SUPPRESSION OF BURSTING SYNCHRONIZATION IN A SCALE-FREE NETWORK

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Abstract
Bursting activity in neuron ensembles can be synchronized, meaning the adjustment of the bursting phases due to coupling. We used a coupling prescription according to a clustered scale-free. In this work, we analysed the possibility to suppress bursting synchronization through applied light pulses.

Key words
neuron, synchronization, suppression, scale-free, chaos

1 Introduction

Synchronized rhythms have been observed for many years in electroencephalograph recordings of electrical brain activity, and are thought to be an important mechanism for neural information processing [Salinas, 2001]. Such synchronized rhythms reflect the hierarchical organization of the connectome since they occur over a wide range of both spatial and temporal scales [Stam, 2004]. Synchronization in complex clustered networks has been investigated [Huang, 2006]. A clustered network is characterized by sparsely linked subnetworks, in other words, clusters. Then, each cluster with its own internal connections, where the clusters may present exactly the same or different topology. As a matter of fact it is of interest for a clustered network the occurrence of global synchronization, in that nodes from different networks are synchronized [Guan, 2008].

Since bursting occur in a longer period than spiking, bursting neurons have to be modelled by systems with a minimal number of variables. Using discrete time systems (maps) for the sake of computational speed one can model bursting neurons with two-dimensional maps, in which one variable stands for the rapidly spiking action potential, whereas the other acts as a modulating factor and introduces the slower bursting timescale [Ibarz, 2011]. Bursting neurons can synchronize due to coupling and it is sometimes undesirable to have such synchronized rhythms, since they may be related to pathologies like essential tremor or Parkinsonian disease. Hence the suppression of synchronized bursting is of potential interest in the neurosurgery field of deep-brain stimulation [Rosenblum, 2004]. Strategies to suppress synchronized bursting have been proposed in the last years, as the introduction of a time-delayed feedback signal in spatially localized cortical areas (or neurons, in the level of the present model) [Batista, 2010]. The suppression of bursting synchronization may also be obtained through neuron control with light [Han, 2007]. Recent researches have showed the possibility of neuronal inhibition or stimulation with light pulses applied on a targeted neuron, which is adapted with microbial light-sensitive proteins [Zhang, 2007]. Such modified neuron are able to produce photosensitive proteins which releases ions when exposed to light, and it is through the injected current formed by these released ions that the neuronal dynamics can be controlled [Lameu, 2012].

This paper is organized as follows: in Section 2, we present the Rulkov neurons in a clustered scale-free neural network, Section 3 deals with the suppression of synchronized bursting rhythms. Our conclusions are left to the last Section.

2 Bursting synchronization in a clustered scale-free

There are a number of mathematical models which emulate neuronal activity, ranging from differential equations to discrete-time maps. We choose the Rulkov model [Rulkov, 2001]

$$\left.\begin{aligned} x_{n+1} &= \frac{\alpha}{1+x_n^2} + y_n \\ y_{n+1} &= y_n - \sigma x_n - \beta, \end{aligned}\right\} \qquad (1)$$

where n is the discrete time, x_n is the fast and y_n is the slow dynamical variable. Furthermore, α affects directly the spiking time-scale. Thus, its value is chosen to produce chaotic behaviour for the evolution of the fast variable x_n, characterized by an irregular sequence of spikes, and the parameters σ and β describe the slow time-scale.

We consider the neurons to be non-identical, that is, they possess a mismatch in their internal parameters, we choose α as a mismatch parameter. Complete synchronization in networks of non-identical neurons is not possible. However, weaker synchronization such as phase synchronization may take place [Pikovsky, 2001]. To study phase synchronization, we introduce a phase for the chaotic dynamics of the bursts. This turns out to be a non straightforward task. For chaotic oscillators the phase cannot be defined unambiguous. Different ways to introduce a phase are possible, each one being chosen according to the particular case [Pereira, 2007].

The burst begin when the slow variable y_n, which presents nearly regular saw-teeth oscillations, has a local maximum, in well-defined instants of time (n_k). The duration of the chaotic burst, $n_{k+1} - n_k$, depends on the variable x_n and fluctuates in an irregular fashion as long as x_n undergoes chaotic evolution. We can define a phase describing the time evolution within each burst, varying linearly from n_k to n_{k+1}

$$\phi_n = 2\pi k + 2\pi \frac{n - n_k}{n_{k+1} - n_k}, \; n_k \leq n < n_{k+1} \tag{2}$$

where n_k denotes the time occurrence of the kth burst. The bursts occur in a coherent manner, that is, the time interval between burst have a small deviation.

A diagnostic of bursting synchronization is provided by the order parameter

$$z_n = R_n \exp(\mathrm{i}\Phi_\mathrm{n}) \equiv \frac{1}{\mathrm{N}} \sum_{\mathrm{j=1}}^{\mathrm{N}} \exp(\mathrm{i}\phi_\mathrm{n}^{(\mathrm{j})}), \tag{3}$$

where R_n and Φ_n are the amplitude and angle, respectively, of a centroid phase vector for a one-dimensional network with periodic boundary conditions. If the bursting phases $\phi_n^{(j)}$ are spatially uncorrelated, their contribution to the result of the summation in Eq. (3) is small. However, in a globally phase synchronized state the order parameter magnitude asymptotes the unity.

The time averaged order parameter magnitude is given by

$$\bar{R} = \frac{1}{T} \sum_{n=1}^{T} R_n, \tag{4}$$

when the bursting dynamics is globally synchronized then $\overline{R} \approx 1$.

We generate clustered network composed of M subnetworks each presenting a scale-free property. Scale-free network has few nodes (hubs) connected with a large number of other ones, whereas most of the nodes are connected with only a few nodes. The subnetworks are connected through the hubs. A scale-free network presents the number of connection per node according to a power-law dependence.

The bursting dynamics is governed by

$$\left.\begin{aligned} x_{n+1}^{(i,l)} &= \tfrac{\alpha^{(i,l)}}{1+(x_n^{(i,l)})^2} + y_n^{(i,l)} + \mathcal{C}_n^{(i,l)} \\ y_{n+1}^{(i,l)} &= y_n^{(i,l)} - \sigma x_n^{(i,l)} - \beta, \end{aligned}\right\} \tag{5}$$

where $i = 1, 2, ..., N$, $l = 1, 2, ..., M$, $\sigma = \beta = 0.001$, and the values of $\alpha^{(i,l)}$ are uniformly distributed in the interval [4.1,4.4]. We considered subnetworks prescription with the coupling term of the form

$$\mathcal{C}_n^{(i,l)} = \frac{\varepsilon}{k^{(i,l)}} \sum_{j \in I} x_n^{(j,l)}, \tag{6}$$

where $j \neq i$, $\varepsilon > 0$ is the coupling strength and we assumed that each node i is coupled with a set I comprising $k^{(i,l)}$ other nodes inside the subnetwork, as well as the M subnetworks are globally connected by the hubs with coupling strength ε_h. We numerically analyse the synchronization using the order parameter and the dynamical modularity as a function of the coupling strength. Fig. (1a) presents regions for which the coupled subnetworks have (I) non synchronized behaviour, (II) each subnetwork synchronized (black lines) without global synchronization (red line) and (III) global synchronization. The dynamical modularity for a network with M subnetworks is

$$DM = \frac{\sum_\mu \bar{R}_{\mu\mu}/M}{\sum_{\mu,\mu\neq\xi} \bar{R}_{\mu\xi}/[M(M-1)]}, \tag{7}$$

where $\bar{R}_{\mu\xi}$ is obtained between subnetworks μ and ξ. The dynamical modularity has value close to unity when the network is synchronized. Fig. (1b) exhibits our results for the dynamical modularity. In the region I the onset of the synchronization, we can see that in the region II each subnetwork synchronizes but the coupled networks do not synchronize, and in the region III the coupled networks are synchronized. As a result, the transition to global synchronization occurs with the subnetworks synchronizing first.

3 Suppression of synchronization

We investigate the effect of an external perturbation. A perturbation may produce a global phase locking, and it has been observed in networks with global [Ivanchenko, 2004] and scale-free coupling [Batista, 2007]. We verified that neurons in a network with scale-free subnetworks are able to synchronize their bursting activities. Then, we insert a perturbation in

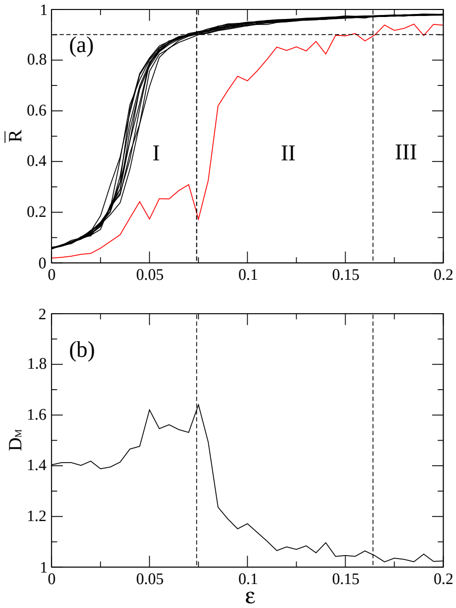

Figure 1. We consider $M = 10$, $N = 230$ and $\varepsilon_h = 0.16$. (a) Time averaged order parameter as a function of the coupling strength ε for each subnetwork (solid black lines) and for the network (red line), (b) dynamical modularity.

the fast variable of one hub to reduce or suppress the global synchronization. The perturbation consists of light pulses applied on a specific neuron hub. An alternating on-off pulse that during a time T_{on} the selected neuron is deactivated by a light pulse, followed by a time T_{off} during that no perturbation is applied.

Perturbation may induce non synchronization, for this reason we analyse the time intervals T_{off} and T_{on}. Fig. (2) shows the time averaged global parameter as a function of the fraction between the time intervals. In accordance with our results we can observe that the fraction represented by a square is a good value. With regard to non synchronization the Fig. (3) exhibits this possibility, since there is not a region III.

4 Conclusion

In conclusion, we presented a neuronal clustered subnetwork model in which the neuron dynamical is described by the Rulkov's map, furthermore the connective architecture presents scale-free property. We demonstrated the possibility of obtaining bursting global synchronization of Rulkov neurons in a clustered networks. Moreover, the network with subnetworks presenting scale-free property and connected by the hubs presents domains of synchronization, as well as we verified that there are unsynchronized nodes while the hubs are synchronized.

Photostimulation of neurons is a non invasive method, in fact it can be targeted to specific classes of neurons. We considered an external perturbation that simulate this behaviour. Moreover, we obtained non synchronization applying a perturbation in one hub.

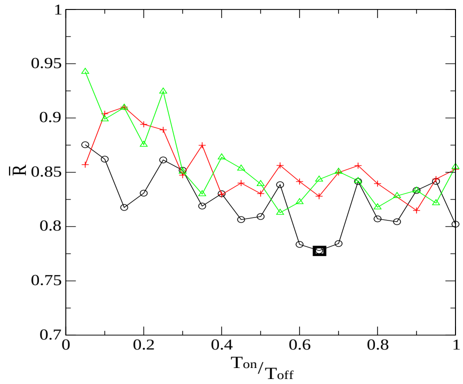

Figure 2. The time averaged global order parameter as a function of the fraction between the period with (T_{on}) and without (T_{off}) perturbation considering $M = 10$, $N = 230$, $\varepsilon_h = 0.16$ and $\varepsilon = 0.2$. We fix $T_{\text{off}} = 100$ iteractions (circles), $T_{\text{off}} = 500$ (crosses) and $T_{\text{off}} = 1000$ (triangles).

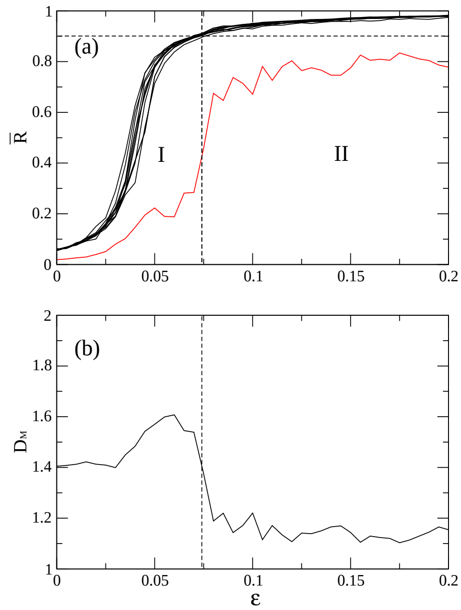

Figure 3. We consider $M = 10$, $N = 230$, $\varepsilon_h = 0.16$ and an external perturbation. (a) Time averaged order parameter versus coupling strength ε for subnetworks (black lines) and for the network (red line), (b) dynamical modularity.

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