 + k_h(y_1 - y_2) + d_h(\dot{y}_1 - \dot{y}_2)$$
$$-0.5\dot{\varphi}d_h(x_1 - x_2) = m_1 a\dot{\varphi}^2\sin\varphi,$$
$$I_0\ddot{\varphi} + m_1 a(\ddot{y}_1\cos\varphi - \ddot{x}_1\sin\varphi) = M, \quad (1)$$
$$m_2\ddot{x}_2 + F_{fr}\,\mathrm{sgn}(\dot{x}_2) + k_h(x_2 - x_1) +$$
$$d_h(\dot{x}_2 - \dot{x}_1) + 0.5\dot{\varphi}d_h(y_2 - y_1) = 0,$$
$$m_2\ddot{y}_2 + F_{fr}\,\mathrm{sgn}(\dot{y}_2) + k_h(y_2 - y_1) +$$
$$d_h(\dot{y}_2 - \dot{y}_1) - 0.5\dot{\varphi}d_h(x_2 - x_1) = 0.$$

Calculated model includes also friction force at the contact of sealing ring with casing $F_{fr} = f\Delta p S_{fr}$, where $\Delta p = p_1 - p_2$ is pressure difference, S_{fr} is friction surface of ring.

If up to nominal rotor speed the rotor disk and floating sealing ring will be solid bodies, then announced problem may be solved in the context of classic impact theory.

Let's simulate the impact interaction by setting two parameters. It is coefficients of restitution e and friction f. Let's describe the vibroimpact motion of rotor and ring (when their relative displacement exceeds the clearance value $\Delta = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \geq \delta_0$) by equations system, where independent variable is normal component of impact pulse S_u:

$$\begin{cases} S_v = -fS_u,\ \dot{u}_2^+ - \dot{u}_1^+ = -e(\dot{u}_2^- - \dot{u}_1^-), \\ m_1(\dot{u}_1^+ - \dot{u}_1^-) = S_u,\ m_2(\dot{u}_2^+ - \dot{u}_2^-) = -S_u, \\ m_1(\dot{v}_1^+ - \dot{v}_1^-) = S_v,\ m_2(\dot{v}_2^+ - \dot{v}_2^-) = -S_v, \\ I_0(\dot{\varphi}^+ - \dot{\varphi}^-) = S_u \times \mathrm{O_1G} - S_v \times \mathrm{CG}, \end{cases} \quad (2)$$

Here $\dot{u}_1^-, \dot{u}_1^+$, ($\dot{v}_1^-, \dot{v}_1^+$) are projections of vibratory velocities of rotor (its centre $\mathrm{O_1}$ in figure 2) on the normal (tangent) before an impact and after it. Respective values of ring (its centre $\mathrm{O_2}$) are designated by an index 2.

Set of equations (2) determines normal and tangential vibratory velocities of rotor and ring after an impact:

$$\dot{u}_1^+ = \frac{(1-\mu e)\dot{u}_1^- + \mu(1+e)\dot{u}_2^-}{1+\mu},\ \dot{u}_2^+ = \frac{(\mu - e)\dot{u}_2^- + (1+e)\dot{u}_1^-}{1+\mu}$$

$$\dot{v}_1^+ = \dot{v}_1^- + f\mu\frac{(1+e)(\dot{u}_1^- - \dot{u}_2^-)}{1+\mu},\ \dot{v}_2^+ = \dot{v}_2^- - f\frac{(1+e)(\dot{u}_1^- - \dot{u}_2^-)}{1+\mu}$$

It defines also change of frequency of rotation:

$$\dot{\varphi}^+ = \dot{\varphi}^- - \frac{m_1 m_2(a + fr)(1+e)(\dot{u}_1^- - \dot{u}_2^-)}{I_0(m_1 + m_2)} \quad (3)$$

where I_0 is moment of rotor inertia, $r \approx \mathrm{CG}$ is radius of rotor disk, $a = \mathrm{O_1G}$ is eccentricity of rotor disk.

Transition from velocities $\dot{x}, \dot{y}$ to velocities $\dot{u}, \dot{v}$ is carried out by means of formulas:

$$\dot{u} = \dot{x}\cos\theta + \dot{y}\sin\theta$$
$$\dot{v} = -\dot{x}\sin\theta + \dot{y}\cos\theta$$

The impact angle θ is defined from geometrical reasoning: $\mathrm{tg}\theta = -\frac{y_1}{x_1}$.

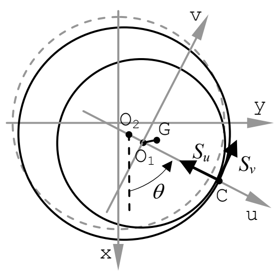

Figure 2. The Projection of a shock pulse of system "rotor - ring"

Thus, motion of rotor and ring in intervals between impacts is described by the differential equations (1), and at the moment of impacts is considered as change of their velocities and frequency of rotation. However this simple approach in a combination with two-mass model of rotor system allows obtaining following results.

3 Results of simulation

So, the features of vibroimpact regimes between unbalanced rotor and floating sealing ring consist of:

1) There is a fixed quantity of impacts between rotor and ring which occur during of the compelled oscillation of rotor or in process of one its revolution. All revealed steady vibroimpact regimes are four-impact.

2) Character of vibroimpact motions is determined only by value of radial clearance.

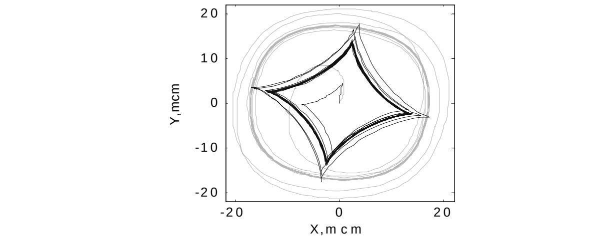

Figure 3. Oscillations, spectrum and trajectories of rotor (light) and ring when $0 < \delta_0/\Delta < 0.4$

If $0 < \delta_0/\Delta < 0.4$ then rotor and ring move in-phase (Figure 3). At that we may to insist that in plane two impacts arise. Given regime is analyzed analytically in paper [Banakh and Nikiforov, 2007]. Circular precession of rotor is practically kept because the rotor mass is much more than the ring mass. At the same time the motion trajectory of ring gets a rhombus form (Figure 3). Floating sealing ring is slave element.

If $0.4 < \delta_0/\Delta < 0.6$ then the ring lags behind of rotor at impacts. Its period of motion equals three periods of rotor oscillations (Figure 4). In process of one ring oscillation arises twelve impacts with rotor. It conducts to X-shaped orbit of its motion (Figure 4).

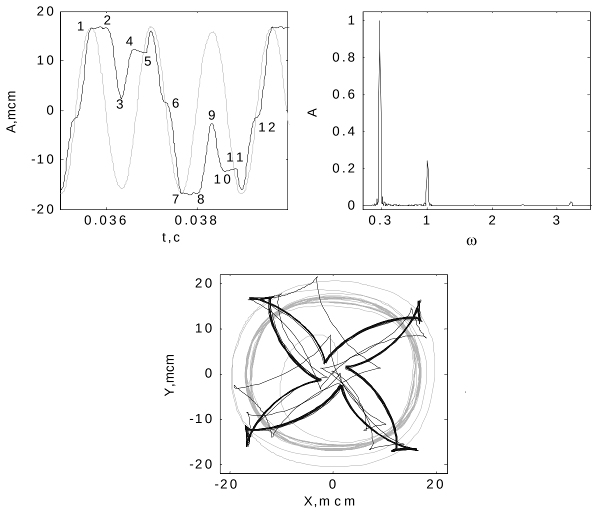

Figure 4. Oscillations, spectrum and trajectories of rotor (light) and ring when $0.4 < \delta_0/\Delta < 0.6$

If $0.6 < \delta_0/\Delta < 0.8$ then vibroimpact motion of rotor and ring occurs in-phase but with unequal clear (from impacts) flyby of ring in mutually perpendicular planes (Figure 5). In this case the centre of ring describes an elliptic trajectory (Figure 5).

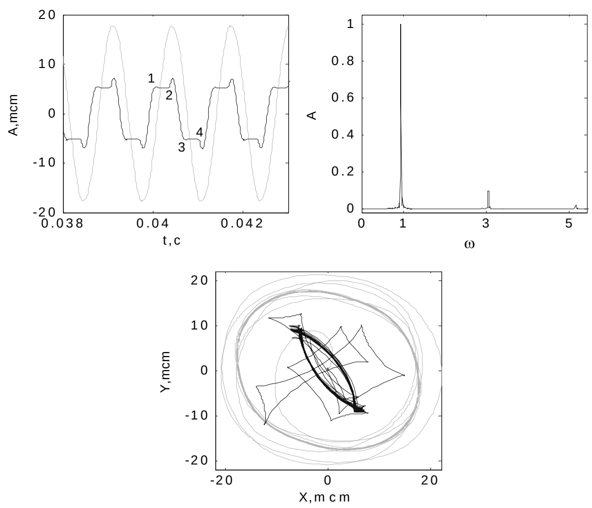

Figure 5. Oscillations, spectrum and trajectories of rotor (light) and ring when $0.6 < \delta_0/\Delta < 0.8$

If radial clearance commensurable with relative displacement of rotor and ring ($0.8 < \delta_0/\Delta < 1$) then period of vibroimpact motion for ring equals three period of rotor oscillations (Figure 6). And again the ring bumps twelve times with rotor during the orbital motion which under the form reminds four-bladed propeller (Figure 6).

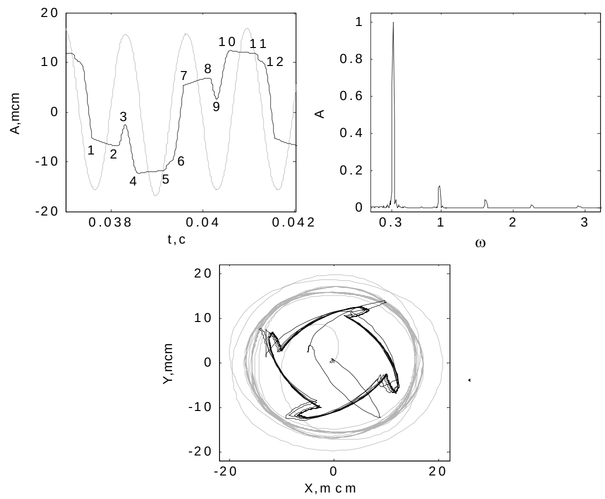

Figure 6. Oscillations, spectrum and trajectories of rotor (light) and ring when $0.8 < \delta_0/\Delta < 1$

3) Impact oscillations of system "rotor - ring" have the special frequency spectrum with harmonics of rotation frequency which are multiple to three. At in-phase vibroimpact motion superharmonious oscillations of rotor and a ring with frequency 3ω are excited. But their power composes only the one tenth from power of oscillations with frequency ω (Figure 3) and (Figure 5). In case of out-of-phase vibroimpact motion the powerful subharmonic oscillations of ring with frequency $\omega/3$ are excited (Figure 4) and (Figure 6).

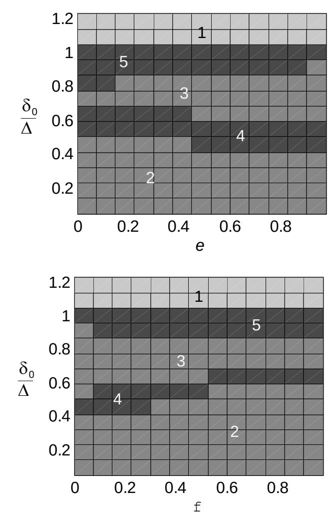

Figure 7. Regimes domains of system "rotor - ring" nonimpact-1, and vibroimpact: in-phase rhombic-2 and elliptic-3, out-of-phase X-shaped-4 and four-bladed -5

4) Coefficients of restitution e and friction f influence only on width of existence domains of revealed vibroimpact regimes (Figure 7).

5) Vibroimpact interaction brings to the amplitude increase. Level of rotor and ring oscillations grows on an average on 10%. Calculated data for various regimes with frequency of rotation $\dot{\varphi} = \omega = 0.8\sqrt{k/m_1}$ are submitted in table 1. Values of amplitudes show that the effect of impact damping of rotor oscillations by means of ring is absent.

Table 1

δ_0/Δ	1.1	0.9	0.7	0.5	0.3
$\sqrt{x_1^2 + y_1^2}$ μm	15.7	16.9	16.4	16.9	17.5
Regime	Nonimpact	Vibroimpact			

6) Considerable change of rotation frequency at the contact doesn't occur because ring mass is small in comparison with rotor mass. In particular it is seen out of formula (3). At scaling up of ring mass the deceleration of rotation frequency grows. It is seen out of limit case for infinite heavy ring mass which corresponds vibroimpact motion of rotor inside fixed sealing ring:

$$\dot{\varphi}^{+} = \dot{\varphi}^{-} - \frac{m_1}{I_0}(a + fr)(1 + e)\dot{u}_1^{-}$$

4 Analysis of out-of-phase vibroimpact motion

It is above that in-phase motion of system "rotor - ring" was analyzed in details [Banakh and Nikiforov, 2007]. In case of their out-of-phase oscillations the character of vibroimpact regimes is transformed seriously (Fig.4,6). The analysis of oscillations in a plane shows (Fig.8) that in case of regimes 4,5 against regimes 2,3 there is quick secondary impact of rotor with ring (with the opposite party of ring at the moment T/2). Oscillations in other plane are similar and occur to shift of phases $\pi/2$ by virtue of circular system symmetry.

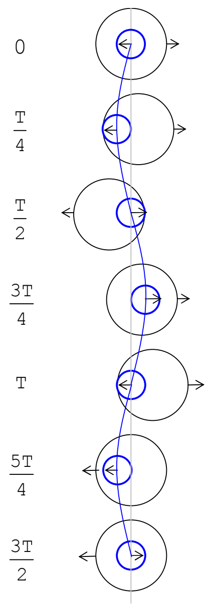

Figure 8. Planar model of system "rotor-ring" for vibroimpact regime 5 ($0.8 < \delta_0/\Delta < 1$)

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