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EXPERIMENTAL STUDY OF UNBALANCED ROTORS SYNCHRONIZATION OF THE MECHATRONIC VIBRATION SETUP

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Abstract

In the paper the results of experimental examination of self-synchronization and Sommerfeld effect both in the open-loop and the closed-loop control modes are presented. The experiments are fulfilled on the novel Multiresonance Mechatronic Laboratory Setup (MMLS), which includes the unbalanced vibro-actuators, mounted on the spring-suspended platform, sensors, electrical motors and the computer interface facilities. It is shown that the closed-loop control makes it possible to stabilize the rotation speed more accurately, than the open-loop motors control. Some additional effects, like low-frequency self-oscillations, may arise due to the integral (I) component of the closed-loop control action.

Key words

control of oscillations, experimental methods, synchronization, vibration, Sommerfeld effect, laboratory setup, technology, nonlinear dynamics

1 Introduction

Vibrational units with unbalanced (eccentric) rotors are widely used in the industry. A typical problem for control of vibration units is passing through resonance at start-up mode of vibroactuators, in the case of the operating modes belonging to a post-resonance zone. Such a problem arises in the case when the power of a motor is not sufficient for passage through resonance zone due to the so-called *Sommerfeld effect*, cf. (Sommerfeld, 1902; Blekhman, 1988; Blekhman *et al.*, 1995; Blekhman, 2000; Blekhman, 2016; Kovriguine, 2012). This effect consists of influence of the lateral motion of the unbalanced shaft support on its rotation, mainly in the vicinity of a resonance. At resonance vibrational motion provides just an "energy sink", or, as Sommerfeld put it "the plant owner spends expensive coal not to rotate his shaft, but rather to shake the foundation" (Sommerfeld, 1902; Dimentberg *et al.*, 1997). It is important to reduce the maximum power of the driving motor achieved during the spin-up mode (Fradkov *et al.*, 2011; Blekhman, 2000). The decrease of the spin-up power leads to decrease of nominal power or the weight and the size of the motor. Solution of problems for control of a rotor speed in a broad range including both pre-resonance and post-resonance regions and passage through resonance is highly important for developing vibrational equipment with improved technological characteristics. The analysis of near-resonance behavior is a challenging task, which attracts attention of researchers for about 60 years, see (Quinn *et al.*, 1995; Blekhman, 2000; Blekhman, 1988; Leonov, 2008; Leonov *et al.*, 1996; Cvetićanin, 2010; Tomchin *et al.*, 2015) for mentioning a few.

The generalized form of a non-ideal system which contains a pure nonlinear oscillator and a non-ideal energy source is studied in (Cvetićanin and Zukovic, 2015). Significant attention is paid to the steady-state motion and the Sommerfeld effect. The authors proposed the procedure for determination of the parameters for suppression of the Sommerfeld effect of the non-ideal system.

A practical problem of synchronization of vibrating actuators was posed and investigated by means of computer simulations in (Blekhman *et al.*, 2002; Blekhman and Fradkov, 2004) based on definitions introduced in (Blekhman *et al.*, 1997). It has been shown that using the proposed control algorithms enable the system not only to achieve both simple and multiple synchronization but also provide a way to ensure synchronization of a desired multiplicity.

Hou *et al.* (Hou *et al.*, 2016) studied the synchronization phenomenon for two co-rotating rotors, interacting via the vibrating body in the far-resonance vibration system by means of the *energy balance* method. Two rotors excited by induction motors installed in a vibrating body are considered. The synchronous zone, synchronous state, synchronous stability are determined in accordance to the energy balance method. It is observed that the dynamic characteristics of the vibrating body are related to the synchronous state of the system.

The problem of controlled passage through resonance zone for mechanical systems with several degrees of freedom based on the *speed-gradient* method (Andrievskii *et al.*, 1996) is analyzed in (Gorlatov *et al.*, 2015) by simulations of two-rotor vibration setup.

Due to the high complexity of vibrational mechanical systems and presence of the unmodeled dynamics and nonlinearities, the design and application of various experimental setups for scientific research, industry and education are very topical. Some experimental laboratory setups are described in the literature: pendulum-like oscillators (Fradkov *et al.*, 2014; La Hera *et al.*, 2009; Oud *et al.*, 2006), multiple DOF systems (Fradkov *et al.*, 2012; Mayr *et al.*, 2015), etc. The experimental and analytical results on a rotational-pendulum vibration absorber are presented in (Wu *et al.*, 2011). The characteristic frequencies of the absorber are tuned dynamically by the rotational speed adjusting in an open-loop manner. The device is coupled to the primary structure through a mechanical spring, thus possessing two natural modes of vibrations in the vertical plane. Experimental results confirm the theoretical statements and demonstrate the efficiency of the proposed scheme.

In (Panovko *et al.*, 2015) the testbed consisting of a rigid rectangular metal (platform) is installed horizontally on 14 identical springs, which are mounted on a fixed base. Two identical unbalanced vibro-actuators are also symmetrically and mutually parallel installed on the platform. Each vibro-actuator consists of asynchronous three-phase AC motor with a fixed disk, eccentrically mounted on the rotor. The motors are fed by the common frequency converter of three-phase current, providing their rotation in the opposite directions. The setup is supplied by three piezoelectric accelerometers, oriented in the vertical and horizontal directions for measuring the platform oscillations. The angular position of rotors and the shaft rotation speed are measured by the optical encoders. Additionally, the phase shift between the rotors may be visualized by means of the stroboscopic lighting. The authors described the results of experimental studies of oscillations in a classical single-mass oscillation system on a spring suspension in a wide range of excitation frequencies. It is shown that an effect of self-synchronization of unbalanced rotors arises when changes in the oscillation modes of the lifting body, depending on the excitation frequency.

The setup of (Panovko *et al.*, 2015) is a useful laboratory tool for studying an excitation of vibration by unbalanced rotors in the open-loop contour. Significantly wider range of the research may be fulfilled with the help of the *Multiresonance Mechatronic Laboratory Setup* (MMLS), developed in the IPME RAS jointly with the Mekhanobr Engineering JSC (Andrievsky *et al.*, 2016*a*; Andrievsky *et al.*, 2016*b*). The MMLS makes it possible to study vibrational motion of the platform with vibroactuatos both in open-loop and closed-loop modes. The complex may be used in the various fields of the research, education and industrial vibration machines design as follows :
– investigation of problems of dynamics of vibrating machines, such as vibratory maintenance of rotation of an unbalanced rotors, vibration braking of rotation, passing through resonance during start-up and rundown of the unbalanced actuator (the Sommerfeld effect), self-synchronization of vibrating actuators, sustainability and stabilization of natural unstable synchronous rotation, vibration isolation systems with natural and man-made disturbances, stabilization of the resonant modes controls;
– the impact of vibration on the solids, liquids, vibrorheological phenomena, the effects of vibrational displacement; vibrating processes, optimization of machine tool parameters for the given objective function (Blekhman, 1988; Blekhman *et al.*, 2008; Andrievsky *et al.*, 2016*a*; Tomchina *et al.*, 2011; Fradkov *et al.*, 2011; Tomchin *et al.*, 2015).

The focus of this paper is application of the MMLS for experimental examination of the self-synchronization and Sommerfeld effects both for the open-loop and the closed-loop control modes of the unbalanced vibroactuators, mounted on the spring-suspended platform.

The rest of the paper is organized as follows. The MMLS is briefly described in Section 2. The experimental results on unbalanced rotors synchronization are presented in Section 3. Concluding remarks are given in Section 4.

2 Setup Description

The *Multiresonance Mechatronic Laboratory Setup* (MMLS) has been developed on the basis of many years of experience on creating vibrating stands at the Mekhanobr Engineering JSC and the IPME RAS (Blekhman, 2000; Tomchin and Fradkov, 2007; Blekhman *et al.*, 2002; Tomchina *et al.*, 2011; Fradkov *et al.*, 2011; Tomchin *et al.*, 2015; Andrievsky *et al.*, 2016*a*).

The MMLS includes the vibrational stand, electrical engines, sensors, and personal computer (PC). All the devices constitute an integrated system, where the electrical and mechanical processes are inextricably linked

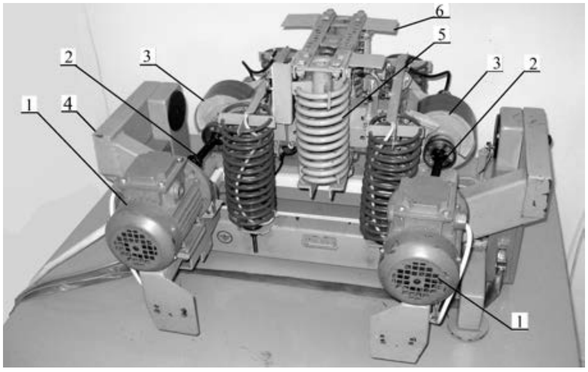

Figure 1. Photo of the two-rotor vibrational stand. 1 – AC induction motor, 2 – propeller shaft, 3 – unbalanced rotor, 4 – support frame, 6 – additional frame, 5 – springs.

Figure 2. Photo of the MMLS

each other, which gives a basis to call the setup a *mechatronic* one. The mechanical part of the MMLS is an electrically driven vibrational device, see Fig. 1.

The key part of the stand is a pair of the unbalanced (centrifugal) actuators. Each actuator includes three-phase AC induction motor 1) with computer-controlled rotation velocity, the propeller shaft 2) and the exciter as such 3) representing an unbalanced rotor, which rotates on the motor shaft in a vertical plane on the stand carrier body. Unbalance of the rotor is provided by the eccentrically located weight. The drive shafts and the anti-vibration screw springs repeatedly reduce the stand table vibration transmission to the support frame 4) and to the basis, where the frame is located. The frame 6) is mounted on the stand table on the springs 5) for installing an additional weight. The control circuit includes the *Schneider Electric* frequency converter *Altivar ATV12H018M2* for each motor. The inputs of the frequency converter are fed by the dimensionless digital control signals u_i from the range of $[0, 65000]$ ($i = \{l, r\}$ for, conventionally, "left" and "right" motors). Photo of the MMLS, including the vibrational stand, electrical engines and the personal control computer is depicted in Fig. 2.

The mathematical, methodical and software support, covering both issues of mathematical modeling of phenomena and effects, and their reproduction in the real-world conditions have been developed for efficient use of the MMLS in the research. The stand can operate in two vibroactuator modes: the *self-synchronization* mode and the *controlled synchronization* one. Using the vibroactuators self-synchronization phenomenon makes it possible to obtain various types of vibrations of the stand frame by changing the rotors turning direction.

However, the self-synchronization mode for some cases is not quite stable: random deviations of the motor parameters and settings, as well as fluctuations of technological load can cause large changes of the phase difference of the rotor turning from the values which ensure the desired mode of oscillation. In other cases, the required phasing of rotor rotation is unstable. In these situations the problem of motor control arises, where the measurement of the rotor angles is also needed, and an influence of the relative phase shift of rotation, i.e. operation in the controlled synchronization mode should be taken into account.

In more details, description of the MMLS may be found in (Andrievsky *et al.*, 2016*a*; Andrievsky *et al.*, 2016*b*).

3 Experiments on Unbalanced Rotors Synchronization

3.1 Open-loop control

Two different stepwise control voltages are applied to the motors, which should lead to different speeds of the "left" and "right" (conventionally) motors.

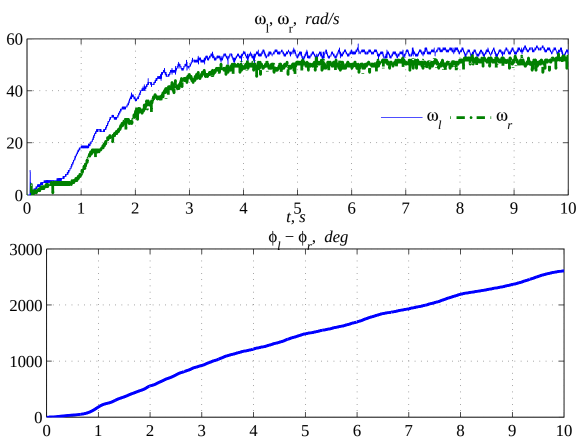

Figure 3. Open-loop control. Experiment #1. Time histories of ω_l, ω_r (upper plot) and $\Delta\phi$ (lower plot). No synchronization occurs.

Figure 3 demonstrates the time histories of rotation speeds ω_l, ω_r and discrepancy $\Delta\phi(t) = \phi_l(t) - \phi_r(t)$

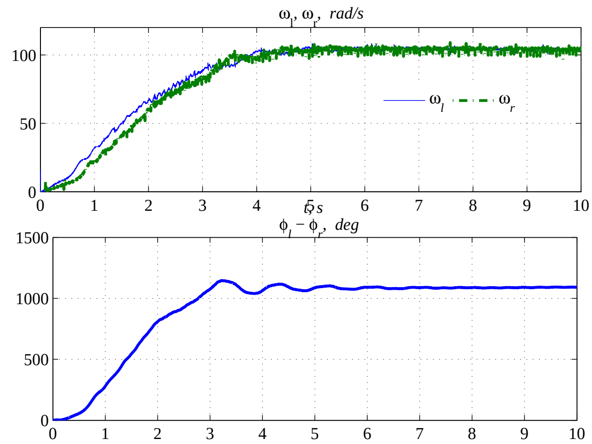

Figure 4. Open-loop control. Experiment #2. Time histories of ω_l, ω_r (upper plot) and $\Delta\phi$ (lower plot). Synchronization takes place.

between the rotor angles $\phi_l(t)$, $\phi_r(t)$ for the case when the voltages applied to the motors correspond to the desired rotation speeds as 65 and 60 rad/s for the "left" and the "right" motors, respectively (experiment #1). Since for the given velocities the system is far from the mechanical resonance, no synchronization occurs and the rotors speeds are close to the desirable ones. The experiment #2 (Fig. 4) is refereed to the case of the motor voltages corresponding to the rotation speeds as 115 and 110 rad/s. In this case, the system is close to the mechanical resonance and the self-synchronization appears. The rotor speeds are close each other, which makes an effect of synchronous rotation with asymptotically constant phase shift between the rotors, as is seen in Fig. 4, the lower plot.

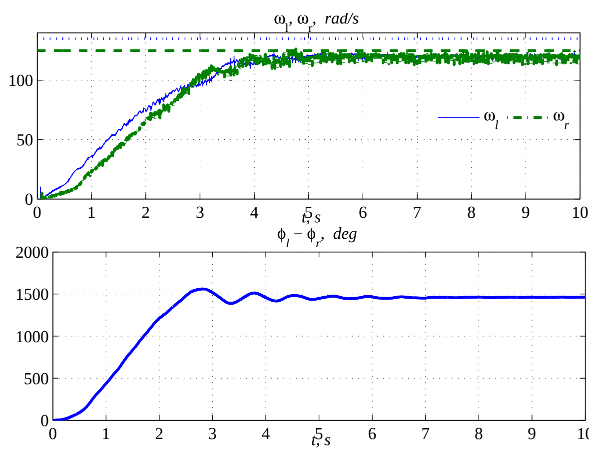

Figure 5. Open-loop control. Experiment #3. Time histories of ω_l, ω_r (upper plot) and $\Delta\phi$ (lower plot). Reference values: $\omega_l^* = 135$ rad/s, $\omega_r^* = 125$ rad/s. Steady-state errors: $\Delta\omega_l \approx 19$ rad/s, $\Delta\omega_r \approx 4$ rad/s. Synchronization and Sommerfeld effect.

For the experiment #3 the rotation speed reference values are taken as $\omega_l^* = 135$ rad/s, $\omega_r^* = 125$ rad/s. The self-synchronization and Sommerfeld effect are observed: the rotors have a tendency to synchronous rotation, and the actual rotation speeds are below the prescribed ones – the steady-state errors as $\Delta\omega_l \approx 19$ rad/s and $\Delta\omega_r \approx 4$ rad/s appear. The similar result is obtained in the experiment #4. The reference value of the right motor speed $\omega_r^* = 125$ is taken as 120 rad/s. This is a minimal speed of the right motor, for what the self-synchronization is observed. As above, the self-synchronization and the Sommerfeld effect take place, see Fig. 6.

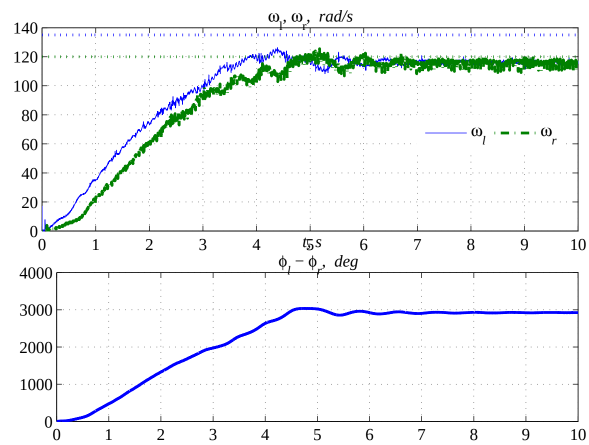

Figure 6. Open-loop control. Experiment #4. Time histories of ω_l, ω_r (upper plot) and $\Delta\phi$ (lower plot). Reference values: $\omega_l^* = 135$ rad/s, $\omega_r^* = 120$ rad/s. Steady-state errors: $\Delta\omega_l \approx -19$ rad/s, $\Delta\omega_r \approx -4$ rad/s. Synchronization and Sommerfeld effect.

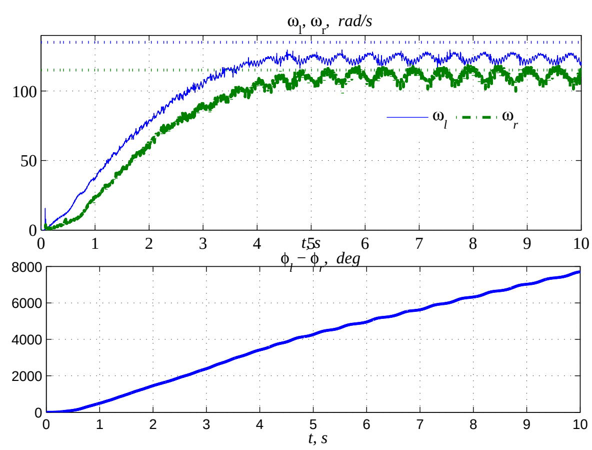

Figure 7. Open-loop control. Experiment #5. Time histories of ω_l, ω_r (upper plot) and $\Delta\phi$ (lower plot). Reference values: $\omega_l^* = 135$ rad/s, $\omega_r^* = 120$ rad/s. Averaged steady-state errors: $\Delta\omega_l \approx -12$ rad/s, $\Delta\omega_r \approx -5$ rad/s. No synchronization. Sommerfeld effect appears.

The experiment #5 is made for the reference values of the rotation speeds as $\omega_l^* = 135$ rad/s, $\omega_r^* = 115$ rad/s. For this case, the self-synchronization does not appear, but due to the mechanical resonance, the Sommerfeld effect take place: the averaged velocities of the rotors lie below the reference values, as is shown in Fig. 7.

3.2 Closed-loop control

The next series of the experiments has been fulfilled in the closed-loop control contour. The rotation speeds have been measured and used in the feedback of the controller for comparison with the reference (the desired) values. The error signals have been used to form the control actions, applied to the motors.

Let the following Proportional-Integral (PI) control be used in the angular velocity feedback loop of each ("left" and "right") motors:

$$\begin{aligned} &u_i(t) = k_P e_i(t) + k_I \sigma_i(t), \quad i = \{l, r\}, \\ &\dot{\sigma}_i(t) = e_i(t), \\ &e_i(t) = \omega_i^*(t) - \omega_i(t), \end{aligned} \tag{1}$$

where k_P, k_I are the proportional and integral controller gains (respectively), ω_i, i={l,r}, are the left and right rotors velocities, ω_i^* are the corresponding reference signals, e_i denote tracking errors, u_i, i={l,r}, denote dimensionless control actions applied to the motors. In what follows, controller gains $K_P = 200$ s/rad, $K_I = 150$ 1/rad are taken.

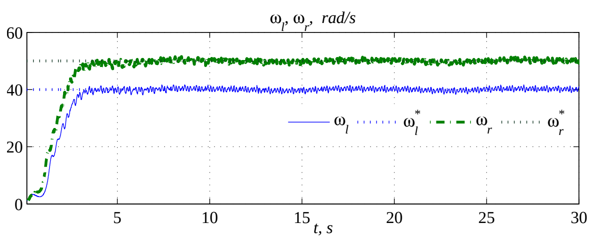

Figure 8. Closed-loop PI control (1). Experiment #6. Time histories of ω_l, ω_r. Reference values: $\omega_l^* = 40$ rad/s, $\omega_r^* = 50$ rad/s. Steady-state errors are about 1 rad/s in a magnitude.

In the experiment #6, reference values are taken as $\omega_l^* = 40$ rad/s, $\omega_r^* = 50$ rad/s. The control aim is achieved, the steady-state errors are about 1 rad/s in a magnitude. No self-synchronization and Sommerfeld effect arise, see Fig. 8.

For the experiment #7 reference values are taken as $\omega_l^* = 120$ rad/s, $\omega_r^* = 115$ rad/s. The rotor speeds and close each other and the system is near the mechanical resonance, see Fig. 9. Rotors self-synchronization appears. Both rotors have a medium speed of about 117 rad/s. PI controllers produce the control actions for bringing the motor speeds to the vicinities of reference values. The left motor control signal u_l increases,

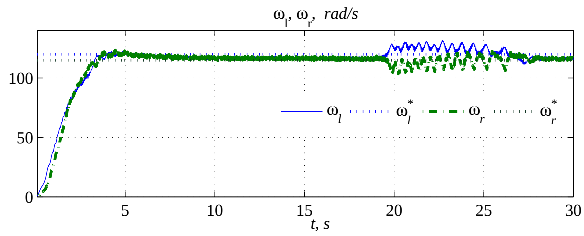

Figure 9. Closed-loop PI control (1). Experiment #7. Time histories of ω_l, ω_r. Reference values: $\omega_l^* = 120$ rad/s, $\omega_r^* = 115$ rad/s.

while the right one u_r decreases. Approximately after the interval of 20 s rotor synchronization is violated, the left rotor speed increases, the right one decreases. After about 7 s, due to changing the control signal, self-synchronization effect is observed again. One can suppose that after some time slot, the synchronization will be lost again. It should be noted that in the absence of self-synchronization, error of each rotor speed is approximately ± 10 rad/s.

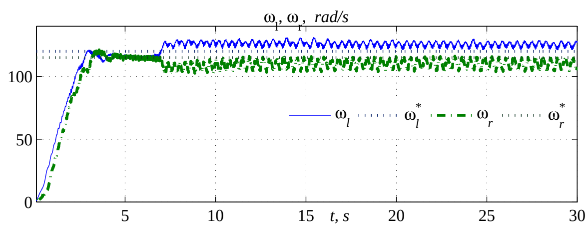

Figure 10. Closed-loop PI control (1). Experiment #8. Time histories of ω_l, ω_r. Reference values: $\omega_l^* = 125$ rad/s, $\omega_r^* = 110$ rad/s.

In the experiment #8 in the closed-loop contour, reference values ω_l^* ω_r^* are taken as 125 rad/s, 110 rad/s, respectively. The rotors self-synchronization is observed during a certain initial stage (about 7 s) of work. PI controllers lead to the synchronization loss and bring the rotor velocities to a certain vicinity of the reference values. Because of the strong mechanical interaction, the rotors speed "swinging" noticeably with respect to the reference values with the magnitude of about 6 rad/s.

4 Conclusions

In the paper the results of the experimental study of self-synchronization and Sommerfeld effects fulfilled on the Multiresonance Mechatronic Laboratory Setup are described. Both the open-loop and the closed-loop control modes are considered. The areas of appearance of the self-synchronizations and the Sommerfeld effect are found. It is shown that the closed-loop control makes it possible to stabilize the rotation speed more accurately, than it is possible in the case of the open-loop motors control. However, in the vicinity of the mechanical resonance, some additional effects, like low-frequency self-oscillations may arise due to the presence of the integral (I) component of the control action.

The article helps to better disclose the feedback effect on the resonance phenomena in physical systems based on the experiments with the vibrational mechatronic setup.

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References

Andrievskii, B.R., P.Yu. Guzenko and A.L. Fradkov (1996). Control of nonlinear vibrations of mechanical systems via the speed gradient method. *Autom. Rem. Control* **57**(4), 456–467.

Andrievsky, B., I. I. Blekhman, L. I. Blekhman, V. I. Boikov, V. B. Vasil'kov and A. L. Fradkov (2016*a*). Education-research mechatronic setup for studying vibrational devises and processes. *J. Mach. Manuf. Reliab.* (to appear).

Andrievsky, B., V. I. Boikov, A. L. Fradkov and R. E. Seifullaev (2016*b*). Mechatronic laboratory setup for study of controlled nonlinear vibrations. In: *Proc. 6th IFAC Int. Workshop on Periodic Control Systems (PSYCO 2016)*. IFAC. Eindhoven, The Netherlands.

Blekhman, I. I. (1988). *Synchronization in Science and Technology*. ASME Press. New York.

Blekhman, I. I. (2000). *Vibrational Mechanics*. World Scientific. Singapore. (Translated form original Russian text published in 1994).

Blekhman, I. I. (2016). Synchronization in nature and technology: Theory and applications. p. 350. (Translated from Russian ed., Nauka, Moscow, 1977).

Blekhman, I. I., A. L. Fradkov, H. Nijmeijer and A. Y. Pogromsky (1997). On self-synchronization and controlled synchronization of dynamical systems. *Systems & Control Letters* **31**(5), 299–306.

Blekhman, I. I., A. L. Fradkov, O. P. Tomchina and D. E. Bogdanov (2002). Self-synchronization and controlled synchronization: General definition and example design. *Mathematics and Computers in Simulation* **58**(4–6), 367–384.

Blekhman, I. I., P. S. Landa and M. G. Rosenblum (1995). Synchronization and chaotization in interacting dynamical systems. *Applied Mechanics Reviews* **48**(11), 733–752.

Blekhman, I.I. and A.L. Fradkov (2004). On general definitions of synchronization. In: *Selected Topics in Vibrational Mechanics* (I.I. Blekhman, Ed.). pp. 179–188. World Scientific. Singapore.

Blekhman, I.I., D.A. Indeitsev and A.L. Fradkov (2008). Slow motions in systems with inertial excitation. *J. Mach. Manuf. Reliab.* **37**(1), 21–27.

Cvetićanin, L. (2010). Dynamics of the non-ideal mechanical systems: A review. *J. Serbian Society for Computational Mechanics* **4**(2), 75–86.

Cvetićanin, L. and M. Zukovic (2015). Non-ideal mechanical system with an oscillator with rational nonlinearity. *J. Vibration and Control* **21**(11), 2149–2164.

Dimentberg, M. F., L. McGovern, R. L. Norton, J. Chapdelaine and R. Harrison (1997). Dynamics of an unbalanced shaft interacting with a limited power supply. *Nonlinear Dynamics* **13**, 171–187.

Fradkov, A. L., B. Andrievsky and K. B. Boykov (2012). Multipendulum mechatronic setup: Design and experiments. *Mechatronics* **22**(1), 76 – 82.

Fradkov, A. L., B. Andrievsky and M. S. Ananyevskiy (2014). State estimation and synchronization of pendula systems over digital communication channels. *Eur. Phys. J. Special Topics* **223**, 773–793.

Fradkov, A.L, O.P. Tomchina and D.A. Tomchin (2011). Controlled passage through resonance in mechanical systems. *J. Sound and Vibration* **330**(6), 1065–1073.

Gorlatov, D. V., D. A. Tomchin and O. P. Tomchina (2015). Controlled passage through resonance for two-rotor vibration unit: Influence of drive dynamics. In: *Proc. 1st IFAC Conference on Modelling, Identification and Control of Nonlinear Systems (MICNON 2015), IFAC-PapersOnLine*. Vol. 48 (11). pp. 313–318. IFAC. Saint Petersburg, Russia.

Hou, Y., P. Fang, Y. Nan and M. Du (2016). Synchronization investigation of vibration system of two co-rotating rotor with energy balance method. *Advances in Mechanical Engineering* **8**(1), 1–19.

Kovriguine, D. A. (2012). Synchronization and Sommerfeld effect as typical resonant patterns. *Arch. Appl. Mech.* **82**, 591–604.

La Hera, P. X., L. B. Freidovich, A. S. Shiriaev and U. Mettin (2009). New approach for swinging up the Furuta pendulum: Theory and experiments. *Mechatronics* **19**(8), 1240–1250.

Leonov, G.A. (2008). The passage through resonance of synchronous electric motors mounted on an elastic base. *J. Applied Mathematics and Mechanics* **72**, 631–637.

Leonov, G.A., D.V. Ponomarenko and V.B. Smirnova (1996). *Frequency-domain Methods for Nonlinear Analysis. Theory and applications*. World Scientific. Singapore, New Jersey.

Mayr, J., F. Spanlang and H. Gattringer (2015). Mechatronic design of a self-balancing three-dimensional inertia wheel pendulum. *Mechatronics* **30**, 1–10.

Oud, W. T., H. Nijmeijer and A. Yu. Pogromsky (2006). A study of Huijgens' synchronization: Experimental results. In: *Group Coordination and Cooperative Control; Lecture Notes in Control and Information Sciences* (K. Pettersen, J. Gravdahl and H. Nijmeijer, Eds.). Vol. 336. pp. 191–203. Springer. Berlin.

Panovko, G. Ya., A. E. Shokhin and S. A. Eremeikin (2015). Experimental analysis of the oscillations of a mechanical system with self-synchronized inertial vibration exciters. *J. Mach. Manuf. Reliab.* **44**(6), 492–496. (translated from original Russian text published in *Problemy Mashinostroeniya i Nadezhnosti Mashin*, 2015, No. 6, pp. 11–15.).

Quinn, D., R. Rand and J. Bridge (1995). The dynamics of resonant capture. *Nonlinear Dynamics* **8**(1), 1–20.

Sommerfeld, A. (1902). Beitrage zum dynamischen ausbau der festigkeislehre. *Z. Ver. Deut. Ing.* **46**, 391–394. (in German).

Tomchin, D. A. and A. L. Fradkov (2007). Control of passage through a resonance area during the start of a two-rotor vibration machine. *J. Mach. Manuf. Reliab.* **36**(4), 380–385.

Tomchin, D.A., O.P. Tomchina and A.L. Fradkov (2015). Controlled passage through resonance for flexible vibration units. *Mathematical Problems in Engineering* **2015**(Article ID 839105), 8.

Tomchina, O., V. Galitskaya and A. Fradkov (2011). Algorithm of multiple synchronization for multi-rotor vibration unit. In: *Proc. 7th European Nonlinear Dynamics Conference (ENOC 2011)*. Rome, Italy.

Wu, S.-T., Y.-R. Chen and S.-S. Wang (2011). Two-degree-of-freedom rotational-pendulum vibration absorbers. *J. Sound and Vibration* **330**, 1052–1064.