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# ON-LINE FILTERING OF SUNLIGHT CAUSTIC WAVES IN UNDERWATER SCENES IN MOTION

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**Abstract**

In this work we address the problem of removing sunlight caustic waves in underwater imagery obtained by a monocular camera in motion in shallow water. A heuristics for filter design is developed, from which essential properties of the wavy water surface are exploited. The method employs optical flow techniques and curvature predictions of pixel traces during the motion. The result is a practically clean image which is much more coherent with the optical flow corresponding to the scene motion. Design parameters are presented and guidelines to appropriately tune them are given. The performance of the algorithm is demonstrated in many case-studies involving synthetic video sequences as well as film material of real subaquatic scenarios.

**Key words**

No more than six key words must be given.

## 1 Introduction

Brightness fluctuations of a spatio-temporal scene radiance exists in nature, particularly underwater. Essentially, they are produced by sunlight flickers (caustic noise) and scattering on the sea-bottom structure in an uncontrolled and rather random manner. Patterns of sunlight flickers depends on the surface waves dynamics, depth and wind speed. Typically, the shallow water imagery, is affected by the presence of these dynamic patterns that strongly perturbed the scene [?] [?]. Besides, in computer vision domain, one finds a broad oceanic engineering applications involving vision-based system like 3D shape reconstruction [?], [?] and 3D trajectory recovery of autonomous navigation and SLAM (simultaneous localization and mapping) [?], [?], among a wide list of other research areas. These particular applications have motivated the present paper.

Submerged objects on a dominantly textured sea-bottom are illuminated by a natural random pattern [17, 32] which is spatially and temporally varying.

When estimating camera motion with respect to the bottom structure, one searches for the traces of any physical point of the bottom by tracking its corresponding pixel from frame to frame [?]. Perturbed points, mainly due to sunlight flicker, cause inconsistency on motion analysis. As the path of each object point can be tracked in the temporal sequence, it seems practical to employ temporal filtering to achieve a de-flickered video. The first realistic de-flicker algorithm was developed by Roosmalen in [?]. Generally, almost all of the correspondence-based lighting techniques avoid the difficult problem of tracking each individual moving scene points [?], [?], [?]. On the contrary, [?] focus on speeding up active vision techniques for fast scenes. Employing special hardware, Digital Light Processing (DLP) technology, the approach relies on controlling illumination to modulate the light much faster than the scene motion. The method simultaneously yields camera pose estimation and dense $3D$ scene structure. Underwater, this algorithm is also able to counteract of scattering effects. However, due to the active nature of the sensor, high power requirements are needed in contrast to passive sensors.

The removal of spatio-temporal noise has been studied, for example in [?], mainly to remove brightness error and fluctuation found in old film footage. The method employs motion estimation together with various other techniques in order to restore the footage. As reported, the footage must not be degraded in a manner that motion cannot be calculated with some degree of precision.

Caustic wave filtering in underwater imagery was studied in [?] [?].In [?] the method proposed consists on using the spatial derivatives of the video frames, instead of the raw frames. The idea behind this implementation is that the derivatives of the brightness changes will have sharp edges. This, in combination of temporal averaging (median), would yield better results. This method requires a quasi-static scene, because the median averaging in time. Besides, the approach assumes that the brightness change found in the scene will have sharp edges. Here, it can be noticed that some caustic waves can be found to have smooth spatial derivatives.

In [**?**] the method developed employs a simple motion estimation. It is based on computing the image difference between a given reference frame and the temporal median of a registered set of neighbouring images. A key observation is that this difference will have two components with separable spectral content. One is related to the illumination field (lower spatial frequencies) and the other to the registration error (higher frequencies). The illumination field, recovered by low-pass filtering, is used to correct the reference image. It is claimed preservation of the image sharpness, even in the presence of registration inaccuracies.

Other works like [**?**] [**?**] use a stereo camera set-up. The authors recognize that the caustic wave actually can be thought as a projected pattern in the sea bottom. The stochastic nature of the caustic wave provides sufficient pixel surrounding differentiability, so that the stereo camera set-up can recognize very accurately the same point in the scene. Then using Structure From Motion (SFM) methods [**?**], the sea bottom can be reconstructed and latter the caustic wave can be filtered. This, of course, requires the use of dual camera. Monocular camera implementations are desired in many cases because of its low cost and widely availability.

In this work we address the problem of removing sunlight caustic waves in shallow water images obtained by a monocular camera in motion. A heuristics for filter design is developed, which is based on essential properties of the wavy water surface. The method employs optical flow techniques and curvature predictions of pixel traces during the motion. The result is a clean image with a much more coherent optical flow. Design parameters are presented and guidelines to appropriately tune them are given. The performance of the algorithm is demonstrated in many case studies involving synthetic video sequences as well as film material of real subaquatic scenarios.

## 2 Working Hypothesis

The main hypothesis that is argued in order to outline a strategy for removing sunlight flickers from subaquatic scene imagery, relies on some physical observations of the wave formation mechanism. Additionally the phenomena of reflection, refraction and diffraction of sun rays crossing over the wavy boundary layer that account finally the light flicker in shallow water.

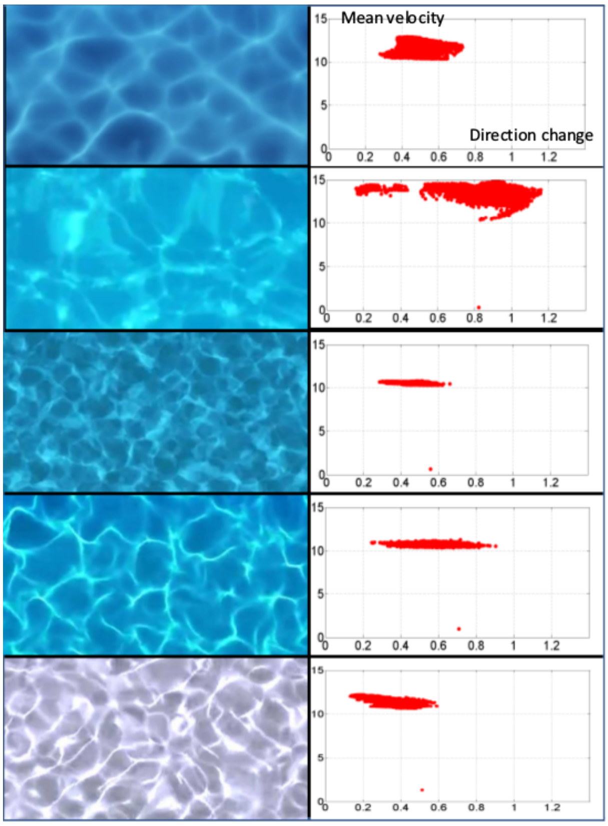


Figure 1. Illustration of dynamics characteristics of different sunlight flicker

The fluctuations of the sea surface depends on many factors, mainly the coastal and oceanic landforms, water depth, and wind and flow conditions. Waves have a certain amount of randomness. They can be described as a stochastic process in combination with the physics governing their generation. The spectrum of ocean waves which might be related to sunlight flickers is roughly limited to the band of 0.1 to 10 Hz., i.e., comprehending ordinary to ultra gravity waves.

Certainly, flickers can be seen as deformable areas of well illuminated points that shrugs or grows (or even vanishes) randomly in time. The kinematics of a randomly moving flicker system on the sea-bottom can be explained by an optical flow field of associated pixels that moves randomly as well. We will conceived this moving flicker system as points with random displacements at different velocities.

In Fig.**??**, some simulation results employing synthetic videos are illustrated. Herein, different sunlight flicker patterns over an uniform background are presented on the left column. Each pattern contains a particular spectrum with different frequency band. Thus, the spectra contribute meaningfully to describe the flicker dynamics from a statistical point of view. In this respect, it is convenient to plot the variables $v(x, y)$ vs. $\overline{\Delta\psi^2}(x, y)$, where $v$ is the mean value in $L_1$-norm of the module of the optical flow at a fixed pixel $(x, y)$ during the whole video. Equivalently, $\overline{\Delta\psi^2}$ is the mean value in $L_1$-norm of the direction change

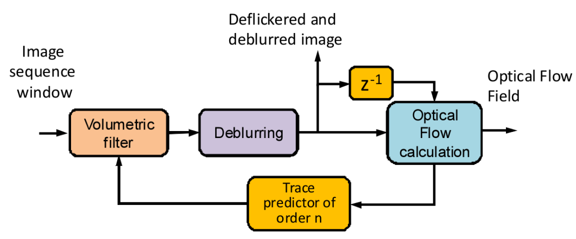


Figure 2. Block diagram of the de-flickering filter

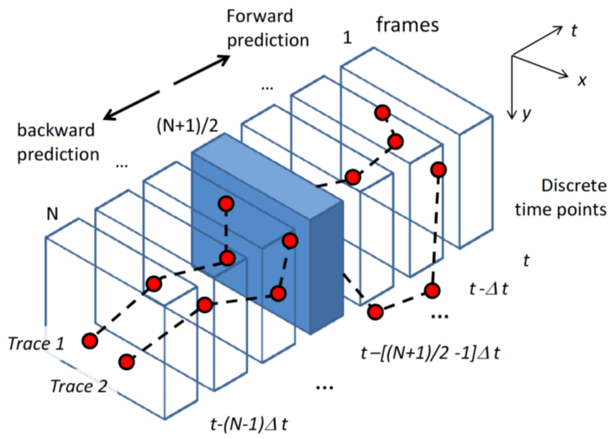


Figure 3. Sliding window with the last $N$ frames and forward-backward predictions of the trace of a physical point

of the flow at $(x, y)$ during the video. Indeed $\overline{\Delta\psi^2} = \Delta[\Delta(\psi/\Delta t)]\Delta t$, i.e., the second derivative of the flow passing through the pixel $(x, y)$ at time $t$. The plots are depicted on the right column of the chart.

In the second column of the figure, one can see the areas formed by the states $v$ and $\overline{\Delta\psi^2}$ for all pixels $(x, y)$ of the frame. The areas become dominantly more widespread in the direction of $\overline{\Delta\psi^2}$ . This characterizes the strong change of direction of the optical flow, which evolves randomly in time. The wider the frequency band of the pattern spectrum the wider the area will be. The second pattern from above is the most changing in direction like a "random walk" stochastic process.

There exists however some patterns of caustic waves that have a marked directional component of velocity which is produced by the wind at the water surface. Accordingly this case is described by an area which is elongated vertically and close to the $v$ axis. This pattern dynamics will be analysed later as a case study.

When the scene imagery stems from a moving camera, two process take place in a superimposed way, namely the moving landscape scene and the moving sunlight flickers. Discarding high frequency vibrations in the camera (e.g.,shakes or sudden turns), the moving image of the first process can be described by an optical flow field that is much more congruent and smooth than corresponding one of the second process.

In summary, the useful characteristic of the complete optical flow field is not just the diversity of both the rate vector modules or directions, but rather the marked changes of their directions. In other words, an optical flow field sequence that shows paths of rapid change of direction, these should correspond to trails of undesired flicker. The remainder of the paper describes the way to exploit this flicker dynamics characteristic as design criteria in order to remove or efficiently attenuate them from the video.

## 3 Algorithm Description

The most basic method of filtering image noise in time is to calculate a time-average of each pixel in the image. More advanced methods are introduced in [**?**], in which instead of using time average, the median in the derivatives is employed yielding better results. A mayor drawback of this kind of methods is that the image sequence must depict a static scene subject to random (in time) noise. If the scene is not static (e.g., as in vision-based navigation systems), the time average of each pixel might produced distorted results, because the pixel in the image now no longer correspond with the same physical point in the scene.

An alternative could be the methods presented in [**?**], [**?**], in which an image sequence, distorted by a low power random noise, is filtered (for instance, employing average) in the dominant direction of the optical flow in an open-loop manner, i.e., without feed-backing the result in order to refine it. Therein, the optical flow is conceived a vector with 3 dimensions, i.e., the motion on the $x - y$ frame and the time direction $t$ that traces the rate of a pixel on its geometric trajectory. Thus, employing this $3D$ vector in a non-static scene, it is possible to know which pixels in the image correspond to each physical point in the scene. In this way, time averaging yields no longer distorted results. An inconvenience of this method is that the noise in the image must be random (in the optical flow direction) and must have relatively low power, enough for achieving accuracy in the optical flow calculation.

In the present work, a method for filtering an image sequence of a highly disturbed, non-static scene is presented. The main difference with respect to the filtering methods referenced so far, is the construction of feedback block containing previous information of the optical flow field in order to refine the result in the next step. This innovation include some design parameters to enhance the stability and robustness in the filter performance.

A block diagram of the algorithm is presented in Fig.**??**.

The block termed volumetric filter can roughly be thought as a volume that contains the traces of almost all the physical points which are tracked in a number of concatenated frames of the image sequence. The structure of the volumetric filter is depicted in Fig.**??**. It is composed of $N$ consecutive frames of the image sequence. They are embraced by a sliding window along the discrete time axis $t$. For the sake of simplicity we chose an odd value for $N$. The first frame in the window corresponds to the actual time point $t$. The frame in the middle is the most important one, because it contains all the pixel $(x, y)$, which will be pivot points for a series expansion.

The second block in order of importance is a trace predictor that helps to estimate the curvature of estimated traces. The block containing an optical flow estimator allows the de-flickering algorithm to establish correspondence between pixels that are concatenated by the trace. In this work the algorithm of Farnebäck is employed for optical flow estimation [**?**]. Finally, a block for de-blurring is included with the purpose of achieving better results, however this inclusion is not determinant for the quality of the results presented in this work.

It is worth noticing in the structure of the Fig.**??** contains a feedback loop of previous information. In this paper no discussions are carried out about the stability of the algorithm. However in a practical sense, the question of stability can be reformulated as a virtuous or vicious cycle problem in the filtering process. Certainly, in the light of the posed rational heuristics and the successful results obtained in this work, such an analysis falls outside of the ends of this paper.

The next description goes into the details of the algorithm working. Fig.**??** will help the description.

The de-flickering algorithm starts with the construction of traces, it is the determination of $N$ pixels $(x, y)$, one pixel for every frame in the window, such that all of them thread a most likely trace of a physical point in the sequence (see the representation of the *Trace 1* in Fig.**??**). The concatenation criterion begins with a pixel $(x, y)$ in the middle frame and predicts $(N-1)/2$ pixels forwardly and $(N-1)/2$ pixels backwardly.

The trace predictor performs the pixel concatenation employing a series expansion, more precisely a Taylor series of order n for pivoting points $(x, y)$ of the middle frame. For evaluating derivatives, it is, ratios of differences up to order n, optical flow fields are useful. These fields will be feed-backed, namely the optical flows at $t, (t-1), ..., (t-N-1)$.

Thus, the frame for $(N+1)/2$, $t_0 = (N+1)/2-1)$, $n = 3$ and the pixel $(x(t_0), y(t_0))$ , the pixel located at one frame backwards, which is concatenated is approximately given by the series expansion:

$$
\begin{aligned}
&x(t_0 - \Delta t) \approx x(t_0) + \frac{1}{2}(\Delta x(t_0 - \Delta t))(-\Delta t) + \\
&\frac{1}{6}(\Delta x(t_0 - 2\Delta t) - (\Delta x(t_0 - \Delta t))(-\Delta t^2) + \\
&\frac{1}{24}((\Delta x(t_0 - 3\Delta t) - (\Delta x(t_0 - 2\Delta t)) - \\
&(\Delta x(t_0 - 2\Delta t) - (\Delta x(t_0 - \Delta t)))(-\Delta t^3) \quad (1)
\end{aligned}
$$

and similarly for $y(t_0 - \Delta t)$. The same is carried out for more concatenated points located backwards, namely $(x(t_0-2\Delta t), y(t_0-2\Delta t))$, ..., $(x(t_0)-((N+1)/21)\Delta t), y(t_0 - ((N+1)/2-1)\Delta t))$. It is noticing that the optical flow fields provide all incremental changes $\Delta(\Delta(...(\Delta x))...)$ and $\Delta(\Delta(...(\Delta y))...)$.

The forward predictions are equally calculated with the same series expansion for times $(t_0 + \Delta t)...(t_0 + ((N+1)/2-1)\Delta t)$.

This concatenation of the $N$ points is calculated exhaustively, embracing all pixels $(x, y)$ of the middle frame.

The final result is a set of traces in the frame volume.

At this stage the volumetric filter will attempt to retrieve the information of the image in the middle without caustic. To this goal, the brightness of each trace will be averaged employing the median value of the trace pixels intensities. The employment of median in the case of a finite $N$ has certain advantage over the mean average because it converges faster than the mean value [**?**]. This median value of the brightness is assigned to the middle frame intensity, leading to a reconstructed image as output. This cleaned image is the basis to calculated the optical flow field at time $t_0$. We have mentioned a step more after the image reconstruction, this is a de-blurring process in order to define a more accentuated sharpness of the frame contours. Nevertheless, this step is not performed in the case studies we will present below.

Clearly, by processing the information in this way, the algorithm will produce a delay of $N/2$ frames. Taking into account the actual the state-of-art in camera technology, for the typical frame rates close to 30 frames per second, this delay is small and will most likely not affect seriously decisions in the most real-time applications of vision-based system, such as for instance in robotics, as a part of the control decisions carried out in a visual servoing system.

## 4 Solution for illogical traces

The calculation of traces by the presented prediction approach do not guarantee that some predicted trace pixels fall outside the volume (see the *Trace 2* in Fig.**??**). Thus, the brightness averaging will fail because there are no brightness value for these non-existent trace points.

These inconsistencies are edge effects and are typical by predictions of high order.

A simple solution to avoid these illogical traces consists in replacing the brightness value for points outside the volume by zero and afterwards in employing the median instead of the mean value. This type of averaging is more realistic to counteract these edges effects.

## 5 Initial conditions

The problem of the initial condition by camera in motion creates an undesired transient. The problem can be partially sorted out employing the following procedure.

Rather than assume a null optical flow at the beginning, one can employed the first optical flow field using the first two dirty images for as long as the first clean optical flow field emerges. Thereafter, the de-flickering algorithm will start working normally.

Thus, the transient behaviour at the beginning of the filtering algorithm can be reduced significantly.

## 6 Assumptions

The efficiency of the algorithm applied to a given subaquatic scenario will depend on three necessary conditions, namely:

I) The scene is globally illuminated and the flickers create dynamic illumination changes over the scene

II) The temporal frame density is high enough in order to capture smooth motions of the scene

III) The caustic waves have certain randomness

Condition I points out the existence of other light sources aside from the refracted sunlight. This may be due to the light backscattering in the water column causes a smooth global illumination. Besides, external illumination and/or multiple reflection of textured objects on the scene contributes to a global illumination. In this way one assumes that there will be a partially visible area in complement with the remaining over illuminated area that compose the scene. Condition I is fulfilled the most of the time.

Condition II is necessary for retrieving the scene motion with approximately the same sharpness as in the original video. Sudden movements of the scene may cause blurred images since the filtering process employs temporal averaging for remove the noise.

Condition III is important to accentuate the reliability of the working hypothesis, under which the filtering objectives can be achieved. Certainly, the randomness is a characteristic of the wavy water surface and not of the scene motion which is more predictable.

## 7 Design parameter

There are two main design parameters to be tuned in order for the algorithm to perform satisfactory and be robust, namely the window size $N$ on the one hand and the order $n$ of the interpolator on the other hand.

additionally, there are proper parameters of the optical flow method particular employed with our methods. These were chosen conforming to the suggested default values.

The first setting for $N$ is related to the frequency band width of the spectrum of the caustic wave process. The parameter $N$ is should be set according to an inverse relation with respect to the band width of the flicker dynamics.

In any case, for a smooth and dominantly slow motion of the scene with respect to the flicker dynamics, a suitable value is $N = 9$. For rapid movements of the scene with a high camera frame rate, $N = 3$ is adequate.

## 8 Case studies

With the goal to illustrate the performance of the method, four case studies are investigated. They could classify into different subaquatic scenarios with a perturbed moving scene.

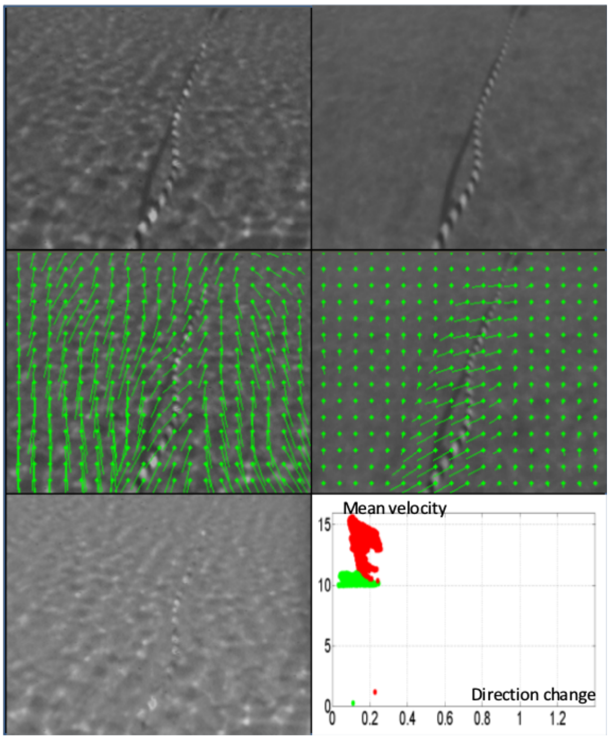


Figure 4. Pipe tracking

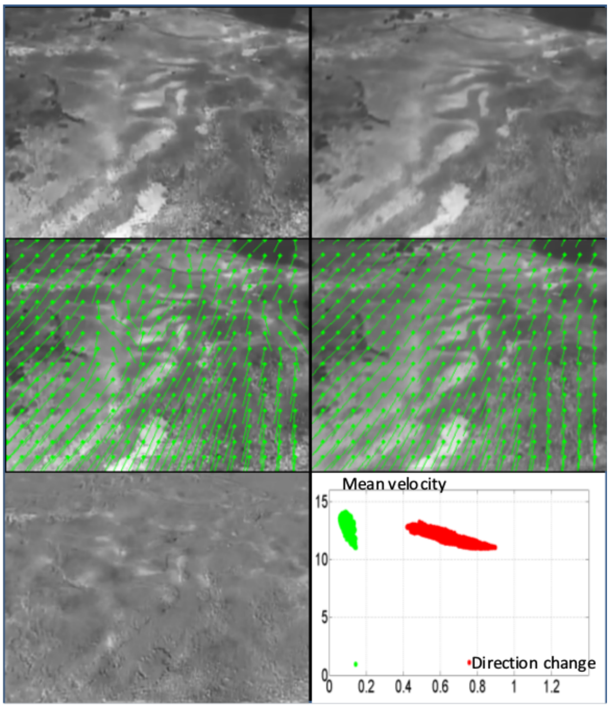


Figure 5. Navigation over ruins underwater

The specific videos contains the navigation over: a) a pipe with uniformly distributed patterns laid on a flat sea bottom, b) ruins of a construction underwater, c) a sequence of an irregular terrain underwater.

In all scenarios, the figures are arranged as follows.

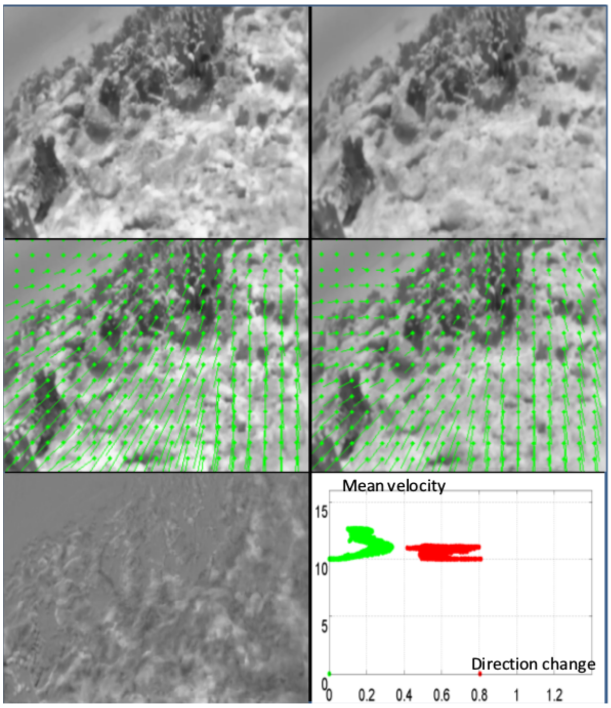


Figure 6. Navigation over ruins underwater

On the first row, a dirty image and the filtered one are depicted side by side. Down below, their optical flow fields. At the foot of the figure on the left, the reconstructed sunlight flickers, which were isolated in an own frame by subtracting the clean image from the dirty one.

On the right of the row, the plot at each coordinate $(x, y)$ of the mean velocity $v(x, y)$ versus the change of direction 2 (x, y) for both the dirty image and the isolated flicker image.

The parameters are set according the guidelines mentioned above. For all runs presented in the paper, following setting was employed: $N = 9$ and $n = 3$. Also for the Gaussian filter employed in a preprocessing step in the optical flow technique [**?**] one selects $\sigma = 2$. The video size was 320x240 pixels.

Now we are able to analyse each case separately.

In fig.**??**, in the illustration for the video of a pipe on the almost flat sea-bed, one notices a practical complete removal of the flickers. This result manifested throughout the video, see [Link to videos]. Also, the posterior flicker reconstruction seems to correspond to the real caustic. Some evidence of this efficiency is given from the different optical flow fields, where that corresponding the clean image, contains the kinematics of the pipe only.

The plot of the states mean velocity versus direction changes shows differentiated areas but due to the different velocity ranges but similar range of direction changes. This case is a very particular one because the motion of the flickers have a constant velocity component with constant direction, even though this is a weak one. The filtering success is in this case effective because the scene motion is more dominant than the translation of the flickers.

Fig.**??** depicts a complex case of spatio-temporal radiance changes superimpose with white spots of the seabed. Even though the scene spots is in motion the filter is able to recognize the flickers and the spots. This is seen in the state plot, where the areas for the scene and flickers are markedly differentiated, if not in the velocity range, though they are quite different in the direction changes. The results can also be appreciated in [**?**]. One notices on the optical flow field of the perturbed image, marked changes of velocity vectors. They are the evidence of the caustic waves presence, that are completely remove by the algorithm as it can be observed on the right in the corresponding optical flow field.

The results of case c) are illustrated in Fig.**??**. The difference between flicker kinematics and scene motion is appreciated in the last plot. Despite the irregularities in the terrain, the illumination changes are attenuated and the scene is restored preserving the image quality. The results can also be appreciated in [**?**].

## 9 Conclusions

A real-time approach for de-flickering videos is presented, in where the scene was perturbed by sunlight caustic wave. Besides, the scene is assumed to be in smooth motion. The approach requires a monocular simple camera. The strategy for filter design was based on useful essential characteristics of the caustic waves that are not directly related to their typical frequency spectra, but rather to the property of diverse and rapid changes of direction of flicker traced pixels, like in a ”random walk ” stochastic process. The method is based on a volumetric filter and a predictor which work together with a block that calculates optical flow fields, which are feed-backed to the volumetric filter. The first result is an almost clean image without flickers and the second one is a much more coherent optical flow field in relation to the pure scene motion.

The coherence of all pixel assignations corresponding to all possible traces of physical points is ensured by a high order predictor inside a sliding window of the volumetric filter. Among all possible traces identified, there exists a small set which is inconsistent with real paths. These edge effects are particularly dealt with in differentiated manner. Thus possible traces contributes to an averaged brightness on the reconstructed image, and inconsistent traces contributes with a smaller brightness average value.

Also guidelines to tune adequately the design parameters are give.

We tested the proposed approach on video sequences with artificially added flicker, as well as on natural real flicker film material. Test results obtained showed that the proposed method produced a very good flicker correction in smooth moving scenes comparable with the movements related to underwater vehicles motions, on which a camera is pinned.

It is worth noticing that even with the employment of large windows sizes in the volumetric filter, the image sequence is reconstructed without mayor distortion. This is not just only due to the smoothness of the camera movements, but also to the accurate reconstruction of the pixel traces provided by the high-order series expansion employed in the predictor.

Some drops of performance occurs by camera shakes or rapid displacements in relation to the camera frame rate. This undesired effects are subject of the present research that include on-line adaptation of the window size for different dynamics of the scene motion and the employment of de-blurring techniques.

Even when the motivation and adaptation of the algorithm was thought to applications in 3D trajectory recovery of autonomous navigation and SLAM (simultaneous localization and mapping), we believe that it can be re-engineered for other areas of the Computer Vision.

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